PHYSICS II

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MULTILOOP CIRCUITS

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

The current into the junction must equal the current out (charge is conserved).

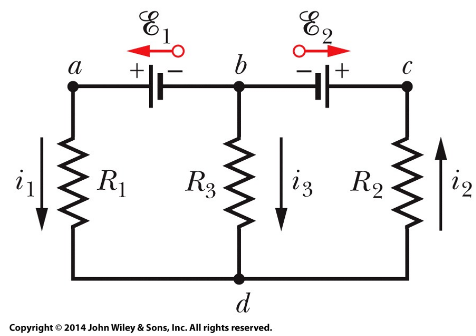


Figure shows a circuit containing more than one loop. If we traverse the left-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0.$$

If we traverse the right-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$$

If we had applied the loop rule to the big loop, we would have obtained (moving counterclockwise from b) the equation

$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$$

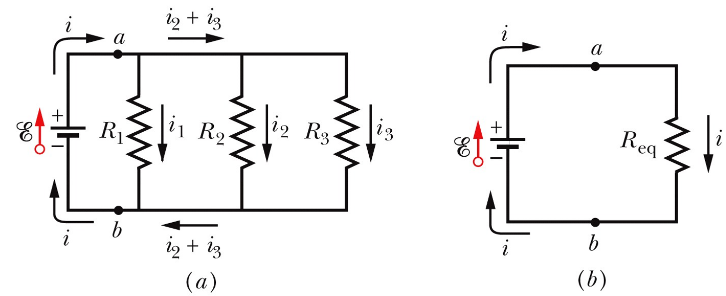
which is the sum of two small loops equations.

MULTILOOP CIRCUITS

Resistances in Parallel

Figure (a) shows three resistances connected in parallel to an ideal battery of *emf* $\mathscr{E}$. The applied potential difference V is maintained by the battery. Fig. b, the three parallel resistances have been replaced with an equivalent resistance R_{eq}.

Parallel resistors and their equivalent have the same potential difference ("par-V").



To derive an expression for R_{eq} in Fig. (b), we first write the current in each actual resistance in Fig. (a) as

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

where V is the potential difference between a and b. If we apply the junction rule at point a in Fig. (a) and then substitute these values, we find

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

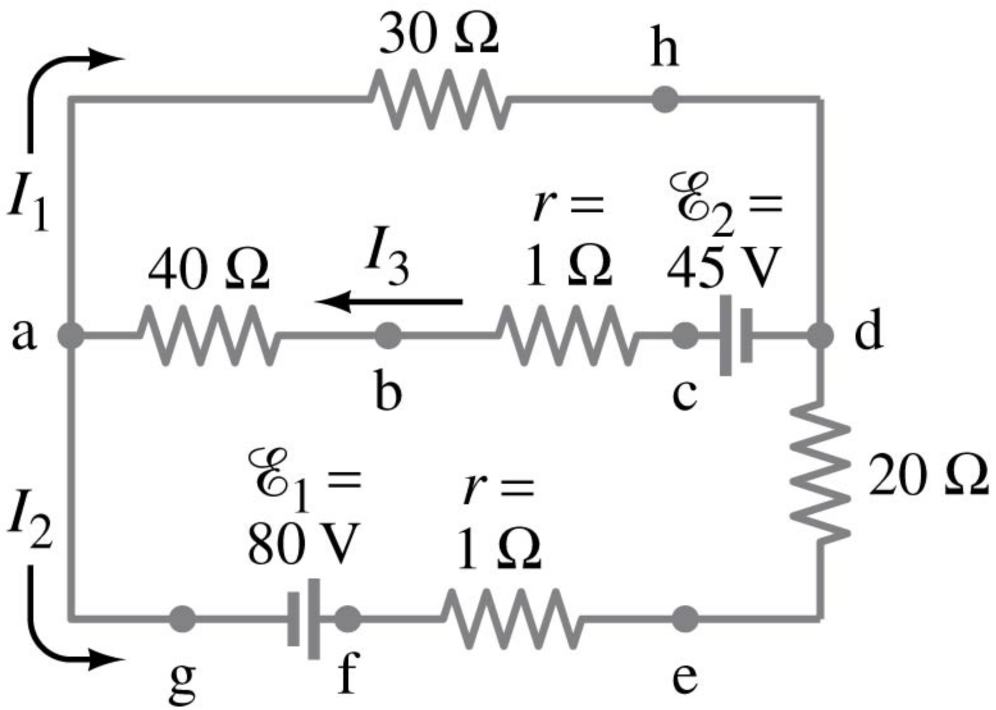
If we replaced the parallel combination with the equivalent resistance R_{eq} (Fig. b), we would have $i = \frac{V}{R_{eq}}$. and thus substituting the value of *i* from above equation we get,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad \text{(}n\text{ resistances in parallel).}$$

