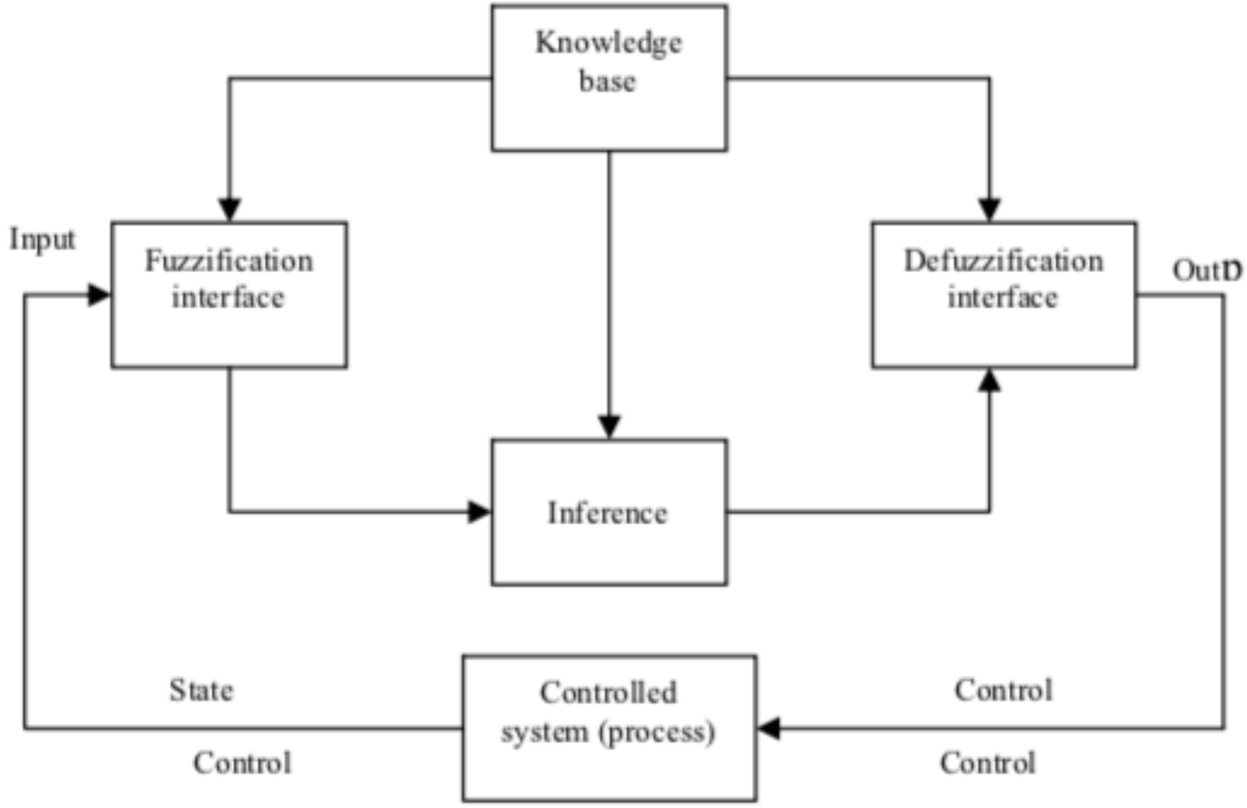
Fuzzy 7

Murat Osmanoglu

Inference

- most popular form of the representation : if-then rule



