PHYSICS I

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Component Method of Adding Vectors

Graphical addition is not recommended when:

- High accuracy is required
- If you have a three-dimensional problem

Component method is an alternative method

- It uses projections of vectors along coordinate axes

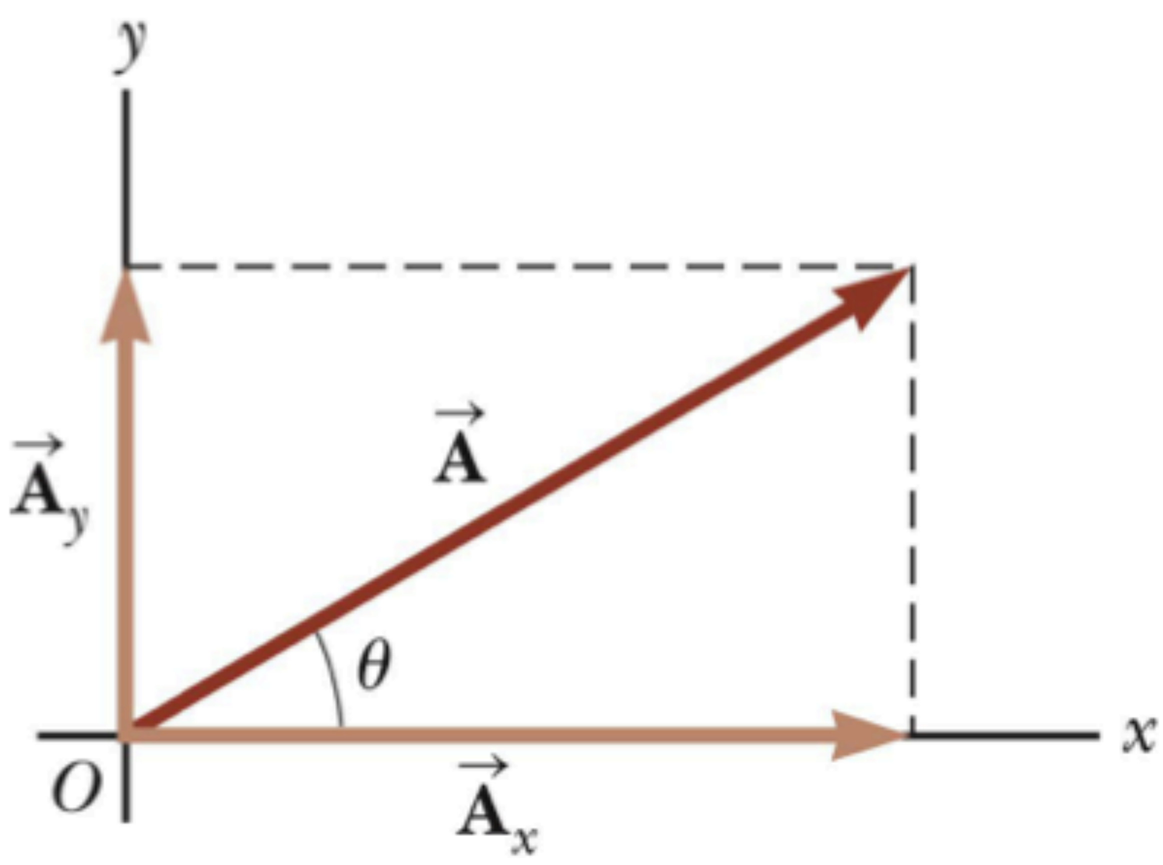
Components of a Vector, Introduction

A **component** is a projection of a vector along an axis.

- Any vector can be completely described by its components.

It is useful to use **rectangular components.**

- These are the projections of the vector along the x- and y-axes.



Vector Component Terminology

$\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ are the ***component vectors*** of $\vec{\mathbf{A}}$.

- They are vectors and follow all the rules for vectors.

A_x and A_y are scalars, and will be referred to as the ***components*** of $\vec{\mathbf{A}}$.