Kirchhoff's Rules

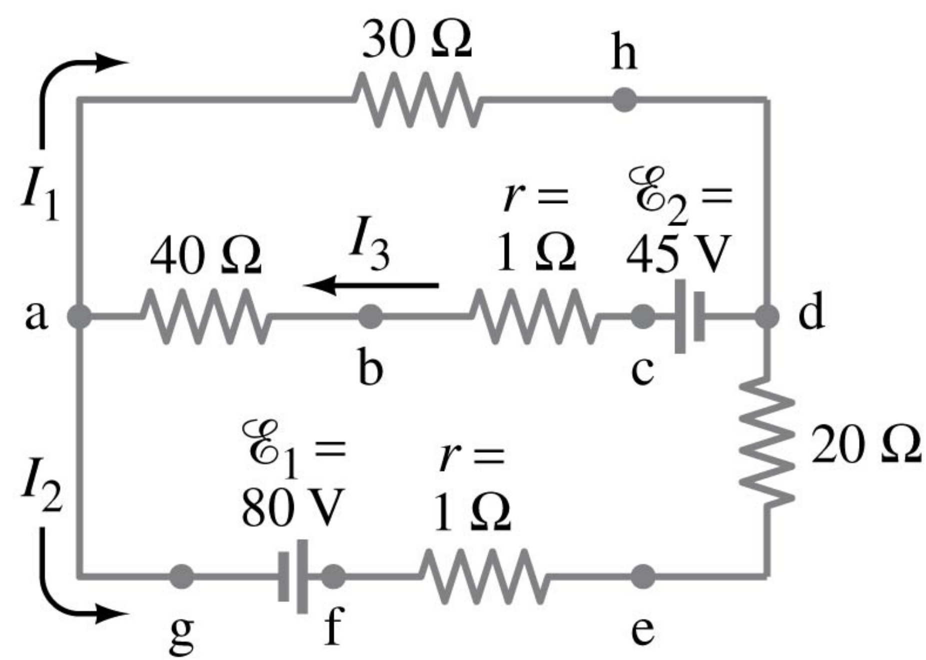
Some circuits cannot be broken down into series and parallel connections.



Kirchhoff's Rules

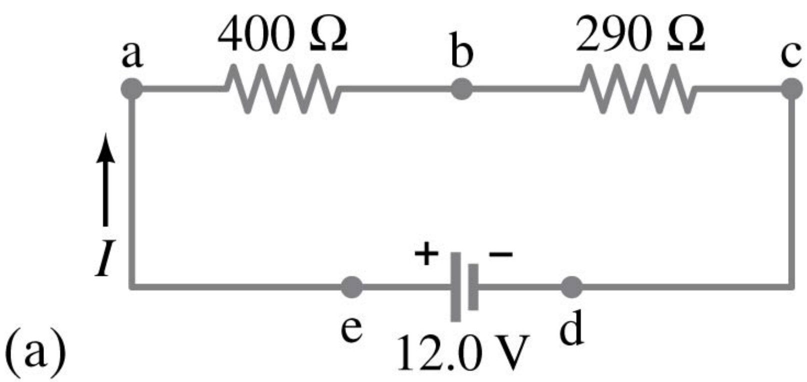
For these circuits we use Kirchhoff's rules.