Inference

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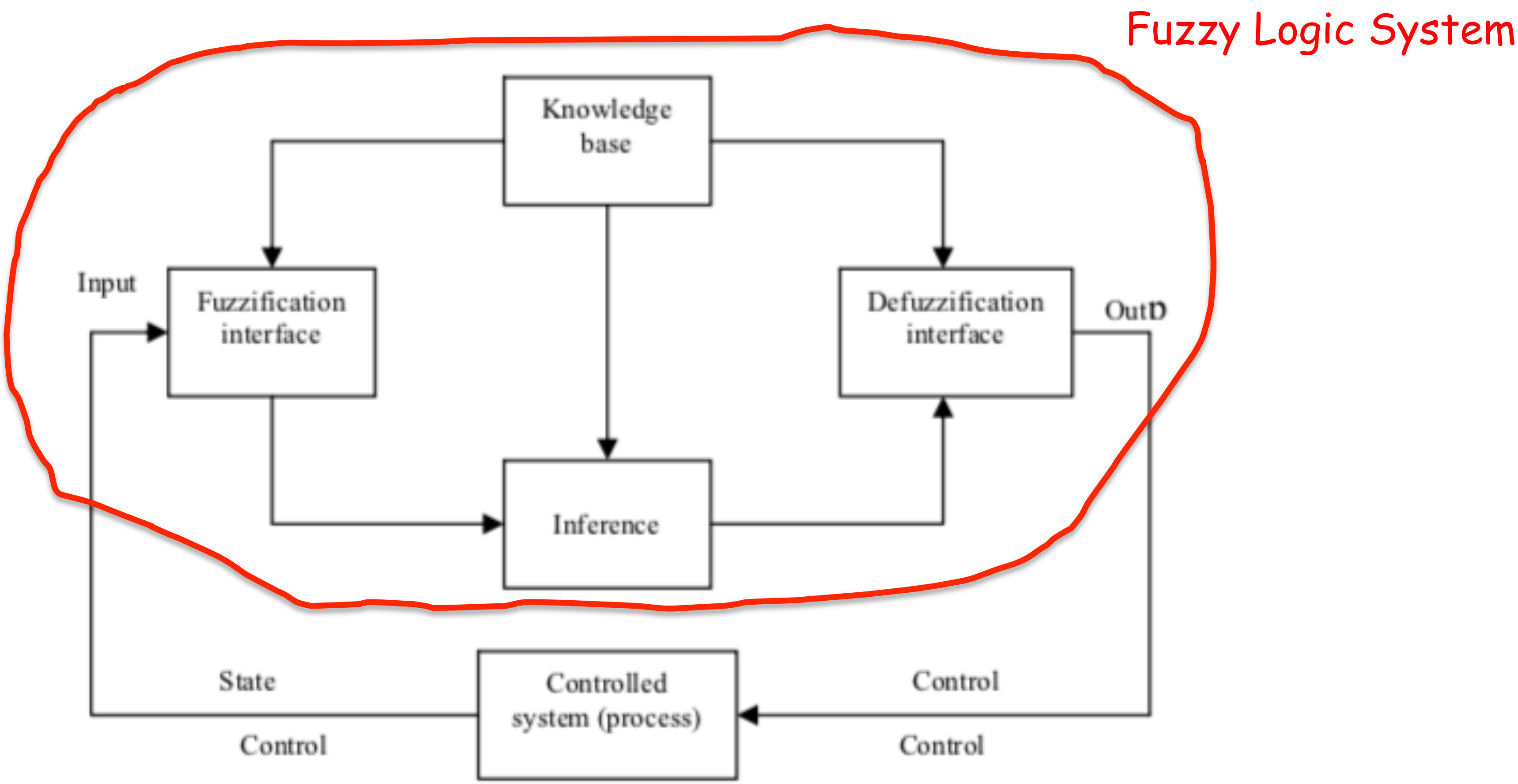