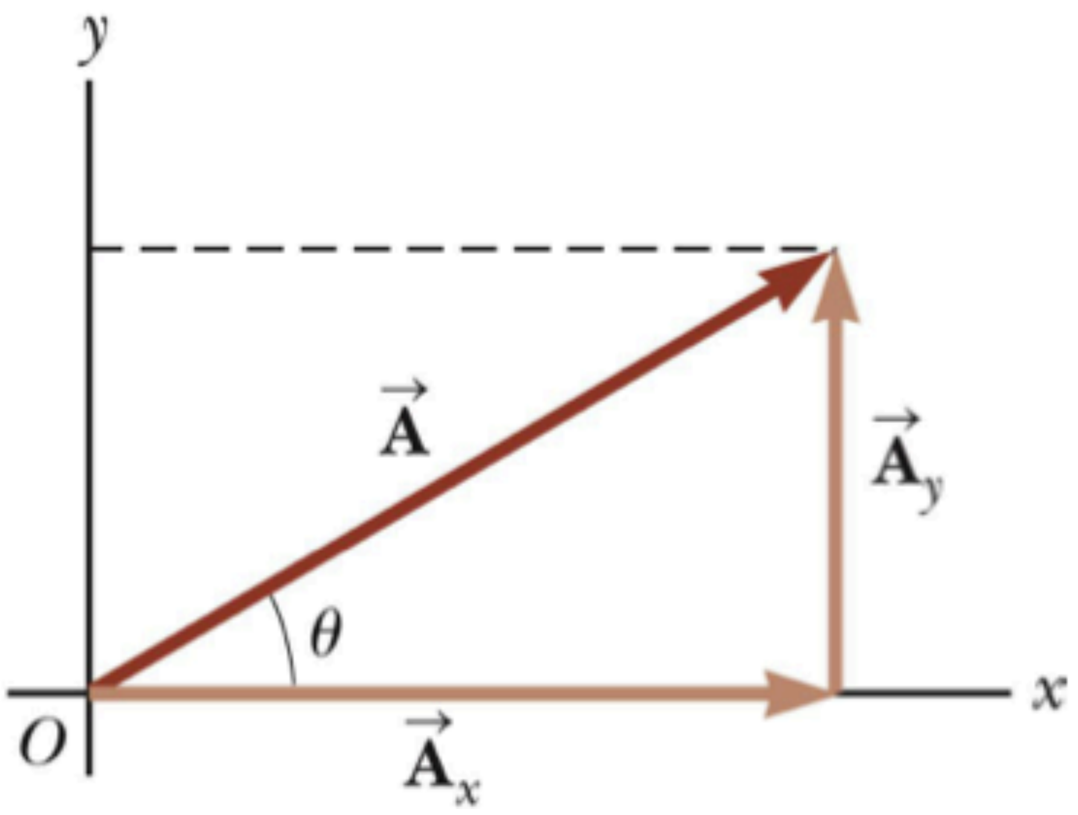
Components of a Vector

Assume you are given a vector $\vec{\mathbf{A}}$

It can be expressed in terms of two other vectors, $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$

These three vectors form a right triangle.

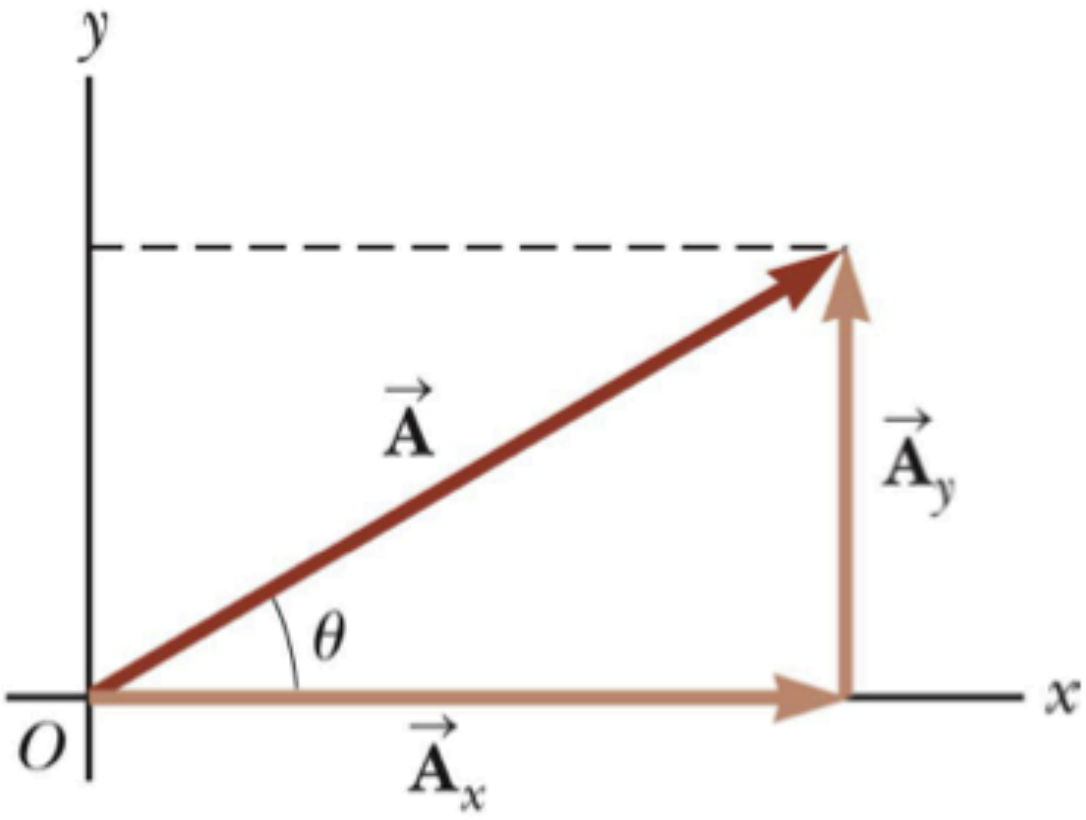


Components of a Vector, 2

The *y*-component is moved to the end of the *x*-component.

This is due to the fact that any vector can be moved parallel to itself without being affected.

- This completes the triangle.



Components of a Vector, 3

The x-component of a vector is the projection along the x-axis.

$$A_x = A\cos\theta$$

The y-component of a vector is the projection along the y-axis.

$$A_y = A\sin\theta$$

This assumes the angle θ is measured with respect to the x-axis.

- If not, do not use these equations, use the sides of the triangle directly.

Components of a Vector, 4

The components are the legs of the right triangle whose hypotenuse is the length of *A*.

