PHYSICS I

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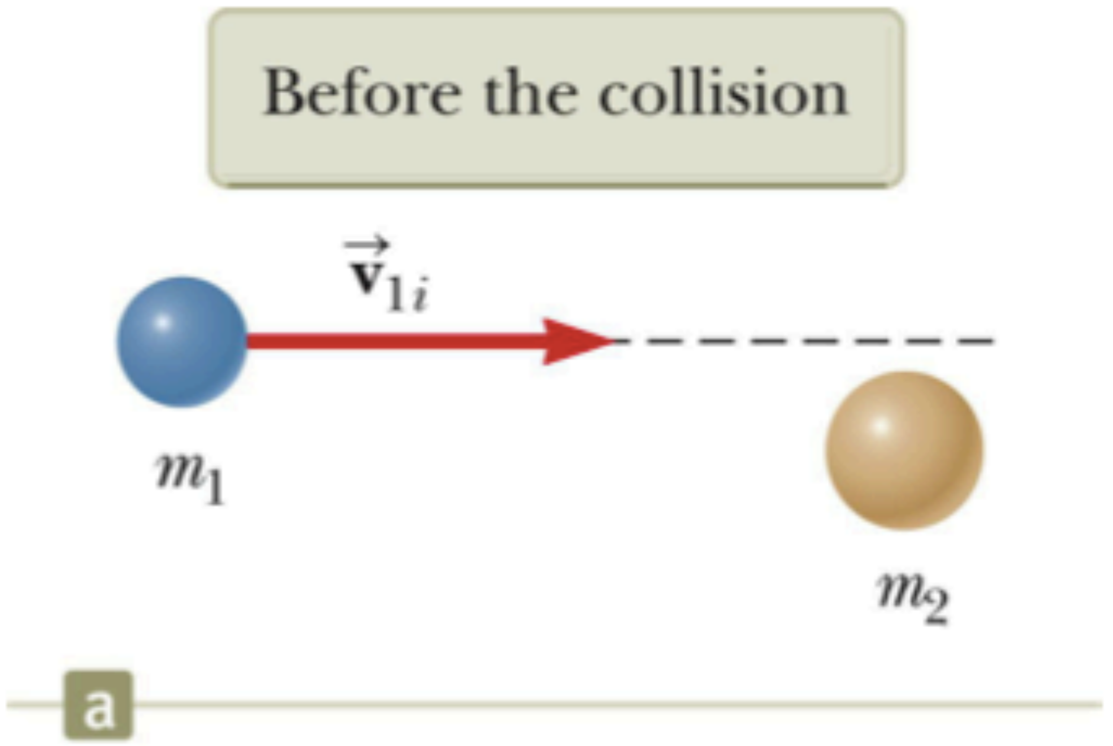
Two-Dimensional Collision

Two-Dimensional Collision, example

Particle 1 is moving at velocity $\vec{\mathbf{v}}_{1i}$ and particle 2 is at rest.

In the *x*-direction, the initial momentum is $m_1 v_{1i}$.

In the *y*-direction, the initial momentum is 0.



Two-Dimensional Collision, example cont.