Inference

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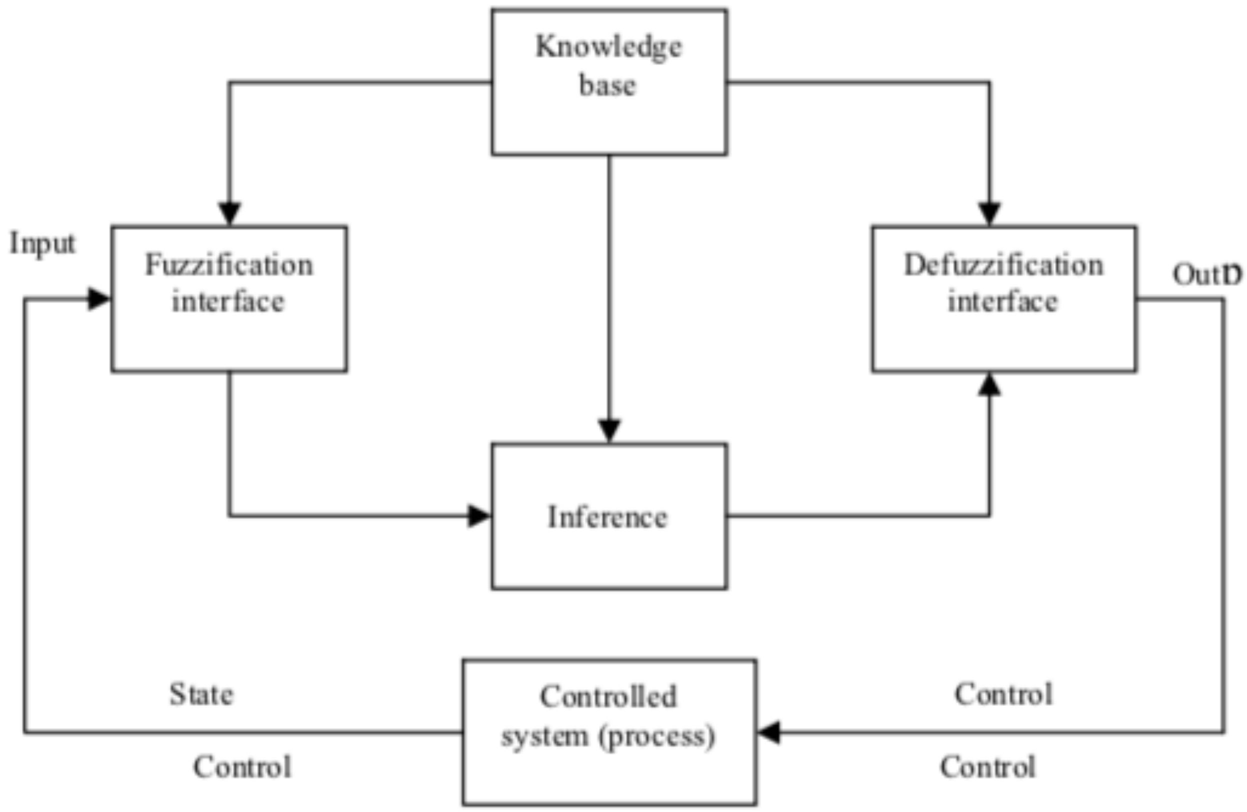

Inference

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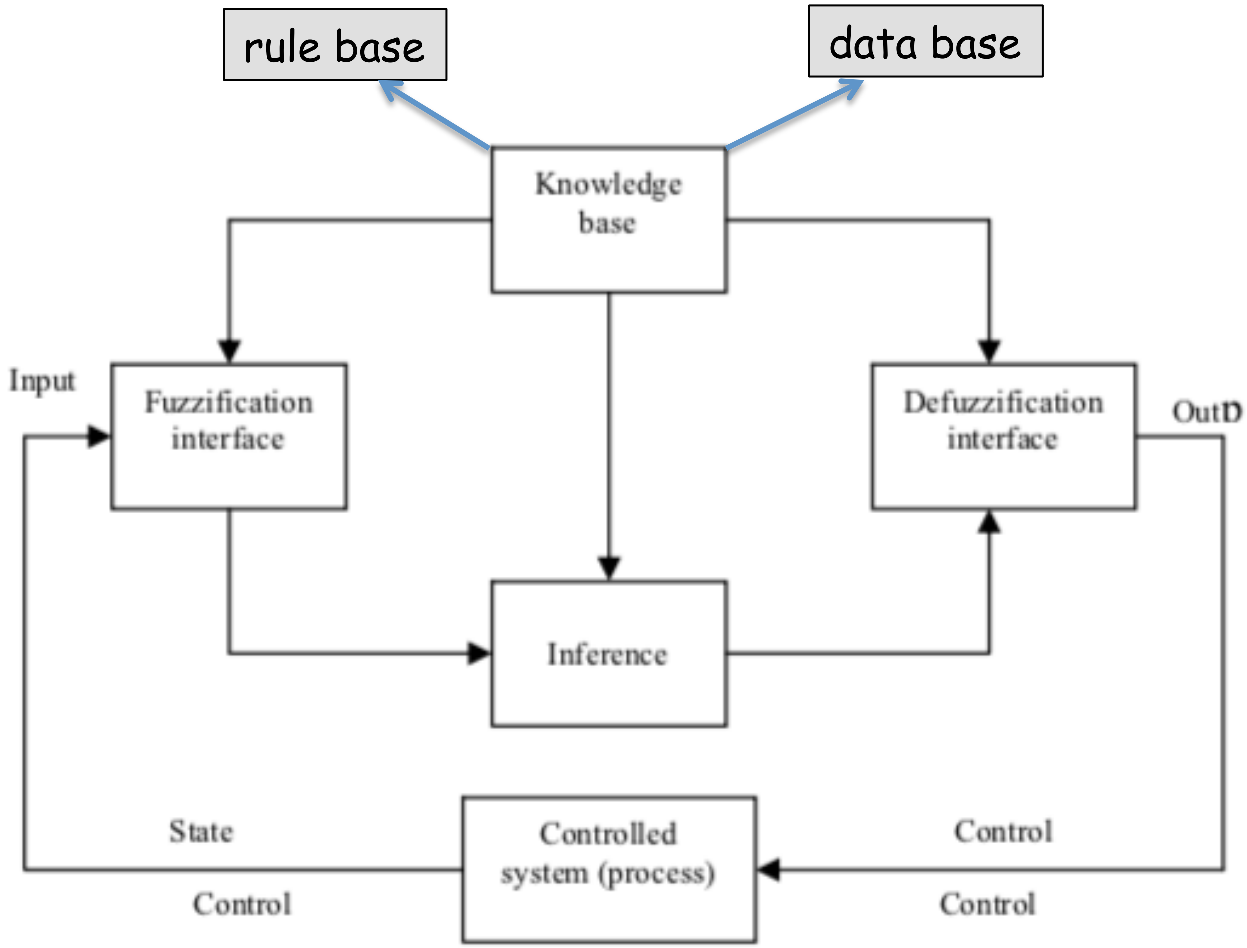