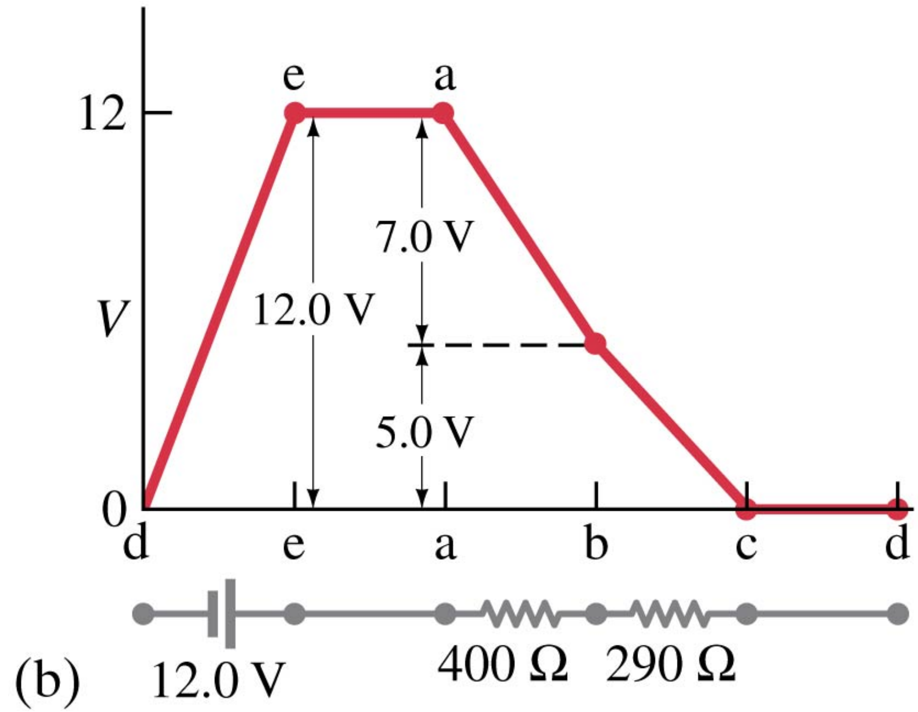
<u>Junction rule:</u> The sum of currents entering a junction equals the sum of the currents leaving it.



Kirchhoff's Rules

Loop rule: The sum of the changes in potential around a closed loop is zero.





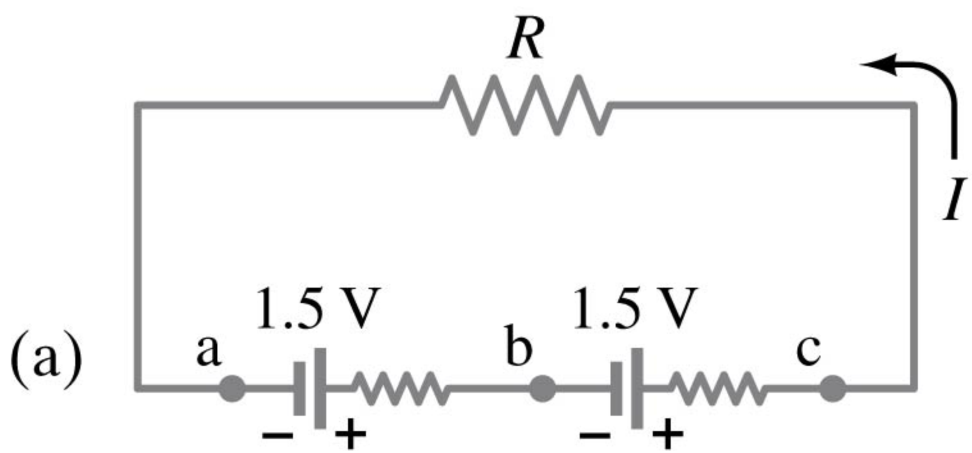
Kirchhoff's Rules

Problem Solving: Kirchhoff's Rules

1. Label each current.
2. Identify unknowns.
3. Apply junction and loop rules; you will need as many independent equations as there are unknowns.
4. Solve the equations, being careful with signs.

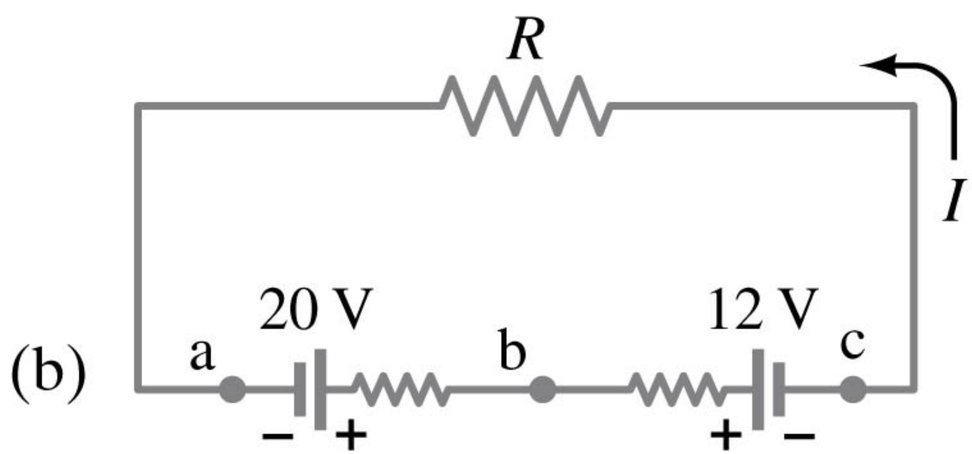
EMFs in Series and in Parallel; Charging a Battery

