Fuzzy 6

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Classical Logic

Proposition : a sentence that states a fact, true or false (not both)
(the thruthness of the sentence can be evaluated)

- Istanbul is the biggest city of Turkey
- 2 + 3 = 5
- 2 + 1 = 4
- Antalya is the capital city of Turkey
- $2 + x = 8$
- Ankara is the best place to live on Earth

letters p, q, r, s are mostly used to represent propositional variables

most of the mathematical statements are constructed by combining one or more propositions using logical operators (connectives)

Classical Logic

Negation (~p) : "it's not the case that p" or "not p" .

- p : 2 + 3 = 5,

  ~p : it is not the case that 2 + 3 = 5

  ~p : $2 + 3 \neq 5$

Classical Logic

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- p : 2 + 3 = 5,

  ~p : it is not the case that 2 + 3 = 5
  ~p : 2 + 3 ≠ 5

p	~p
T	F
F	T

Classical Logic

Conjunction ($p \wedge q$) : "p and q" .

- p : Ali passed the course
  q : Hasan passed the course

  $p \wedge q$: Ali and Hasan both passed the course.

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p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Classical Logic

Disjunction ($p \vee q$) : "p or q" .

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  $p \vee q$: Ali or Hasan passed the course.

Classical Logic

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p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Classical Logic

Conditional Statement ($p \rightarrow q$) : "if p, then q" (p implies q).

- p : it rains
  q : the ground is wet

  $p \rightarrow q$: If it rains, then the ground will be wet.

Classical Logic

Conditional Statement ($p \rightarrow q$) : "if p, then q" (p implies q).

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p	q	p q
T	T	T
T	F	F
F	T	T
F	F	T

Classical Logic

Truth Tables

$p \rightarrow (p \vee q)$

$p \wedge (\sim p \wedge q)$

Classical Logic

Truth Tables

$p \rightarrow (p \vee q)$

$p \wedge (\sim p \wedge q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

Classical Logic

Truth Tables

$p \rightarrow (p \vee q)$ $\quad\quad$ $p \wedge (\sim p \wedge q)$

p	q	p∨q	p ➔ (p∨q)	~p	~p∧q	p ∧ (~p ∧ q)
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

- A compound proposition is called tautology if it's true for all the cases

Classical Logic

Truth Tables

$p \rightarrow (p \vee q)$ $\qquad\qquad$ $p \wedge (\sim p \wedge q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

- *A* compound proposition is called tautology if it's true for all the cases
- *A* compound proposition is called contradiction if it's false for all the cases

Classical Logic

- De Morgan's Low
  $\sim(p \vee q) = \sim p \wedge \sim q$
  $\sim(p \wedge q) = \sim p \vee \sim q$

- $\sim(\sim p) = p$

- $p \wedge 1 = p$
  $p \vee 0 = p$

- $p \wedge 0 = 0$
  $p \vee 1 = 1$

- $p \wedge p = p$
  $p \vee p = p$

- $(p \wedge q) \wedge r = p \wedge (q \wedge r)$
  $(p \vee q) \vee r = p \vee (q \vee r)$

- $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) =(p \vee q) \wedge (p \vee r)$

- $p \wedge \sim p = 0$
  $p \vee \sim p = 1$

- $p \rightarrow q = \sim p \vee q$
  $p \rightarrow q = \sim q \rightarrow \sim p$

Classical Logic

- p : '2 + 3 = 5'

  q : 'my computer is vulnerable to side channel attacks'

- P(x) : 'x + 3 = 5'

  Q(x) : 'computer x is vulnerable to side channel attacks'

Predicate Propositions (or statements) that contains variables

- When a value is assigned to the variable x, then P(x) becomes a proposition and has a truth value.

Fuzzy Logic

Fuzzy Proposition : have its truth value from [0,1]

- Mehmet is young
- Ferrari is expensive
- 3 + 4 = 9

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Operators in Fuzzy Logic

Let a and b be the truth values for the fuzzy propositions p and q

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Operators in Fuzzy Logic

Let a and b be the truth values for the fuzzy propositions p and q

- $\sim p$: $1 - a$
- $p \wedge q$: $\min(a, b)$
- $p \vee q$: $\max(a, b)$
- $p \rightarrow q$: $\max(1 - a, \min(a, b))$

Fuzzy Logic

Fuzzy Proposition : have its truth value from [0,1]

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Operators in Fuzzy Logic

Let a and b be the truth values f

- ~p : 1 – a
- $p \wedge q$: min (a, b)
- $p \vee q$: max (a, b)
- p➔q : max (1 - a, min(a, b))

a = 0.4, b = 0.8

~p : 1 – a : 0.6

$p \wedge q$: min (a, b) : 0.4

$p \vee q$: max (a, b) : 0.8

p➔q : max (1 - a, min(a, b)) : 0.6

Fuzzy Logic

Fuzzy Predicate : its definition contains ambiguity

- x is young
- y is expensive

- young(x) and expensive(y) are fuzzy sets
- the truth value can be considered as the membership function