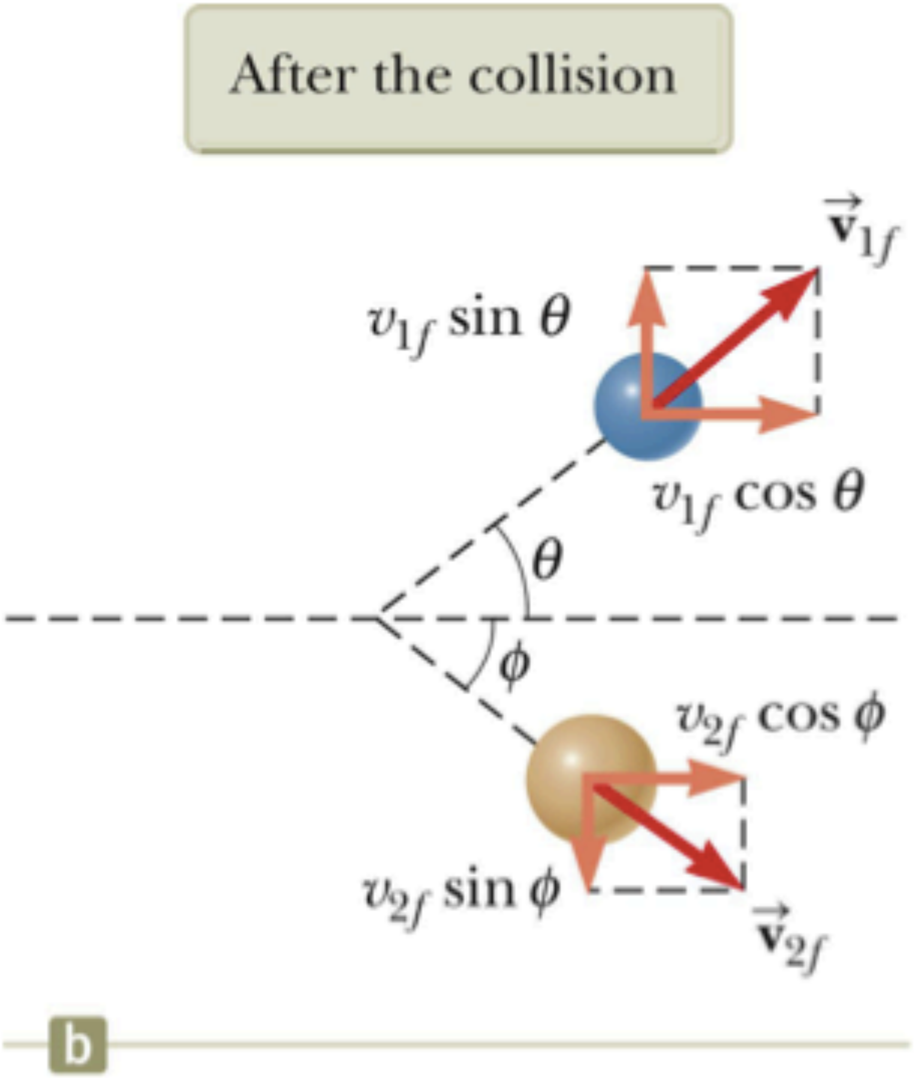
After the collision, the momentum in the x-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$.

After the collision, the momentum in the y-direction is $m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$.

- The negative sign is due to the component of the velocity being downward.

If the collision is elastic, apply the kinetic energy equation.

This is an example of a *glancing collision*.



Problem-Solving Strategies – Two-Dimensional Collisions

Conceptualize

- Imagine the collision.
- Predict approximate directions the particles will move after the collision.
- Set up a coordinate system and define your velocities with respect to that system.
  - It is usually convenient to have the *x*-axis coincide with one of the initial velocities.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.

Categorize

- Is the system isolated?
- If so, categorize the collision as elastic, inelastic or perfectly inelastic.

Problem-Solving Strategies – Two-Dimensional Collisions, 2

Analyze

- Write expressions for the x- and y-components of the momentum of each object before and after the collision.
  - Remember to include the appropriate signs for the components of the velocity vectors.
- Write expressions for the total momentum of the system in the x-direction before and after the collision and equate the two. Repeat for the total momentum in the y-direction.
- If the collision is inelastic, kinetic energy of the system is not conserved, and additional information is probably needed.
- If the collision is perfectly inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknowns.

Problem-Solving Strategies – Two-Dimensional Collisions, 3

Analyze, cont.

- If the collision is elastic, the kinetic energy of the system is conserved.
  - Equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain more information on the relationship between the velocities.

Finalize

- Check to see if your answers are consistent with the mental and pictorial representations.
- Check to be sure your results are realistic.