EMFs in series in the same direction: total voltage is the sum of the separate voltages



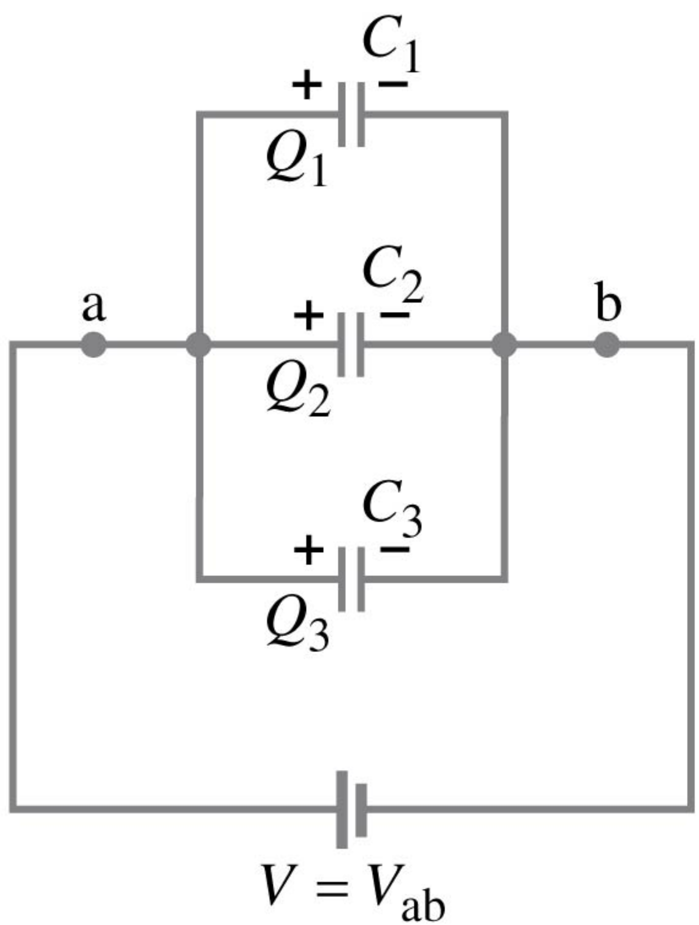
EMFs in Series and in Parallel; Charging a Battery

EMFs in series, opposite direction: total voltage is the difference, but the lower-voltage battery is charged.



Circuits Containing Capacitors in Series and in Parallel

Capacitors in parallel have the same voltage across
each one:



Circuits Containing Capacitors in Series and in Parallel

In this case, the total capacitance is the sum:

