PHYSICS I

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Circular Motion

Uniform Circular Motion, Acceleration

A particle moves with a constant speed in a circular path of radius r with an acceleration.

The magnitude of the acceleration is given by

$$a_c = \frac{v^2}{r}$$

- The centripetal acceleration, $\vec{\mathbf{a}}_c$, is directed toward the center of the circle.

The centripetal acceleration is always perpendicular to the velocity.

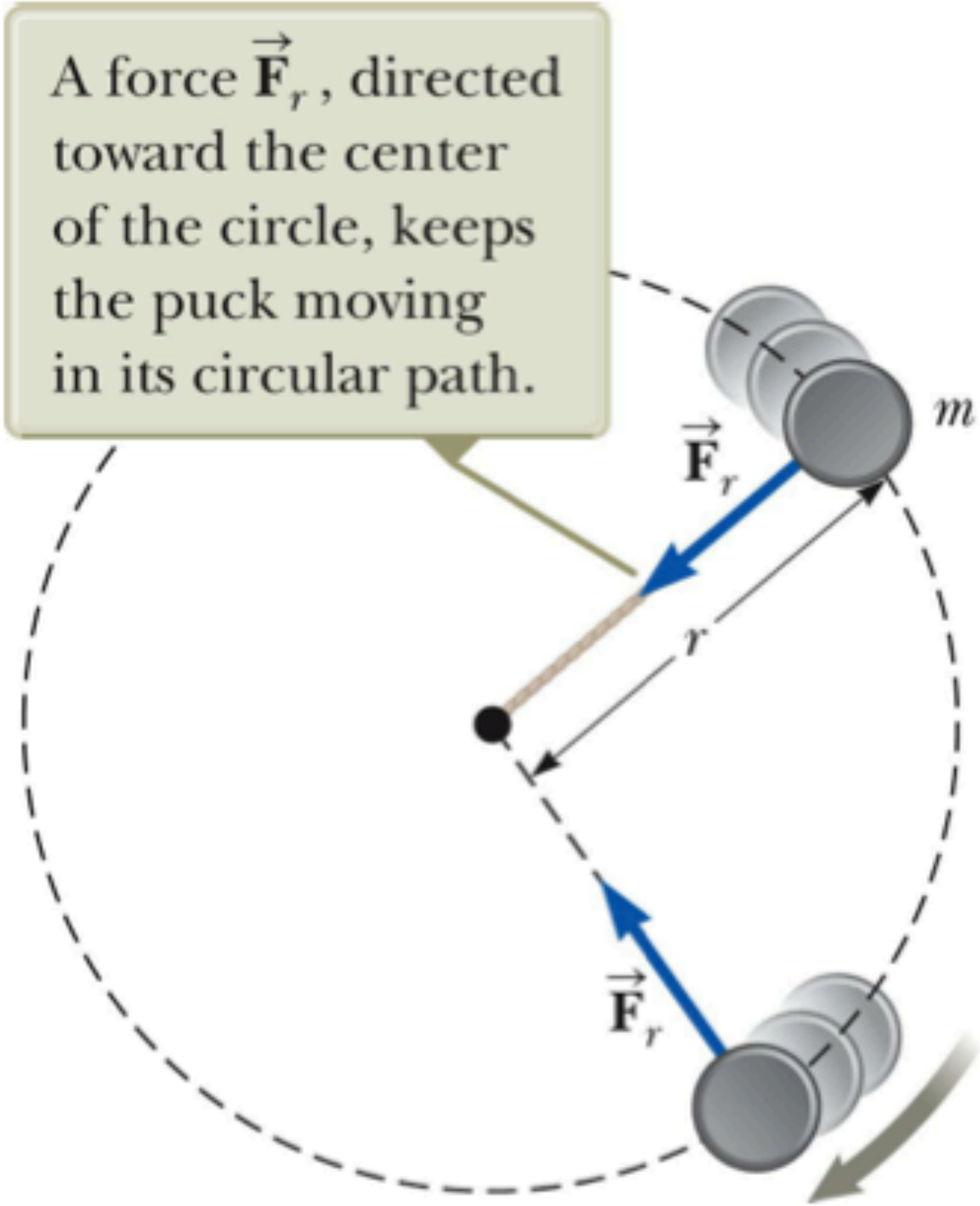
Uniform Circular Motion, Force

A force, $\vec{\mathbf{F}}_r$, is associated with the centripetal acceleration.