Inference

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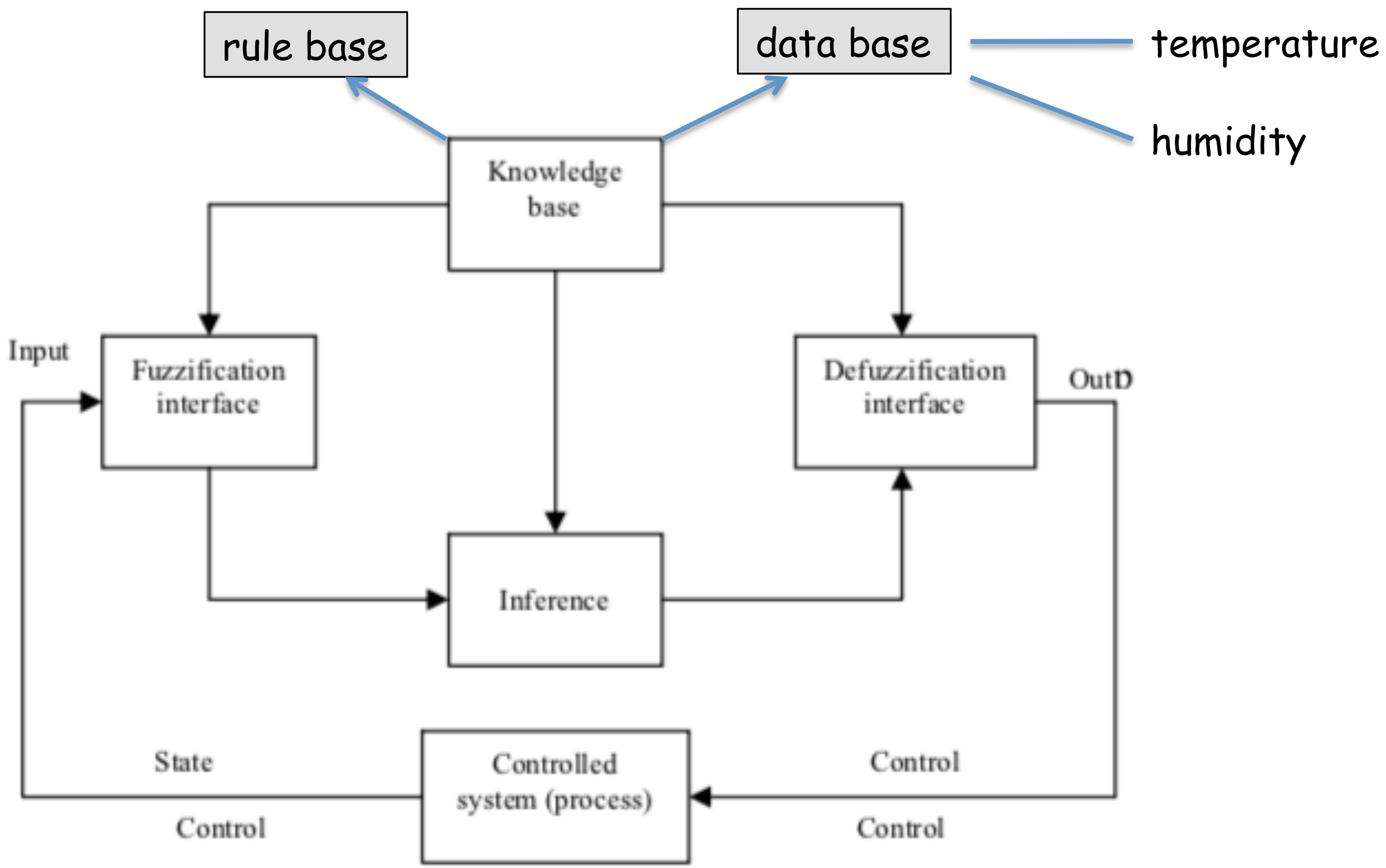