$$Q = C_{\mathrm{eq}} V.$$

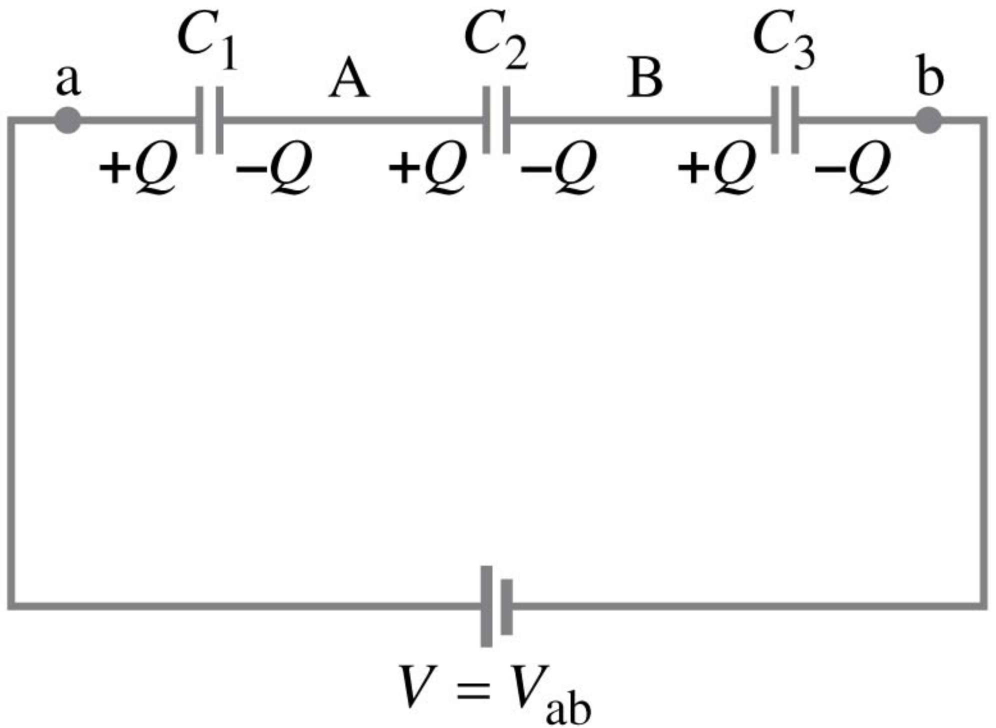
$$C_{\mathrm{eq}} V = C_1 V + C_2 V + C_3 V = (C_1 + C_2 + C_3) V$$

or

$$C_{\mathrm{eq}} = C_1 + C_2 + C_3. \tag{19-5}$$

Circuits Containing Capacitors in Series and in Parallel

Capacitors in series have the same charge:



Circuits Containing Capacitors in Series and in Parallel

In this case, the reciprocals of the capacitances add to give the reciprocal of the equivalent capacitance:

$$V = V_1 + V_2 + V_3.$$

$$\frac{Q}{C_{\text{eq}}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad (19\text{-}6)$$

Circuits Containing Capacitors in Series and in Parallel

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or

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad (19\text{-}6)$$

MULTILOOP CIRCUITS

Resistance and capacitors

Table 27-1 Series and Parallel Resistors and Capacitors

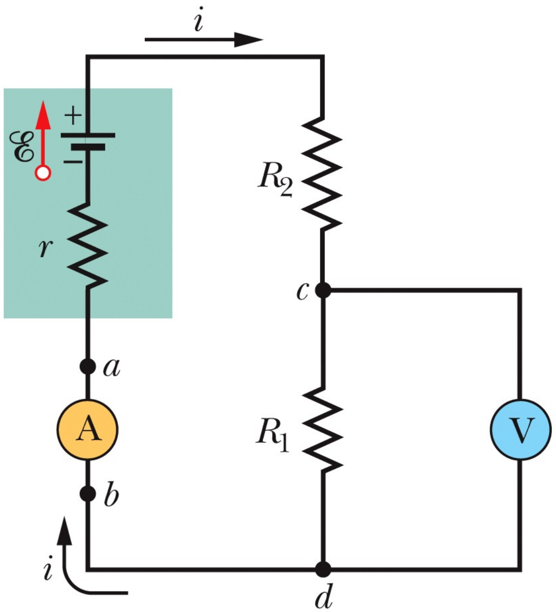
Series	Parallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

Checkpoint 4

A battery, with potential V across it, is connected to a combination of two identical resistors and then has current i through it. What are the potential difference across and the current through either resistor if the resistors are (a) in series and (b) in parallel?

Answer: (a) Potential difference across each resistor: *V/2*
Current through each resistor: *i*
(b) Potential difference across each resistor: *V*
Current through each resistor: *i/2*