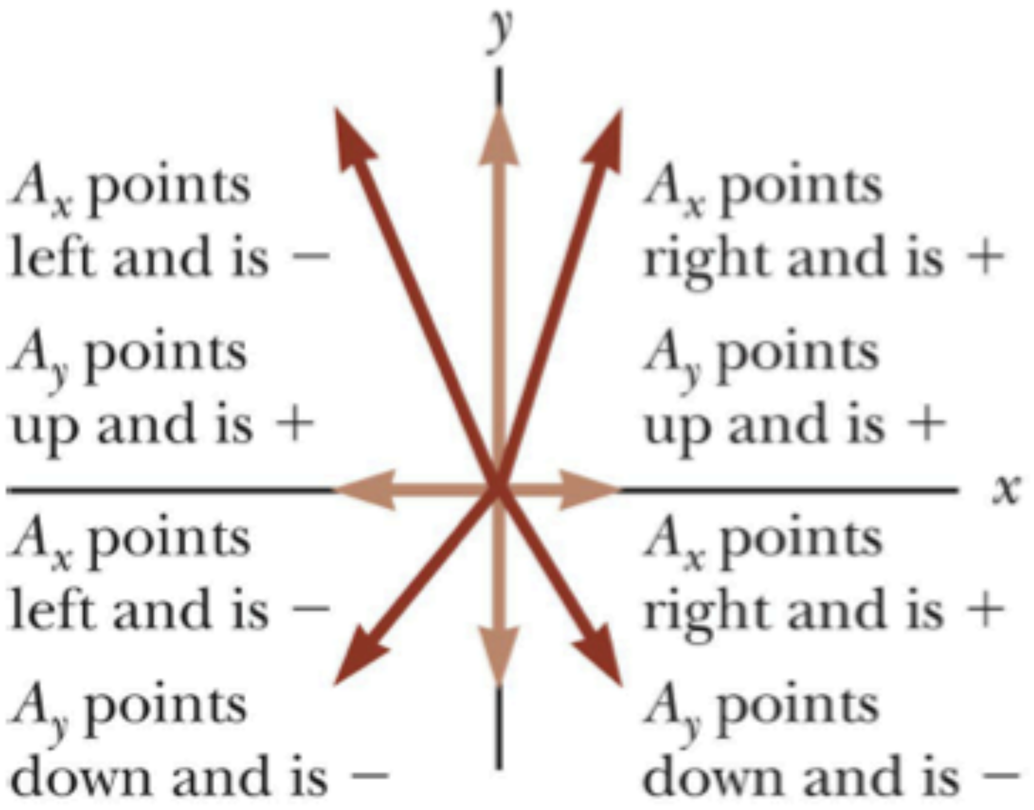
- $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}\frac{A_y}{A_x}$
- May still have to find *θ* with respect to the positive *x*-axis

In a problem, a vector may be specified by its components or its magnitude and direction.

Components of a Vector, final

The components can be positive or negative and will have the same units as the original vector.

The signs of the components will depend on the angle.



Unit Vectors

A ***unit vector*** is a dimensionless vector with a magnitude of exactly 1.

Unit vectors are used to specify a direction and have no other physical significance.

Unit Vectors, cont.

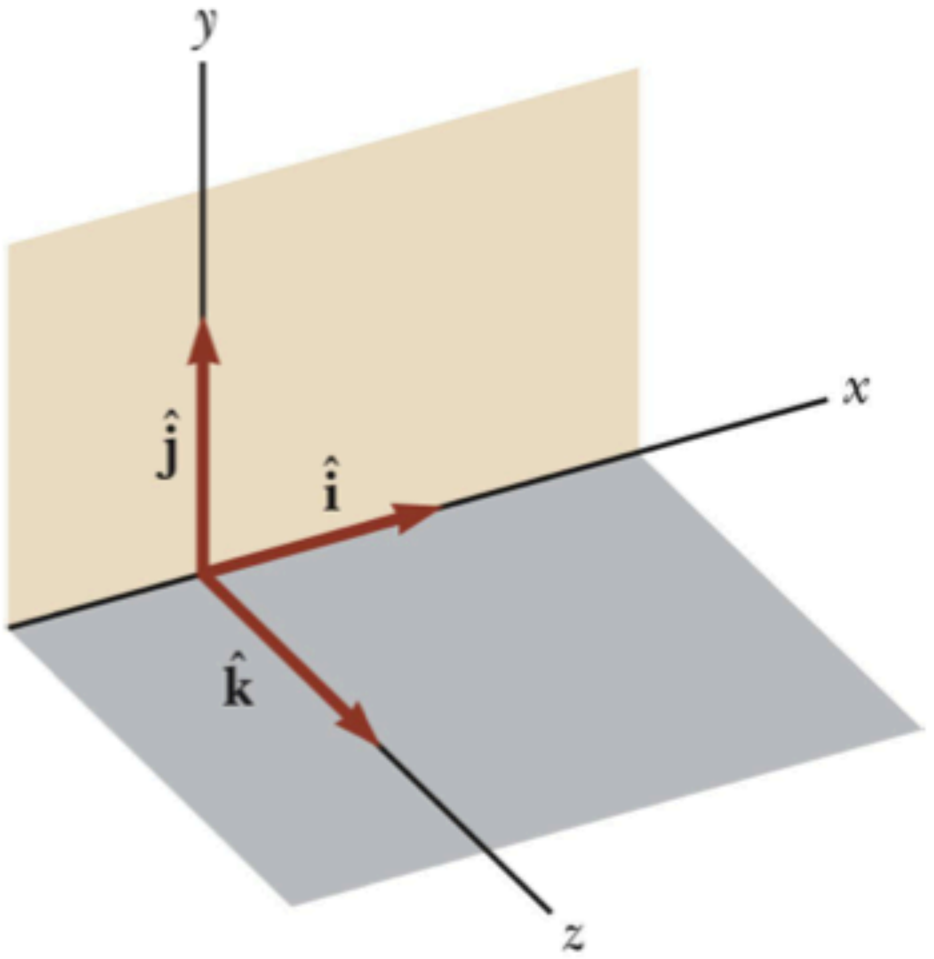
The symbols $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \text{and } \hat{\mathbf{k}}$

represent unit vectors

They form a set of mutually perpendicular vectors in a right-handed coordinate system