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

  the fact is : x is a

  the result is : y is b

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

  the fact is : x is a

  the result is : y is b

- modus ponens

  fact : x is a

  rule : if x is a, then y is b

  result : y is b

- modus tollens

  fact : y is not b

  rule : if x is a, then y is b

  result : a is not b

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

  the fact is : x is a

  the result is : y is b

- modus ponens

fact : x is a

rule : if x is a, then y is b

result : y is b

p	q	$p \rightarrow q$	$[p \wedge (p \rightarrow q)] \rightarrow q$
1	1	1	1
1	0	0	1
0	1	1	1
0	0	1	1

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

  the fact is : x is A′

  the result is : y is B′

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

  If A(x), then B(y)

  the fact is : x is A′

  the result is : y is B′

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

  If $A(x)$, then $B(y)$: $R(x, y)$ $(A(x) \rightarrow B(y))$

  the fact is : x is A'

  the result is : y is B'

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

  If $A(x)$, then $B(y)$: $R(x, y)$ ($A(x) \rightarrow B(y)$)

  the fact is : x is A'

  the result is : y is B'

'if temperature is high, then humidity is fairly high'

High(x) ➔ Fairly_High(y)

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

  If $A(x)$, then $B(y)$: $R(x, y)$ $(A(x) \rightarrow B(y))$

  the fact is : x is A'

  the result is : y is B'

  - modus ponens

fact : x is A'

rule : if x is A, then y is B

result : y is B'

Inference

- most popular form of the representation : if-then rule

- the rule is : If x is A, then y is B
  If A(x), then B(y) : R(x, y) (A(x)→B(y))

  the fact is : x is A′

  the result is : y is B′

  - modus ponens

fact : x is A′ : R(x)

rule : if x is A, then y is B : R(x, y)

result : y is B′ : $R(y) = R(x) \circ R(x, y)$

Fuzzy Composition

- consider the fuzzy rule and the premise given as :

  'x and y are approximately equal'   and   'x is small'

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R(x, y) = ApproximatelyEqual(x, y)

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R(x) = Small(x)

Fuzzy Composition

- consider the fuzzy rule and the premise given as :

  'x and y are approximately equal' and 'x is small'

$R(x, y) = ApproximatelyEqual(x, y)$

$R(x) = Small(x)$

R(x,y)	1	2	3	4
1	1.0	0.5	0	0
2	0.5	1.0	0.5	0
3	0	0.5	1.0	0.5
4	0	0	0.5	1.0

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4	0	0	0.5	1.0

R(x)	1	2	3	4
$\mu_R(x)$	1.0	0.7	0.4	0.1

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$\mu_R(y)$				

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$$R(y) = R(x) \circ R(x, y)$$

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R(y)	1	2	3	4
$\mu_R(y)$				

$$R(y) = R(x) \circ R(x, y)$$

$$\mu_R(y) = \max_x (\min (\mu_R(x), \mu_R(x, y))$$

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  'x and y are approximately equal' and 'x is small'

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R(x) = Small(x)

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$$R(y) = R(x) \circ R(x, y)$$

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Fuzzy Composition

- consider the fuzzy rule and the premise given as :