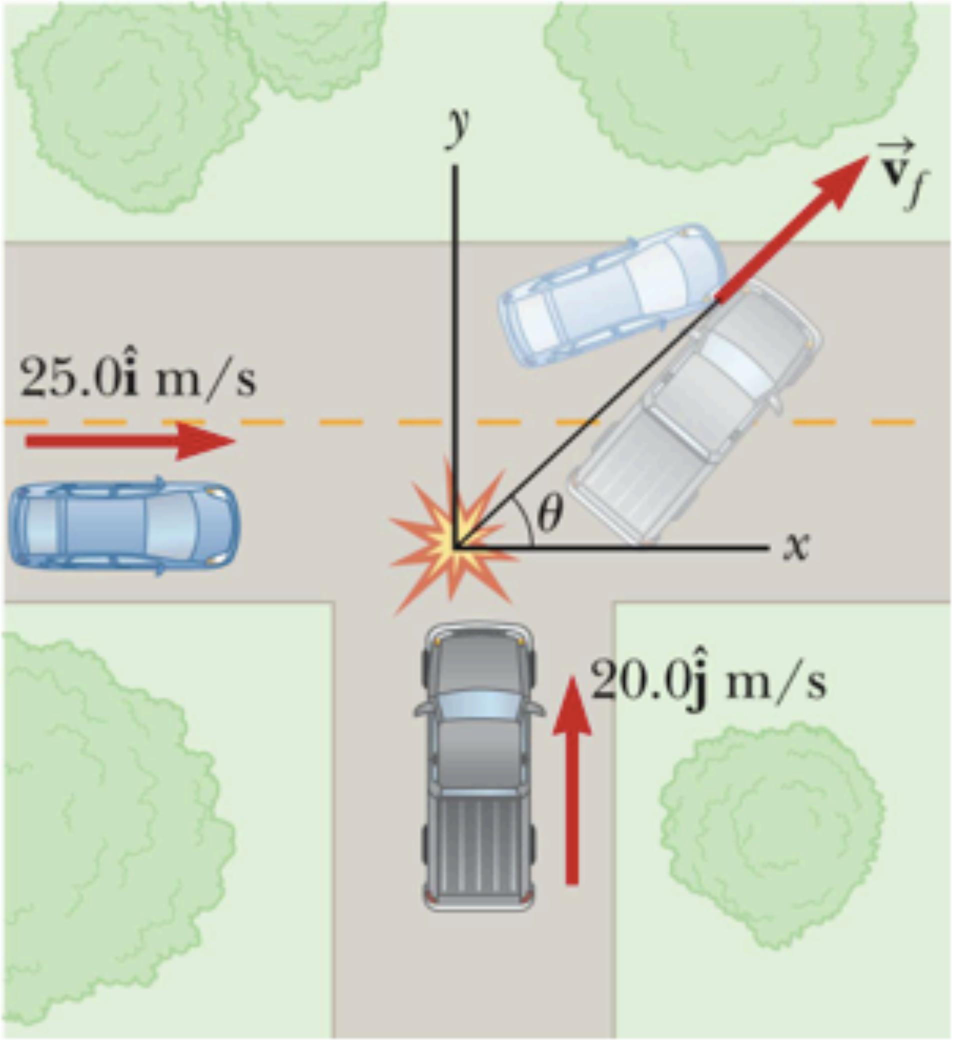
Two-Dimensional Collision Example

Conceptualize

- See picture
- Choose East to be the positive x-direction and North to be the positive y-direction.

Categorize

- Ignore friction
- Model the vehicles as particles.
- Model the system as isolated in terms of momentum.
- The collision is perfectly inelastic.
  - The vehicles stick together.



Two-Dimensional Collision Example, cont.

Analyze

- Before the collision, the car has the total momentum in the x-direction and the truck has the total momentum in the y-direction.
- After the collision, both have x- and y-components.
- Write expressions for initial and final momenta in both directions.
  - Evaluate any expressions with no unknowns.
- Solve for unknowns

Finalize

- Check to be sure the results are reasonable.

The Center of Mass

There is a special point in a system or object, called the ***center of mass***, that moves as if all of the mass of the system is concentrated at that point.

The system will move as if an external force were applied to a single particle of mass *M* located at the center of mass.

- *M* is the total mass of the system.

This behavior is independent of other motion, such as rotation or vibration, or deformation of the system.