The magnitude of each unit vector is 1

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

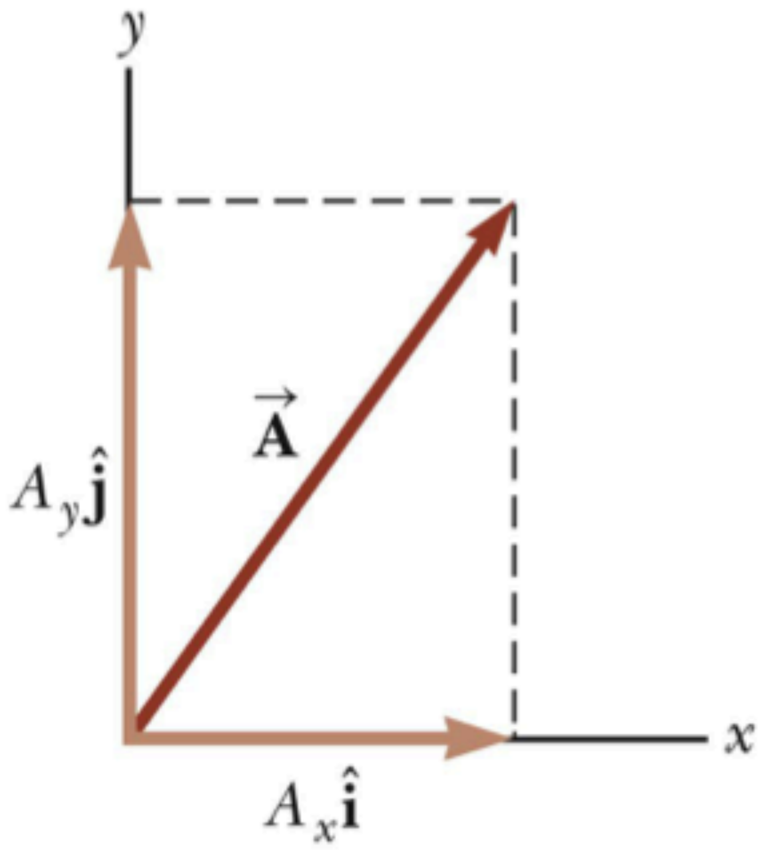


Unit Vectors in Vector Notation

$\mathbf{A_x}$ is the same as $A_x\hat{\mathbf{i}}$ and $\mathbf{A_y}$ is the same as $A_y\,\hat{\mathbf{j}}$ etc.

The complete vector can be expressed as:

$\vec{\mathbf{A}} = A_x\mathbf{i}$洚 $A_y\,\mathbf{j}$



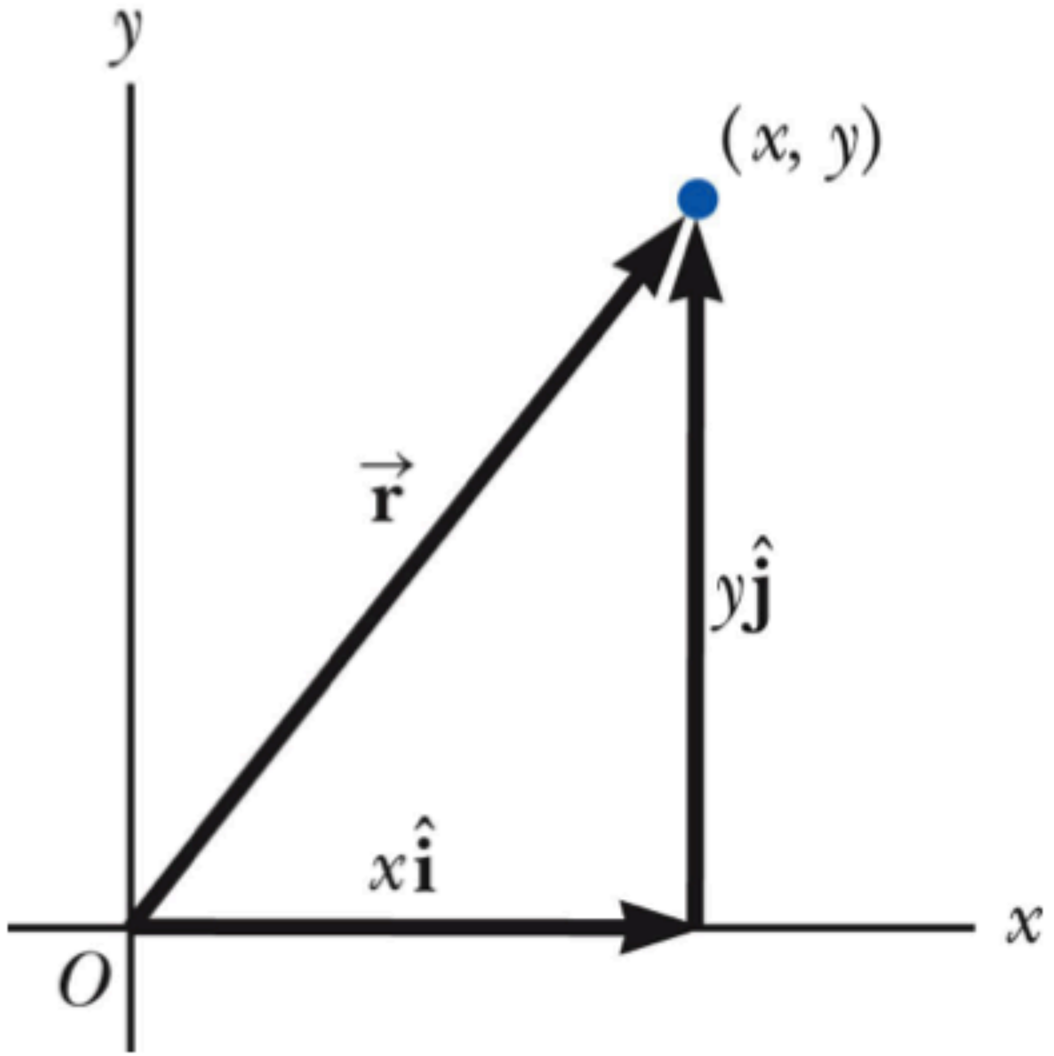
Position Vector, Example

A point lies in the *xy* plane and has Cartesian coordinates of (x, y).

The point can be specified by the position vector.

$$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

This gives the components of the vector and its coordinates.