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2	0.5	1.0	0.5	0
3	0	0.5	1.0	0.5
4	0	0	0.5	1.0

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R(y)	1	2	3	4
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$$R(y) = R(x) \circ R(x, y)$$

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Fuzzy Composition

- consider the fuzzy rule and the premise given as :

  'x and y are approximately equal' and 'x is 2'

$R(x, y) = ApproximatelyEqual(x, y)$

$R(x) = Small(x)$

R(x,y)	1	2	3	4
1	1.0	0.5	0	0
2	0.5	1.0	0.5	0
3	0	0.5	1.0	0.5
4	0	0	0.5	1.0

$$R(y) = R(x) \circ R(x, y)$$

$$\mu_R(y) = \max_x (\min (\mu_R(x), \mu_R(x, y))$$

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1	1.0	0.5	0	0
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3	0	0.5	1.0	0.5
4	0	0	0.5	1.0

R(x)	1	2	3	4
$\mu_R(x)$	0	1.0	0	0

$$R(y) = R(x) \circ R(x, y)$$

$$\mu_R(y) = \max_x (\min (\mu_R(x), \mu_R(x, y)))$$

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3	0	0.5	1.0	0.5
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R(x)	1	2	3	4
$\mu_R(x)$	0	1.0	0	0

R(y)	1	2	3	4
$\mu_R(y)$	0.5	1.0	0.5	0

$$R(y) = R(x) \circ R(x, y)$$

$$\mu_R(y) = \max_x (\min (\mu_R(x), \mu_R(x, y)))$$

Inference

- most popular form of the representation : if-then rule
- The rule is : $R(x, y)\ (A(x) \rightarrow B(y))$

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- The rule is : $R(x, y)\ (A(x) \rightarrow B(y))$

$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y))$$

Inference

- most popular form of the representation : if-then rule
- The rule is : $R(x, y)\ (A(x) \rightarrow B(y))$

$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y))$$

Mamdani

$$f(\mu_A(x), \mu_B(y)) = \mu_A(x) \underline{\triangle} \mu_B(y)$$

Inference

- most popular form of the representation : if-then rule
- The rule is : $R(x, y)\ (A(x) \rightarrow B(y))$

$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y))$$

Mamdani

$$f(\mu_A(x), \mu_B(y)) = \mu_A(x) \underline{\triangle} \mu_B(y)$$

Larsen

$$f(\mu_A(x), \mu_B(y)) = \mu_A(x) \cdot \mu_B(y)$$

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- consider the fuzzy rule given as :

  'if temperature is high, then humidity is fairly high'

  $R(t, h) = A(t) \rightarrow B(h)$ where A in T and B in H

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A = 'high'

A	10	20	30	40
$\mu_A(t)$	0.1	0.2	0.6	0.9

B = 'fairly high'

B	40	60	80	90
$\mu_B(h)$	0.3	0.5	0.8	1.0

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Mamdani

R(t,h)	40	60	80	90
10				
20				
30				
40				

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Mamdani

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20				
30				
40				

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R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

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Mamdani

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10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10				
20				
30				
40				

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A = 'high'

A	10	20	30	40
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Mamdani

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03			
20				
30				
40				

Fuzzy Inference

- consider the fuzzy rule given as :

  'if temperature is high, then humidity is fairly high'

  $R(t, h) = A(t) \rightarrow B(h)$ where A in T and B in H

A = 'high'

A	10	20	30	40
$\mu_A(t)$	0.1	0.2	0.6	0.9

B = 'fairly high'

B	40	60	80	90
$\mu_B(h)$	0.3	0.5	0.8	1.0

Mamdani

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03	0.05		
20				
30				
40				

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- consider the fuzzy rule given as :

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A = 'high'

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$\mu_A(t)$	0.1	0.2	0.6	0.9

B = 'fairly high'

B	40	60	80	90
$\mu_B(h)$	0.3	0.5	0.8	1.0

Mamdani

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03	0.05	0.08	
20				
30				
40				

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  $R(t, h) = A(t) \rightarrow B(h)$ where A in T and B in H

A = 'high'

A	10	20	30	40
$\mu_A(t)$	0.1	0.2	0.6	0.9

B = 'fairly high'

B	40	60	80	90
$\mu_B(h)$	0.3	0.5	0.8	1.0

Mamdani

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03	0.05	0.08	0.1
20				
30				
40				