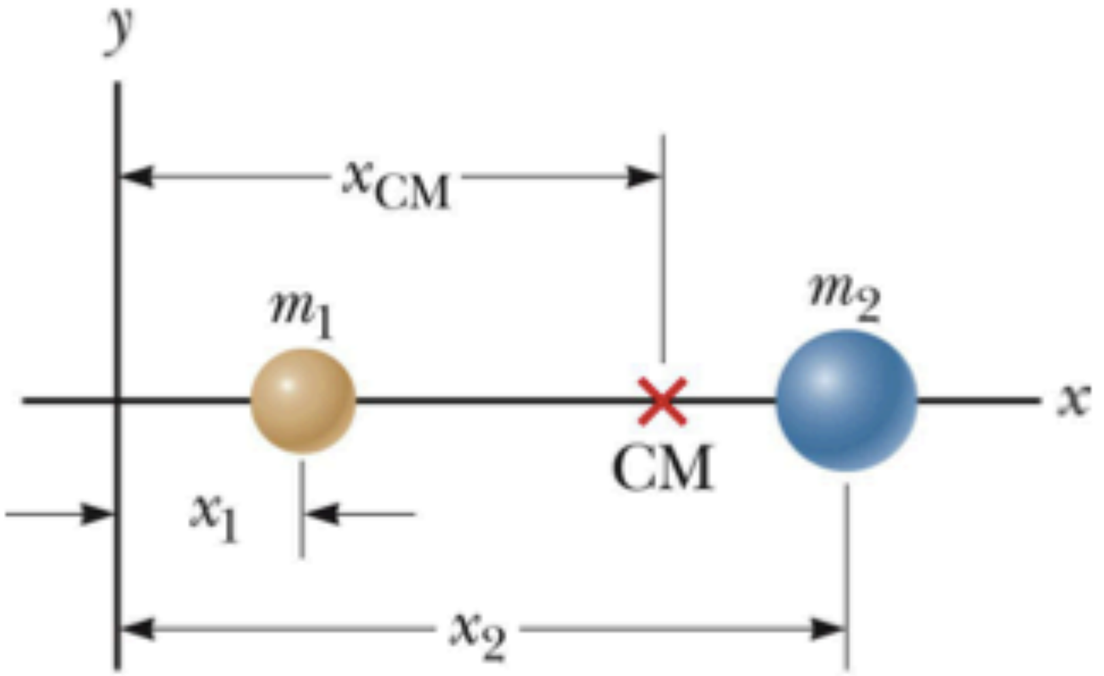
- This is the particle model.

Center of Mass, Coordinates

The coordinates of the center of mass are

$$x_{CM} = \frac{\sum_i m_i x_i}{M} \qquad y_{CM} = \frac{\sum_i m_i y_i}{M}$$

$$z_{CM} = \frac{\sum_i m_i z_i}{M}$$



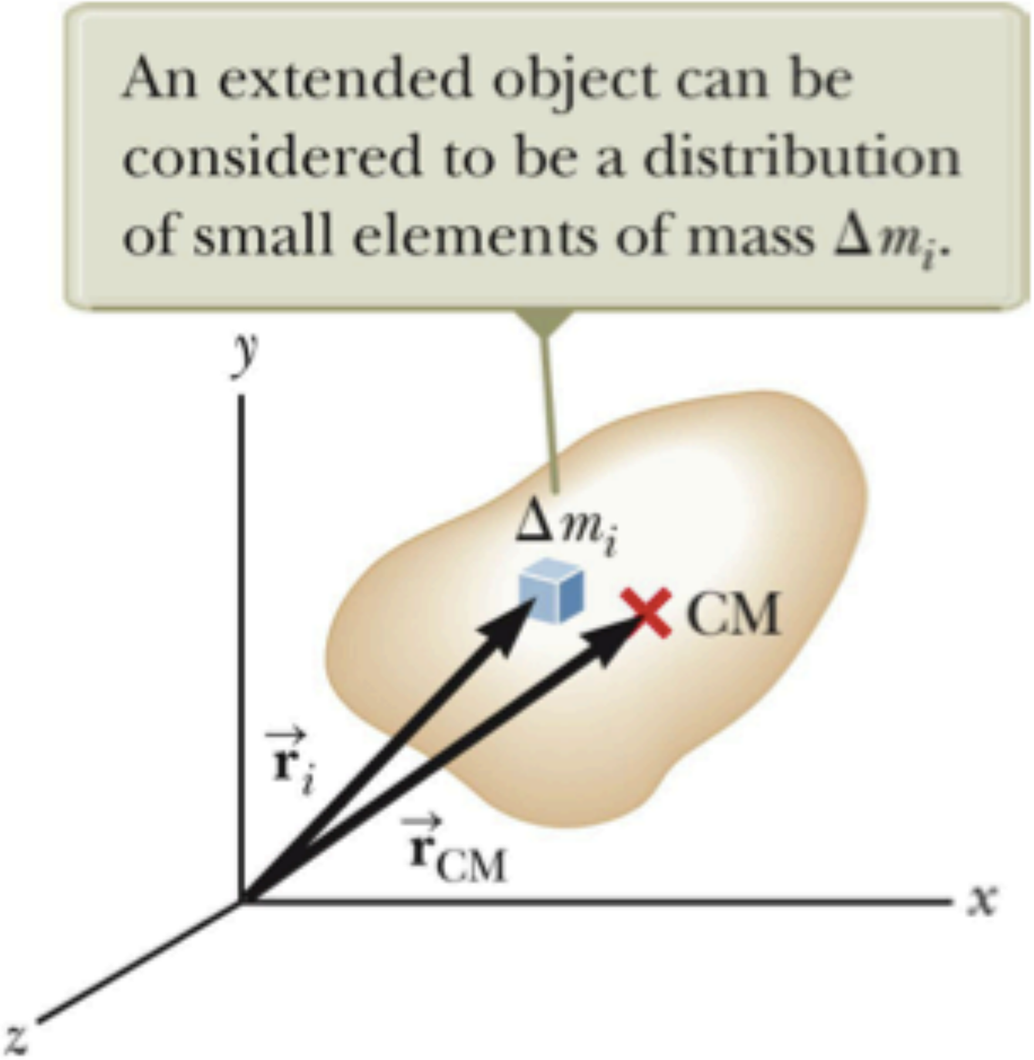
- M is the total mass of the system.
  - Use the active figure to observe effect of different masses and positions.