Adding Vectors Using Unit Vectors

Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

Then

$$\vec{\mathbf{R}} = \left(A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}\right) + \left(B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{R}} = R_x \hat{\mathbf{i}} + R_y \hat{\mathbf{j}}$$

So $R_x = A_x + B_x$ and $R_y = A_y + B_y$

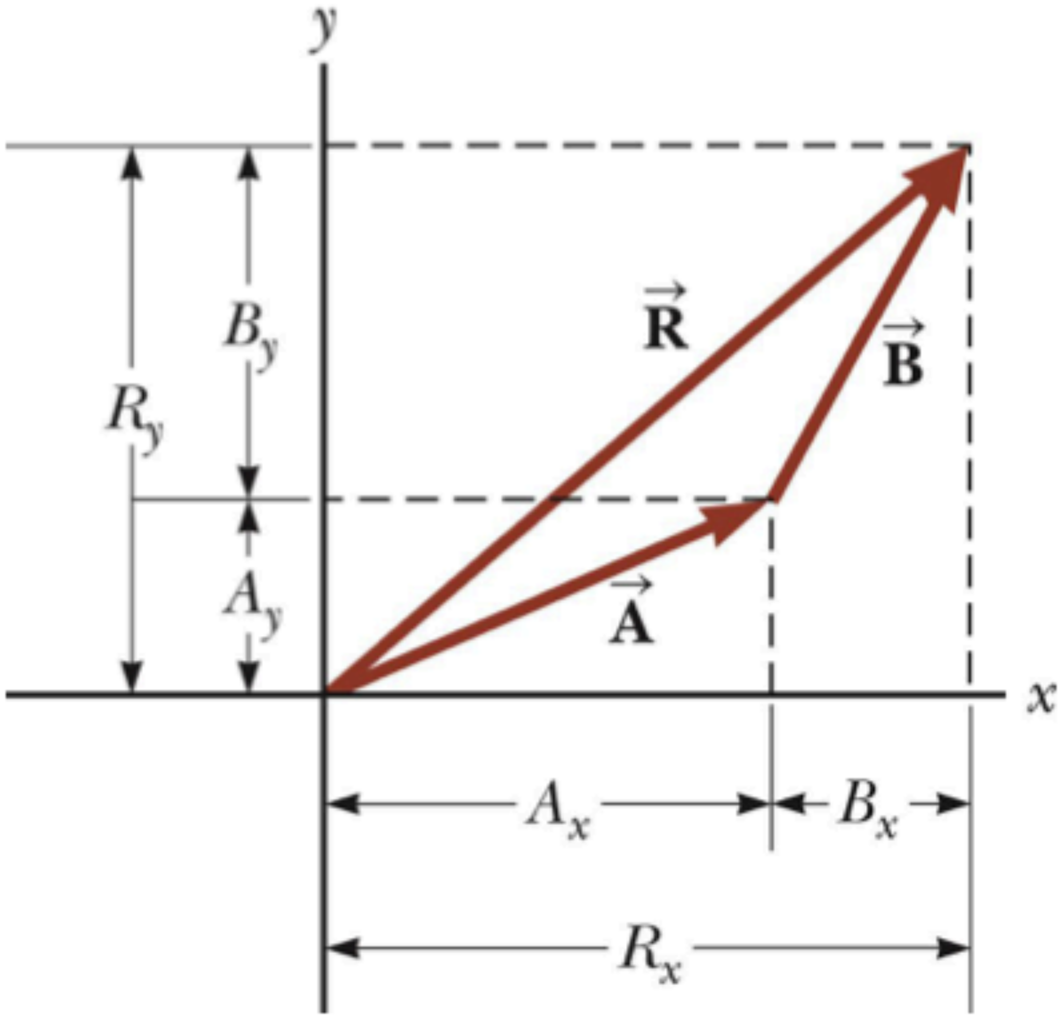
$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Adding Vectors with Unit Vectors

Note the relationships among the components of the resultant and the components of the original vectors.

$R_x = A_x + B_x$

$R_y = A_y + B_y$



Three-Dimensional Extension

Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

Then

$$\vec{\mathbf{R}} = \left(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\right) + \left(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\mathbf{i} + \left(A_y + B_y\right)\mathbf{j} + \left(A_z + B_z\right)\mathbf{k}$$

$$\vec{\mathbf{R}} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k}$$

So $R_x = A_x + B_x$, $R_y = A_y + B_y$, and $R_z = A_z + B_z$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \theta_x = \cos^{-1}\frac{R_x}{R}, \text{ etc.}$$

Adding Three or More Vectors

The same method can be extended to adding three or more vectors.

Assume

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$$

And

$$\vec{\mathbf{R}} = \left(A_x + B_x + C_x\right)\hat{\mathbf{i}} + \left(A_y + B_y + C_y\right)\hat{\mathbf{j}}$$