The force is also directed toward the center of the circle.

Applying Newton's Second Law along the radial direction gives

$$\sum F = ma_c = m\frac{v^2}{r}$$



Conical Pendulum

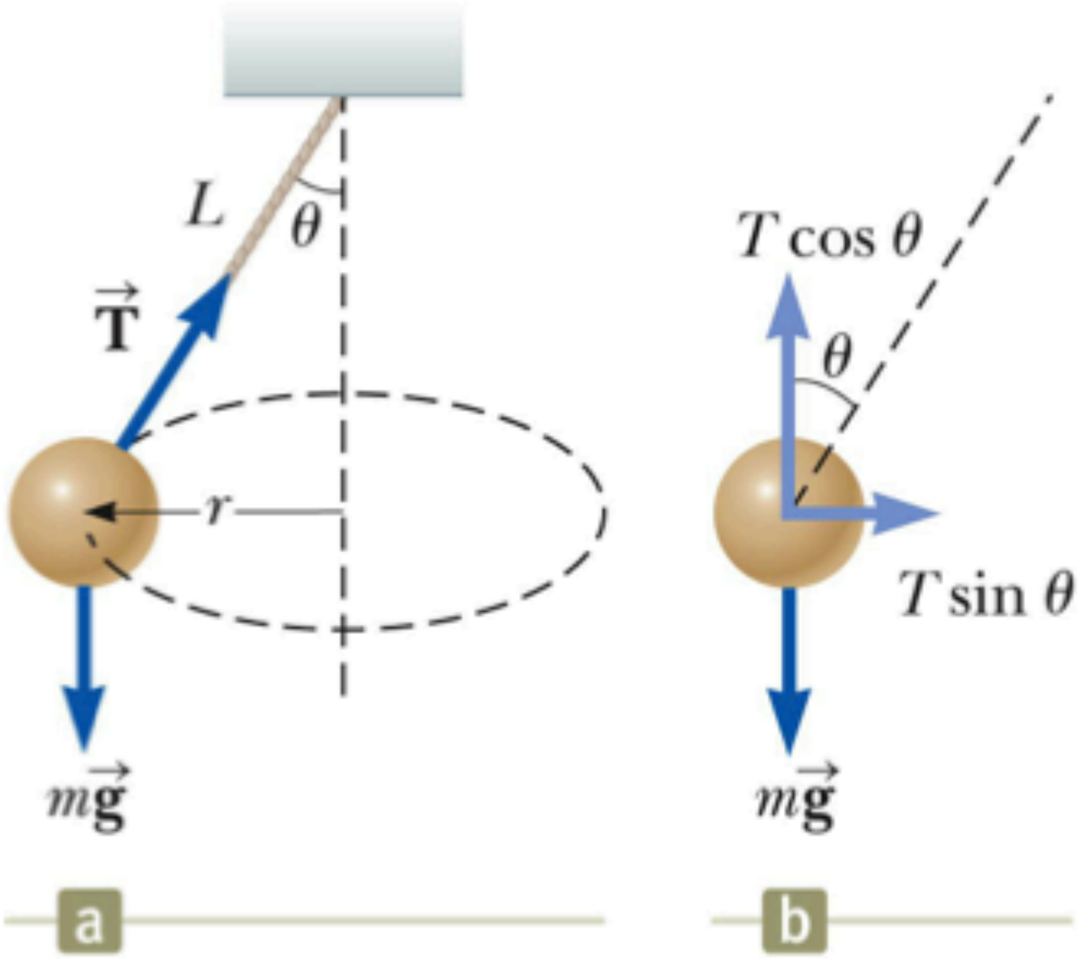
The object is in equilibrium in the vertical direction .

It undergoes uniform circular motion in the horizontal direction.