Center of Mass, Extended Object

Similar analysis can be done for an extended object.

Consider the extended object as a system containing a large number of small mass elements.

Since separation between the elements is very small, it can be considered to have a constant mass distribution.



Center of Mass, position

The center of mass in three dimensions can be located by its position vector, $\vec{\mathbf{r}}_{CM}$.

- For a system of particles,

$$\vec{\mathbf{r}}_{CM} = \frac{1}{M}\sum_i m_i \vec{\mathbf{r}}_i$$

  - $\vec{\mathbf{r}}_i$ is the position of the i^{th} particle, defined by

$$\mathbf{r}_i = x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$$

- For an extended object,

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\, dm$$

Center of Mass, Symmetric Object

The center of mass of any symmetric object of uniform density lies on an axis of symmetry and on any plane of symmetry.

Center of Gravity

Each small mass element of an extended object is acted upon by the gravitational force.

The net effect of all these forces is equivalent to the effect of a single force $M\vec{\mathbf{g}}$ acting through a point called the **center of gravity.**

- If $\vec{\mathbf{g}}$ is constant over the mass distribution, the center of gravity coincides with the center of mass.

Velocity and Momentum of a System of Particles

The velocity of the center of mass of a system of particles is

$$\vec{\mathbf{v}}_{CM} = \frac{d\vec{\mathbf{r}}_{CM}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

The momentum can be expressed as

$$M\vec{\mathbf{v}}_{CM} = \sum_i m_i \vec{\mathbf{v}}_i = \sum_i \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_{tot}$$

The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass.

Acceleration and Force in a System of Particles

The acceleration of the center of mass can be found by differentiating the velocity with respect to time.

$$\vec{\mathbf{a}}_{CM} = \frac{d\vec{\mathbf{v}}_{CM}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{a}}_i$$

The acceleration can be related to a force.

$$M\vec{\mathbf{a}}_{CM} = \sum_i \vec{\mathbf{F}}_i$$

If we sum over all the internal force vectors, they cancel in pairs and the net force on the system is caused only by the external forces.