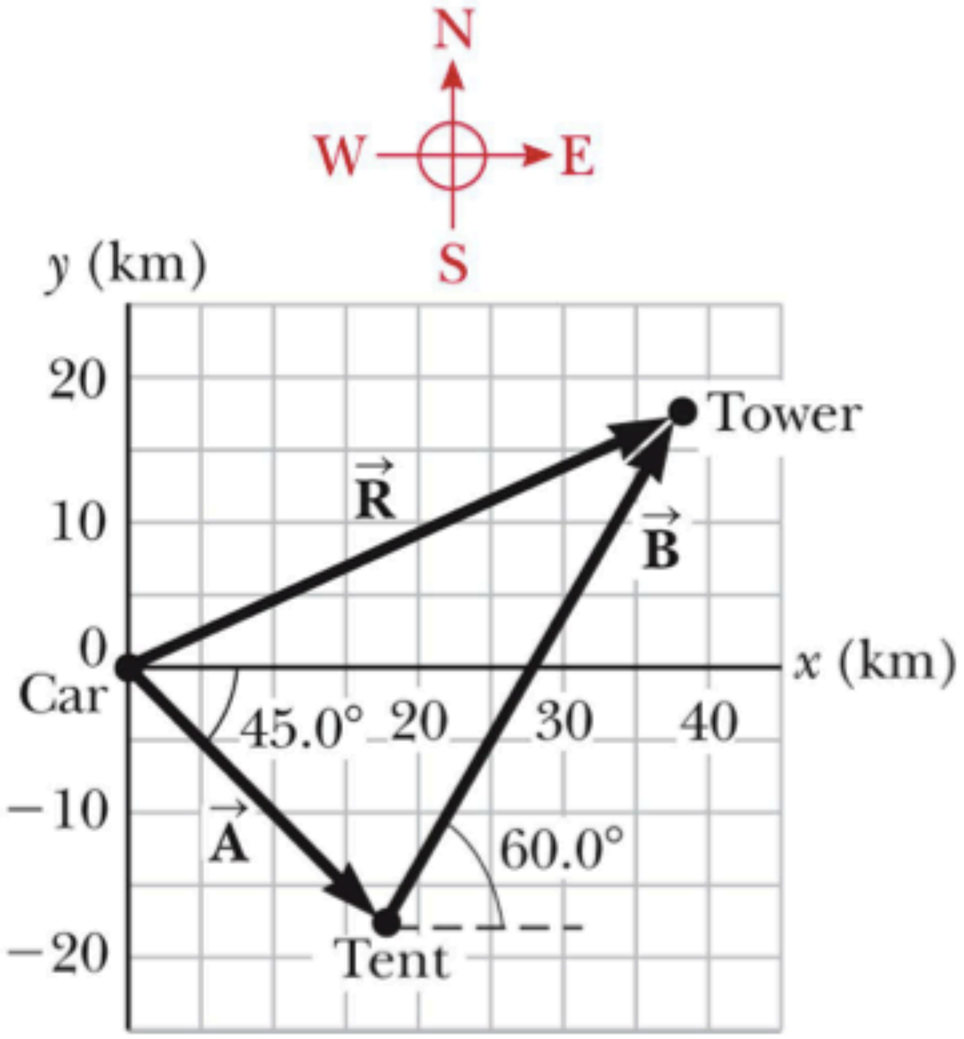
$$+ \left(A_z + B_z + C_z\right)\hat{\mathbf{k}}$$

Example 3.5 – Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

Example 3.5 – Solution, Conceptualize and Categorize

- *Conceptualize* the problem by drawing a sketch as in the figure.
- Denote the displacement vectors on the first and second days by $\vec{A}$ and $\vec{B}$ respectively.
- Use the car as the origin of coordinates.
- The vectors are shown in the figure.
- Drawing the resultant $\vec{R}$, we can now *categorize* this problem as an addition of two vectors.



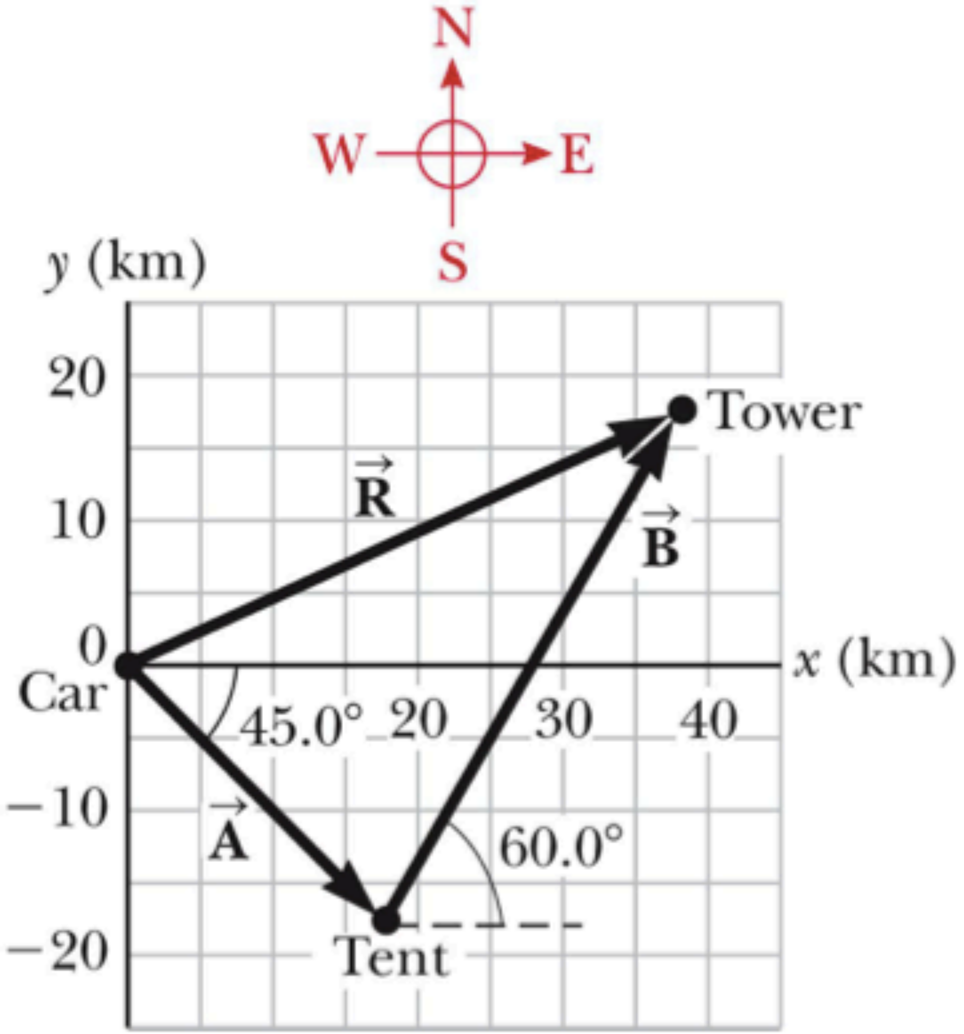
Example 3.5 – Solution, Analysis

Analyze this problem by using our new knowledge of vector components.

The first displacement has a magnitude of 25.0 km and is directed 45.0° below the positive *x* axis.