- $\sum F_y = 0 \rightarrow T \cos \theta = mg$
- $\sum F_x = T \sin \theta = m\, a_c$

v is independent of *m*

$$v = \sqrt{Lg \sin\theta \tan\theta}$$



Horizontal (Flat) Curve

Model the car as a particle in uniform circular motion in the horizontal direction.

Model the car as a particle in equilibrium in the vertical direction.

The force of static friction supplies the centripetal force.

The maximum speed at which the car can negotiate the curve is:

$$v = \sqrt{\mu_s g r}$$

Non-Uniform Circular Motion

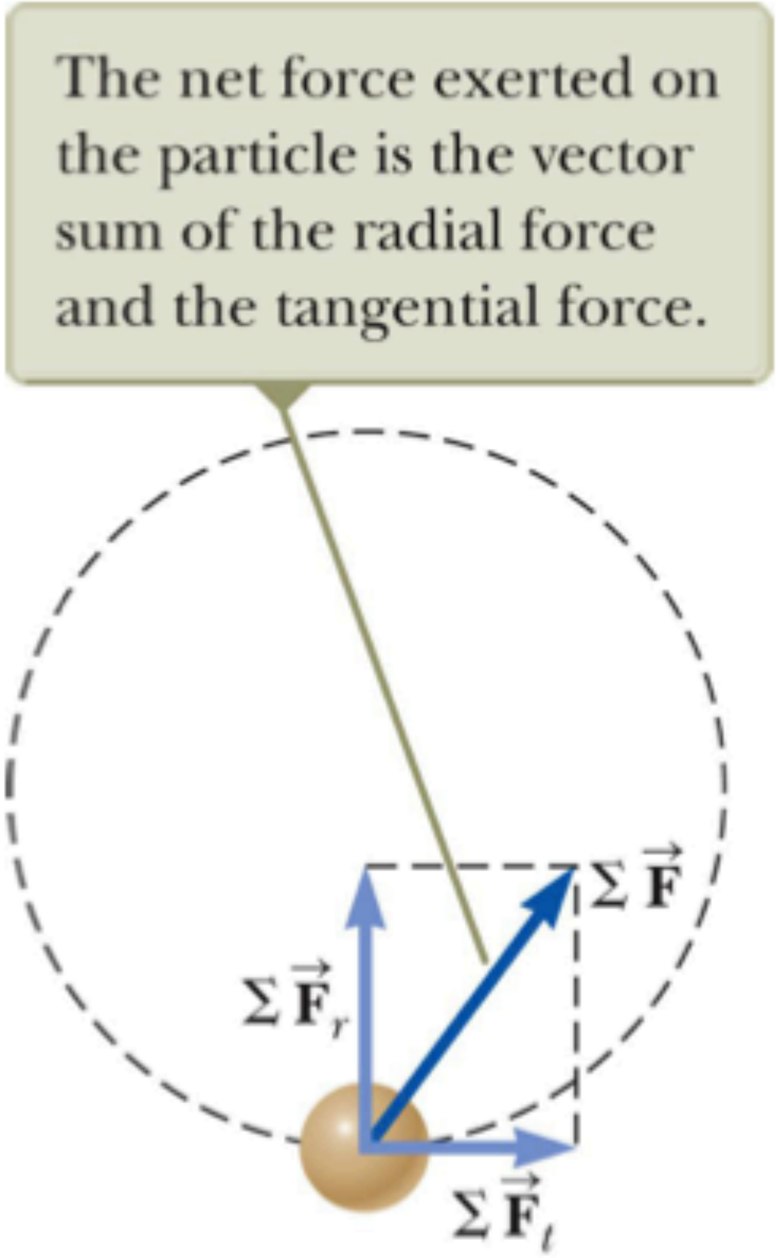
The acceleration and force have tangential components.

$\vec{F}_r$ produces the centripetal acceleration

$\vec{F}_t$ produces the tangential acceleration

The total force is

$$\sum\vec{F} = \sum\vec{F}_r + \sum\vec{F}_t$$



Vertical Circle with Non-Uniform Speed

The gravitational force exerts a tangential force on the object.