The Center of Mass

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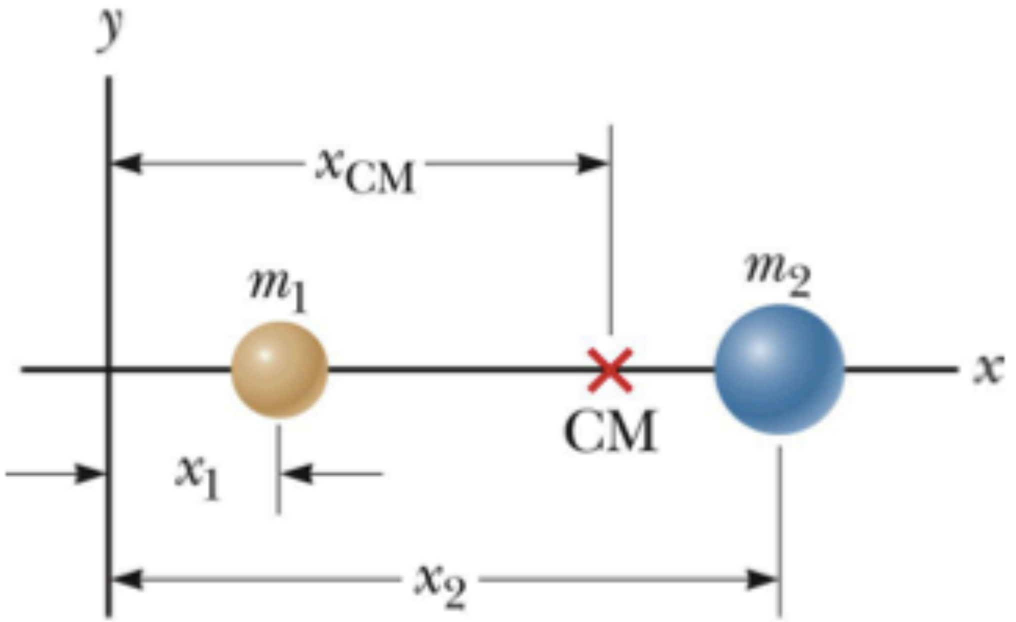

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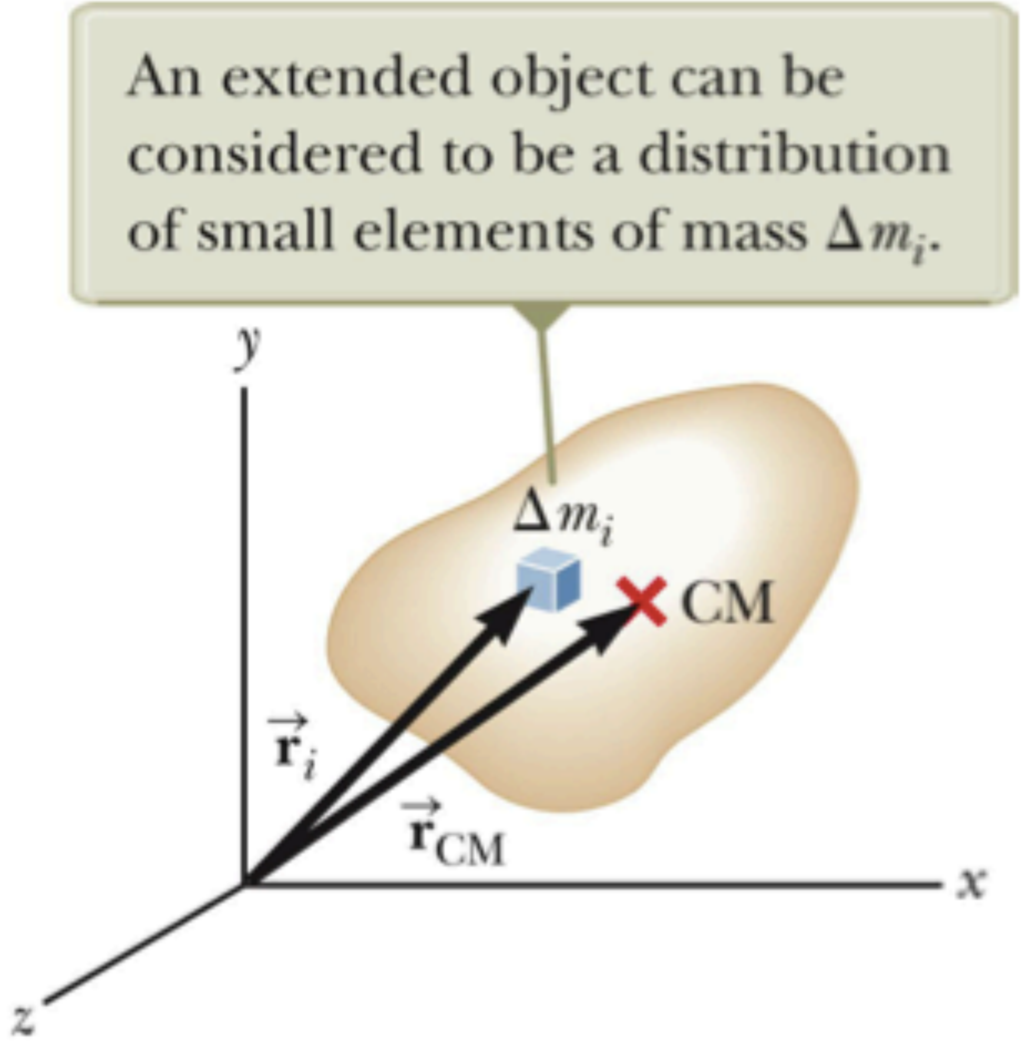