Its components are:

$$A_x = A\cos(-45.0°) =$$
$$(25.0 \text{ km})(0.707) = 17.7 \text{ km}$$
$$A_y = A\sin(-45.0°)$$
$$= (25.0 \text{ km})(-0.707) = -17.7 \text{ km}$$



Example 3.5 – Solution, Analysis 2

The second displacement has a magnitude of 40.0 km and is 60.0° north of east.

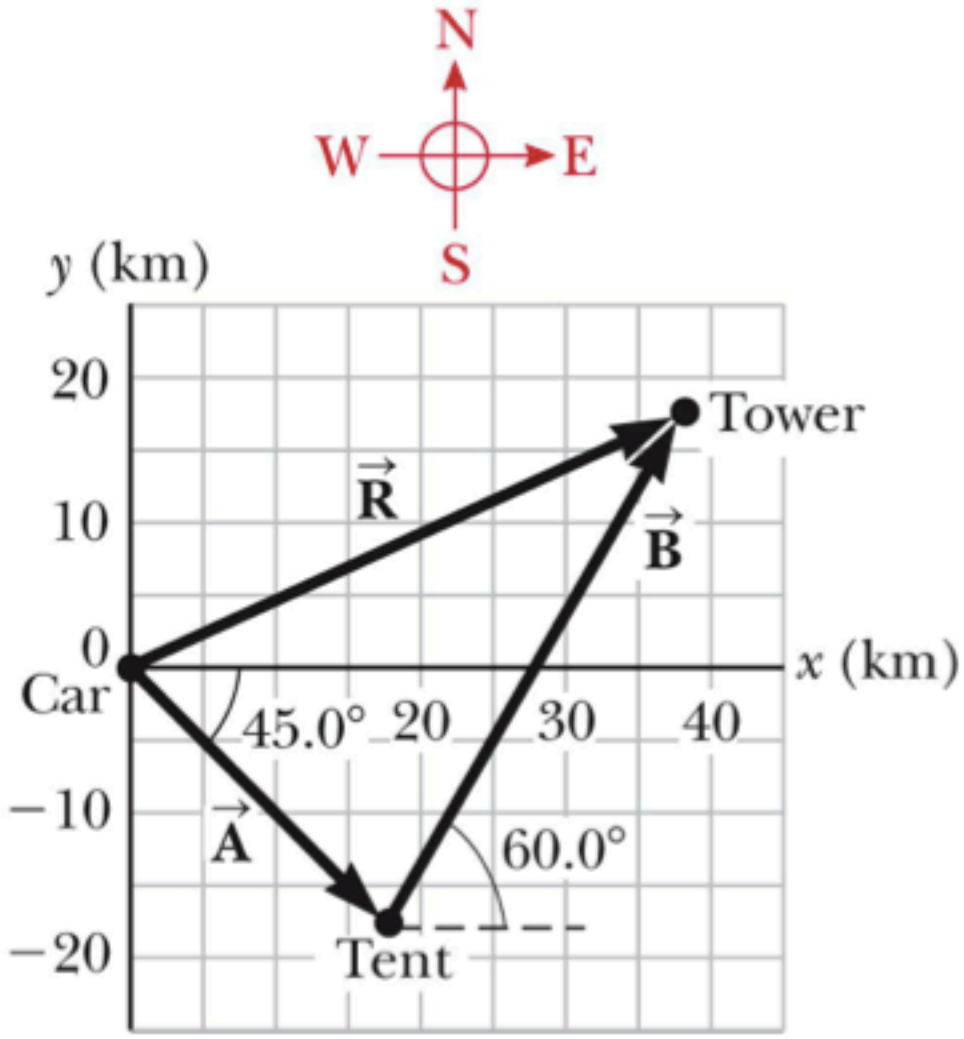
Its components are:

$$B_x = B\cos 60.0° =$$

$$(40.0 \text{ km})(0.500) = 20.0 \text{ km}$$

$$B_y = B\sin 60.0°$$

$$= (40.0 \text{ km})(0.866) = 34.6 \text{ km}$$



Example 3.5 – Solution, Analysis 3

The negative value of A_y indicates that the hiker walks in the negative y direction on the first day.

The signs of A_x and A_y also are evident from the figure.

The signs of the components of B are also confirmed by the diagram.

Example 3.5 – Analysis, 4

Determine the components of the hiker's resultant displacement for the trip.