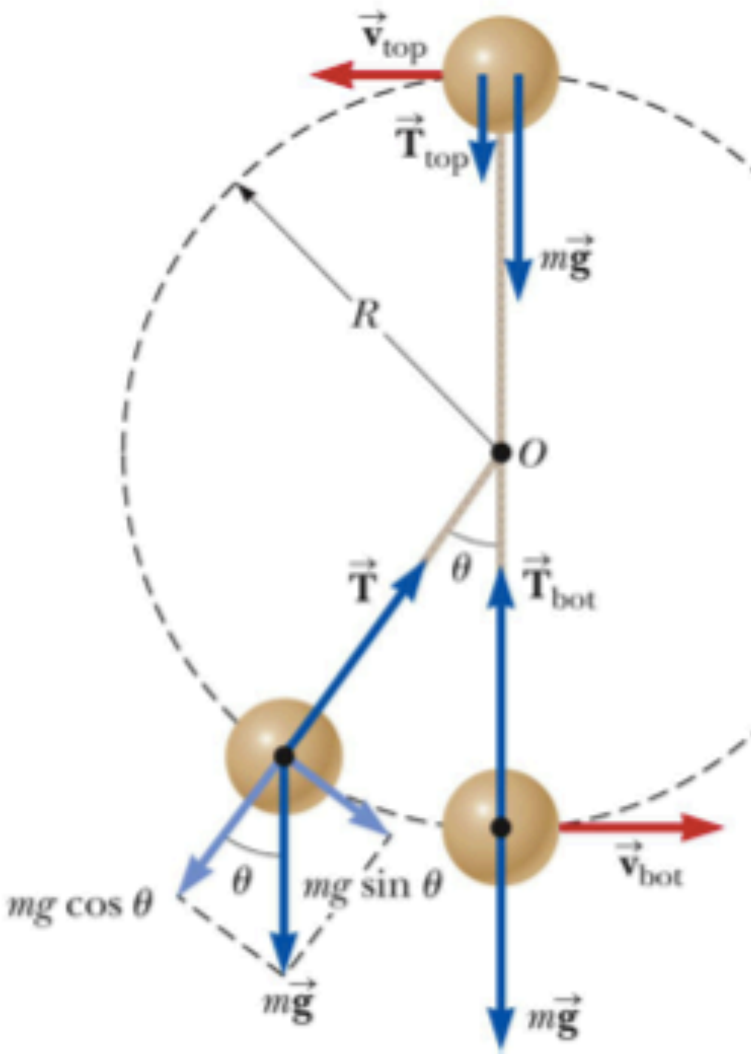
- Look at the components of F_g

Model the sphere as a particle under a net force and moving in a circular path.

- Not uniform circular motion

The tension at any point can be found.

$$T = mg\left(\frac{v^2}{Rg} + \cos\theta\right)$$



Top and Bottom of Circle

The tension at the bottom is a maximum.

$$T = mg\left(\frac{v_{bot}^2}{Rg} + 1\right)$$

The tension at the top is a minimum.

$$T = mg\left(\frac{v_{top}^2}{Rg} - 1\right)$$

If $T_{top} = 0$, then

$$v_{top} = \sqrt{gR}$$

Resistive Force Proportional To Speed, Example

Assume a small sphere of mass m is released from rest in a liquid.