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Impulse and Momentum of a System of Particles

The impulse imparted to the system by external forces is

$$\int \sum \vec{\mathbf{F}}_{ext} dt = M \int d\vec{\mathbf{v}}_{CM} \rightarrow \Delta \vec{\mathbf{p}}_{tot} = \vec{\mathbf{I}}$$

The total linear momentum of a system of particles is conserved if no net external force is acting on the system.

$$M \vec{\mathbf{v}}_{CM} = \vec{\mathbf{p}}_{tot} = constant \quad when \sum \vec{\mathbf{F}}_{ext} = 0$$

For an isolated system of particles, both the total momentum and the velocity of the center of mass are constant in time.

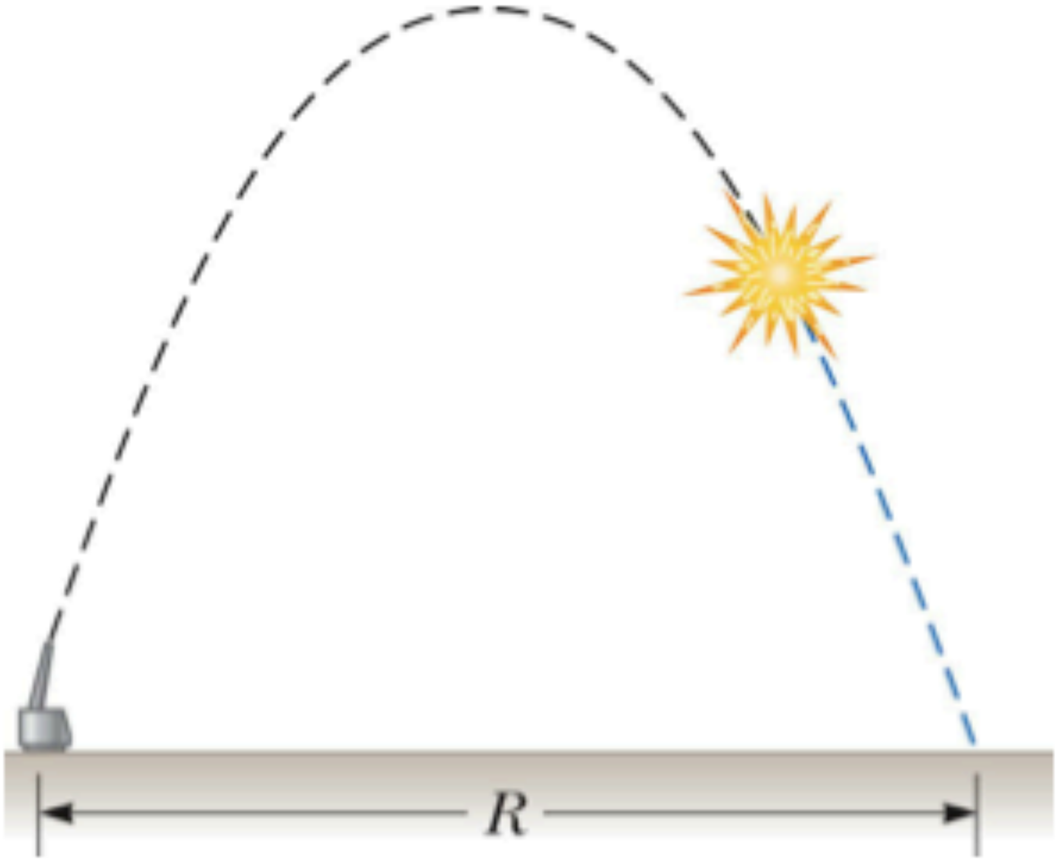
- This is a generalization of the isolated system (momentum) model for a many-particle system.