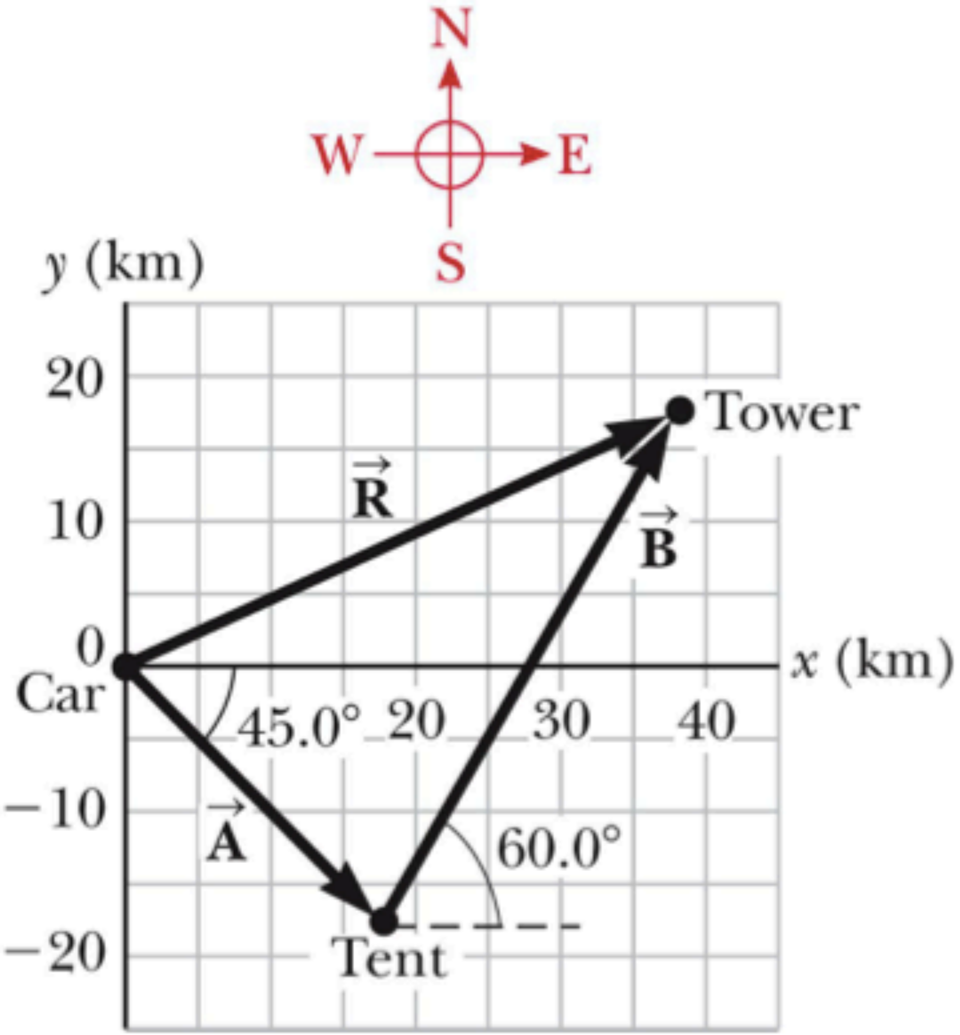
- Find an expression for the resultant in terms of unit vectors.

The resultant displacement for the trip has components given by

- $R_x = A_x + B_x$ = 17.7 km + 20.0 km = 37.7 km
- $R_y = A_y + B_y$ = -17.7 km + 34.6 km = 16.9 km

In unit vector form

$$\vec{R} = (37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}})\text{ km}$$

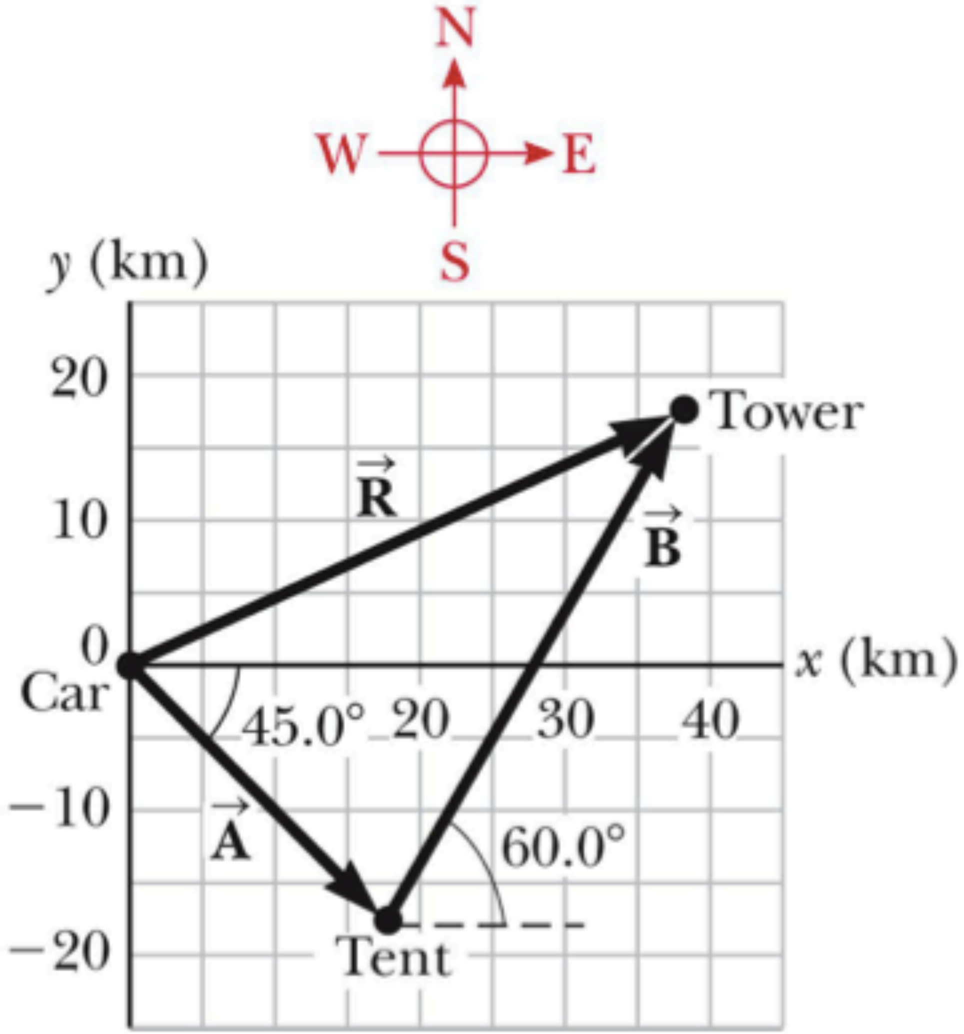


Example 3.5 – Solution, Finalize

The resultant vector has a magnitude of 41.3 km and is directed 24.1° north of east.

The units of $\vec{R}$ are km, which is reasonable for a displacement.

From the graphical representation , estimate that the final position of the hiker is at about (38 km, 17 km) which is consistent with the components of the resultant.



Example 3.5 – Solution, Finalize, cont.

Both components of the resultant are positive, putting the final position in the first quadrant of the coordinate system.

- This is also consistent with the figure.

Scalar & Vector Product

$\vec{a}.\vec{b}$ & $\vec{a} \times \vec{b} = ?$

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$