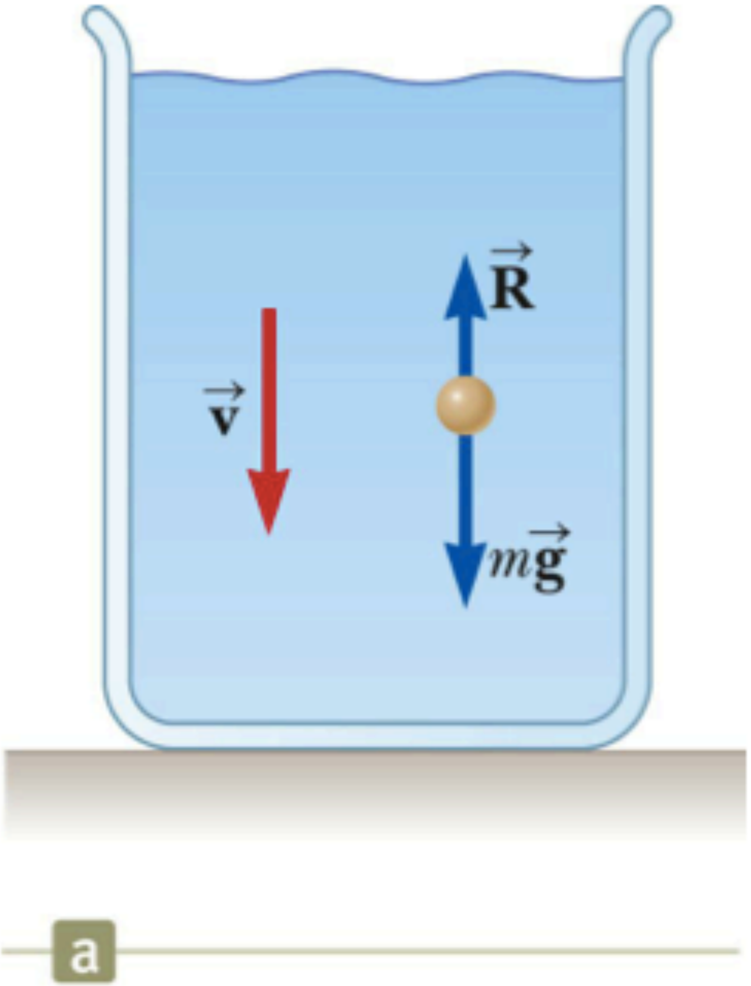
Forces acting on it are:

- Resistive force
- Gravitational force

Analyzing the motion results in

$$mg - bv = ma = m\frac{dv}{dt}$$

$$a = \frac{dv}{dt} = g - \frac{b}{m}v$$



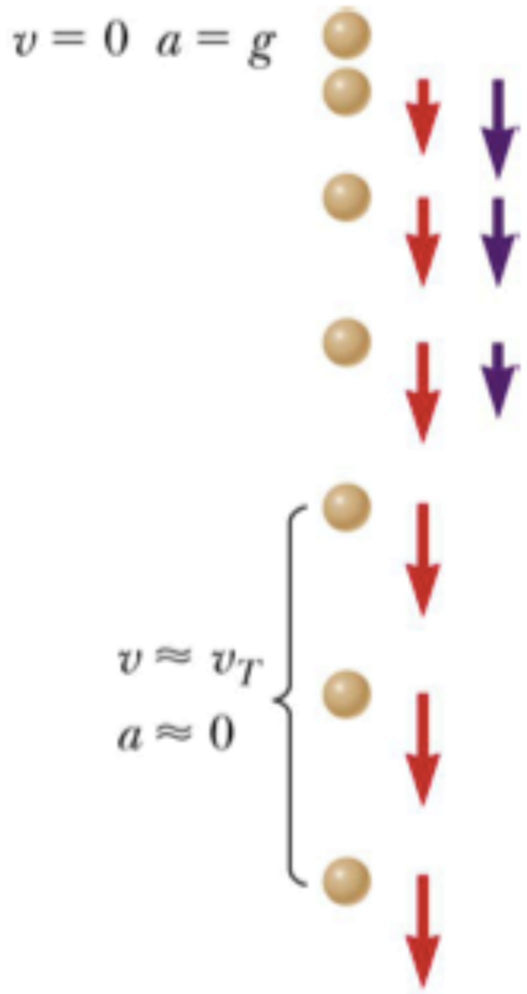
Resistive Force Proportional To Speed, Example, cont.

Initially, $v = 0$ and $dv/dt = g$

As t increases, R increases and a decreases

The acceleration approaches 0 when $R \rightarrow mg$

At this point, v approaches the ***terminal speed*** of the object.