Motion of the Center of Mass, Example

A projectile is fired into the air and suddenly explodes.

With no explosion, the projectile would follow the dotted line.

After the explosion, the center of mass of the fragments still follows the dotted line, the same parabolic path the projectile would have followed with no explosion.



Deformable Systems

To analyze the motion of a deformable system, use Conservation of Energy and the Impulse-Momentum Theorem.

$$\Delta E_{system} = \sum T \rightarrow \Delta K + \Delta U = 0$$

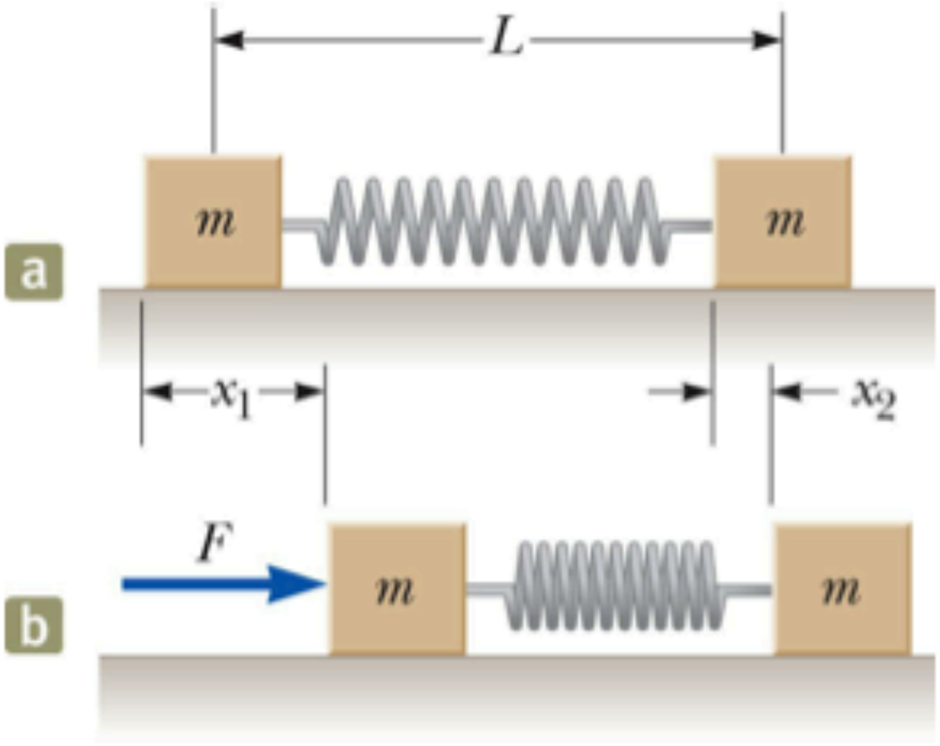
$$\Delta \vec{\mathbf{p}}_{tot} = \vec{\mathbf{I}} \rightarrow m\Delta \vec{\mathbf{v}} = \int \vec{\mathbf{F}}_{ext} dt$$

- If the force is constant, the integral can be easily evaluated.

Deformable System (Spring) Example

Conceptualize

- See figure
- Push on left block, it moves to right, spring compresses.
- At any given time, the blocks are generally moving with different velocities.
- After the force is removed, the blocks oscillate back and forth with respect to the center of mass.



Spring Example, cont.

Categorize

- Non isolated system in terms of momentum and energy.
  - Work is being done on it by the applied force.
- It is a deformable system.
- The applied force is constant, so the acceleration of the center of mass is constant.
- Model as a particle under constant acceleration.

Analyze

- Apply impulse-momentum
- Solve for v_{cm}

Spring Example, final

Analyze, cont.

- Find energies

Finalize

- Answers do not depend on spring length, spring constant, or time interval.

THE END of PHYSICSI

Have a nice day.