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- consider the fuzzy rule given as :

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R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03	0.05	0.08	0.1
20	0.06	0.1	0.16	0.2
30	0.18	0.3	0.48	0.6
40	0.27	0.45	0.72	0.9

Fuzzy Inference

- consider the fuzzy rule given as :

  R(t,h) = 'if temperature is high, then humidity is fairly high'

  A' = ' temperature is fairly high'

Fuzzy Inference

- consider the fuzzy rule given as :

  R(t,h) = 'if temperature is high, then humidity is fairly high'

  A' = ' temperature is fairly high'

A' = 'fairly high'

A'	10	20	30	40
$\mu_A(t)$	0.02	0.15	0.5	0.8

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Fuzzy Inference

- consider the fuzzy rule given as :

  R(t,h) = 'if temperature is high, then humidity is fairly high'

  A' = ' temperature is fairly high'

A' = 'fairly high'

A'	10	20	30	40
$\mu_A(t)$	0.02	0.15	0.5	0.8

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Mamdani

B'	40	60	80	90
$\mu_{B'}(h)$	0.3	0.5	0.8	0.8

Decomposition of Rules

Multiple Input Multiple Output

- R : if x_1 is A_1,x_2 is A_2, …,x_n is A_n, then z_1 is C_1,z_2 is C_2, …,z_m is C_m

Decomposition of Rules

Multiple Input Multiple Output

- R : if x_1 is A_1,x_2 is A_2, …,x_n is A_n, then z_1 is C_1,z_2 is C_2, …,z_m is C_m

R_1 : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_1 is C_1

R_2 : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_2 is C_2

. . .

R_m : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_m is C_m

Decomposition of Rules

Multiple Input Multiple Output

- R : if x_1 is A_1,x_2 is A_2, …,x_n is A_n, then z_1 is C_1,z_2 is C_2, …,z_m is C_m

  R_1 : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_1 is C_1

  R_2 : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_2 is C_2

  . . .

  R_m : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_m is C_m

- a multiple input multiple output fuzzy system can considered as a collection of multiple input single output fuzzy systems

$$R = \{R_1, R_2, . . ., R_m\}$$

Decomposition of Rules

Two Input Single Output

- input : x is A' and y is B'

  R_1 : if x is A_1 and y is B_1, then z is C_1

  R_2 : if x is A_2 and y is B_2, then z is C_2

  . . .

  R_m : if x is A_m and y is B_2, then z is C_m

output : z is C'

Decomposition of Rules

Two Input Single Output

- input : x is A' and y is B'

  R_1 : if x is A_1 and y is B_1, then z is C_1

  R_2 : if x is A_2 and y is B_2, then z is C_2

  . . .

  R_m : if x is A_m and y is B_2, then z is C_m

  output : z is C'

- R_i : if x is A_i and y is B_i, then z is C_i

  R_i : (A_i and B_i) $\rightarrow C_i$

Decomposition of Rules

Two Input Single Output

- input : x is A' and y is B'

  R_1 : if x is A_1 and y is B_1, then z is C_1

  R_2 : if x is A_2 and y is B_2, then z is C_2

  . . .

  R_m : if x is A_m and y is B_2, then z is C_m

  output : z is C'

- R_i : if x is A_i and y is B_i, then z is C_i

  R_i : (A_i and B_i) ➔ C_i

  R_i : (A_i ➔ C_i) and (B_i ➔ C_i)

Decomposition of Rules

Two Input Single Output

- input : x is A' and y is B'

  R_1 : if x is A_1 and y is B_1, then z is C_1

  R_2 : if x is A_2 and y is B_2, then z is C_2

  . . .

  R_m : if x is A_m and y is B_2, then z is C_m

  output : z is C'

- R_i : if x is A_i and y is B_i, then z is C_i

  $R_i : (A_i \text{ and } B_i) \rightarrow C_i$

  $R_i : (A_i \rightarrow C_i) \text{ and } (B_i \rightarrow C_i)$

  $R_i = R_{i,1} \wedge R_{i,2}$

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then z is C

  If A(x), then C(z) : R(x, z) (A(x)→C(z))

  the fact is : x is A'

  the result is : z is C'

Mamdani

R(y) = R(x) o R(x, z)

min for the implication

Larsen

R(y) = R(x) o R(x, z)

product for the implication