Terminal Speed

To find the terminal speed, let $a = 0$

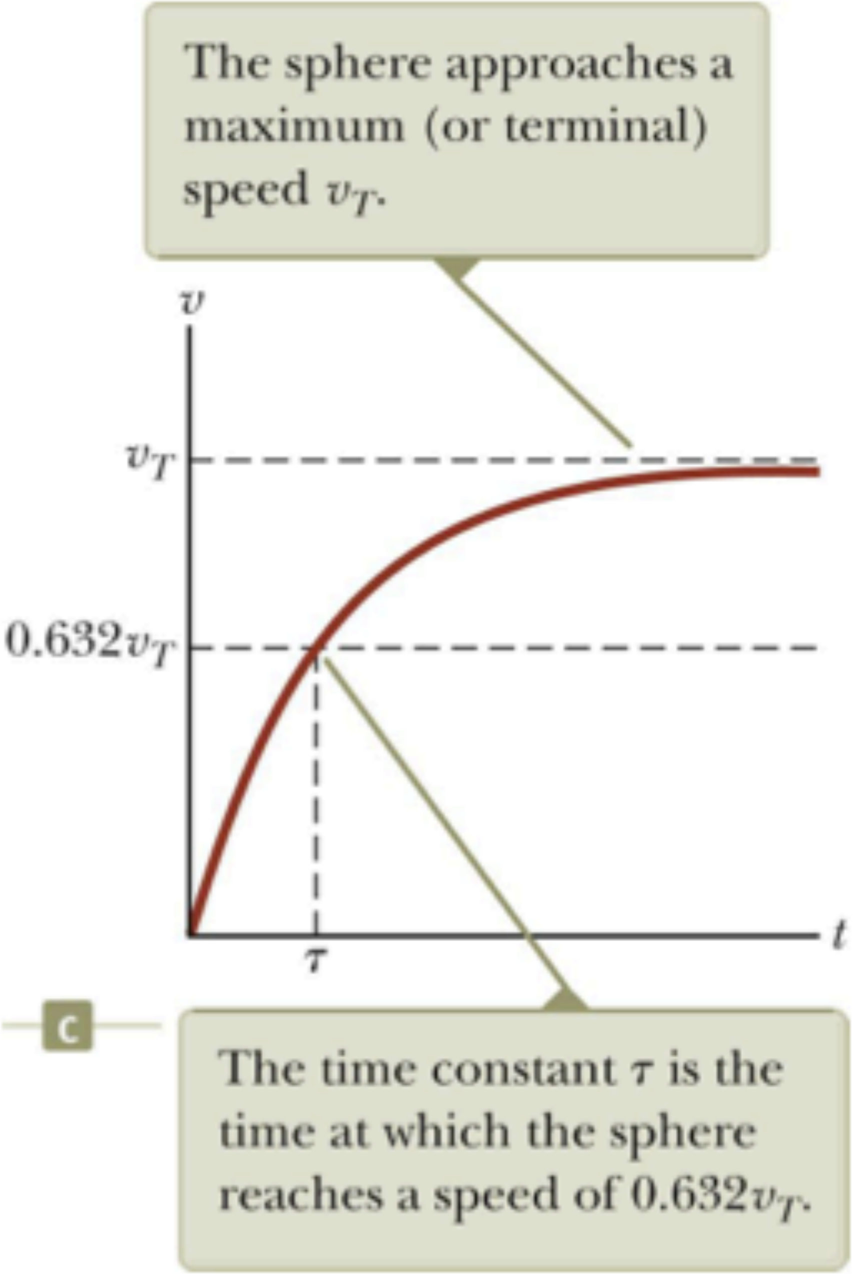
$$v_T = \frac{mg}{b}$$

Solving the differential equation gives

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

τ is the ***time constant*** and

$$\tau = m/b$$



Resistive Force Proportional To v2

For objects moving at high speeds through air, the resistive force is approximately equal to the square of the speed.

$R = \frac{1}{2} DrAv^2$

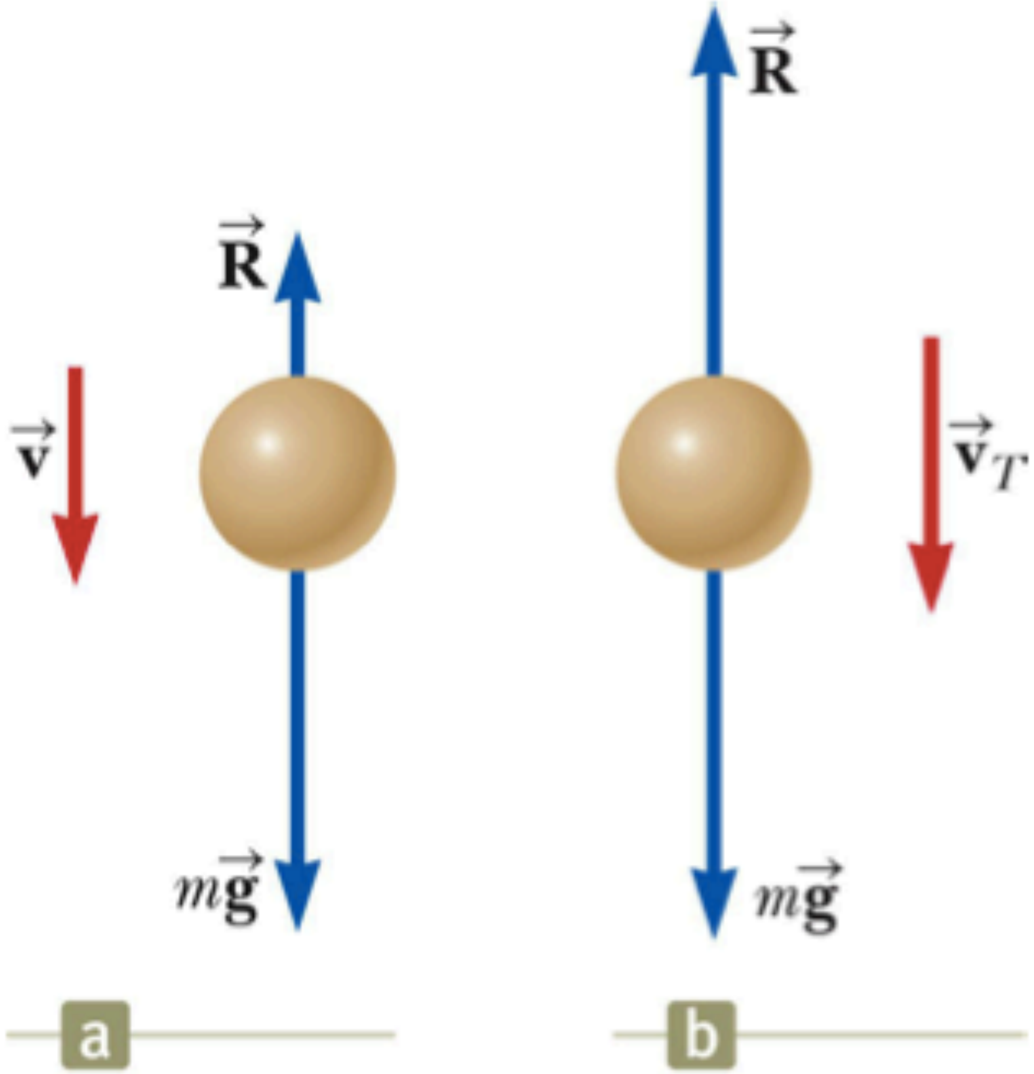
- D is a dimensionless empirical quantity called the drag coefficient.
- r is the density of air.
- A is the cross-sectional area of the object.
- v is the speed of the object.

Resistive Force Proportional To v^2, example

Analysis of an object falling through air accounting for air resistance.

$$\sum F = mg - \frac{1}{2} D\rho A v^2 = ma$$

$$a = g - \left(\frac{D\rho A}{2m}\right) v^2$$



Resistive Force Proportional To v^2, Terminal Speed

The terminal speed will occur when the acceleration goes to zero.

Solving the previous equation gives

$$v_T = \sqrt{\frac{2mg}{D\rho A}}$$