The ammeter and the voltmeter



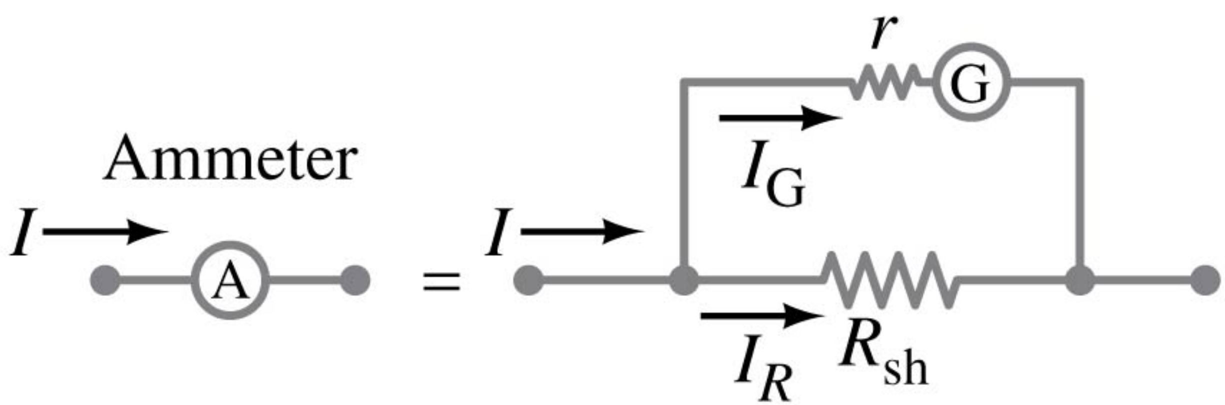
An instrument used to measure currents is called an **ammeter**. To measure the current in a wire, you usually have to break or cut the wire and insert the ammeter so that the current to be measured passes through the meter. In the figure, ammeter A is set up to measure current *i*. It is essential that the resistance R_A of the ammeter be very much smaller than other resistances in the circuit. Otherwise, the very presence of the meter will change the current to be measured.

A meter used to measure potential differences is called a **voltmeter**. To find the potential difference between any two points in the circuit, the voltmeter terminals are connected between those points without breaking or cutting the wire. In the Figure, voltmeter V is set up to measure the voltage across R_1. It is essential that the resistance R_V of a voltmeter be very much larger than the resistance of any circuit element across which the voltmeter is connected. This is to insure that only a negligible current passes through the voltmeter, otherwise, the meter alters the potential difference that is to be measured.

Ammeters and Voltmeters—Measurement Affects the Quantity Being Measured

An ammeter measures current; a voltmeter measures voltage. Both are based on galvanometers, unless they are digital.

The current in a circuit passes through the ammeter; the ammeter should have low resistance so as not to affect the current.

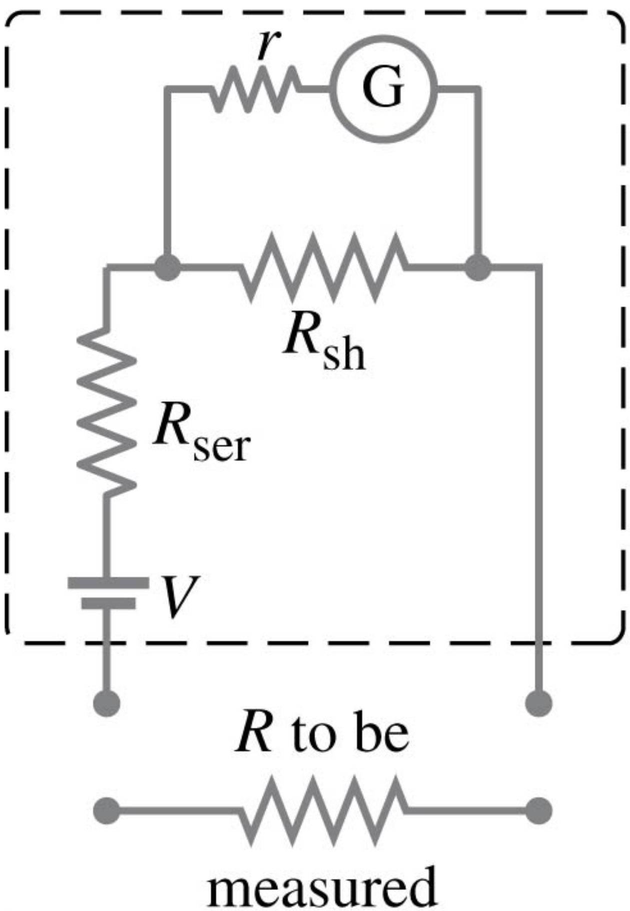


Ammeters and Voltmeters—Measurement Affects the Quantity Being Measured

A voltmeter should not affect the voltage across the circuit element it is measuring; therefore its resistance should be very large.

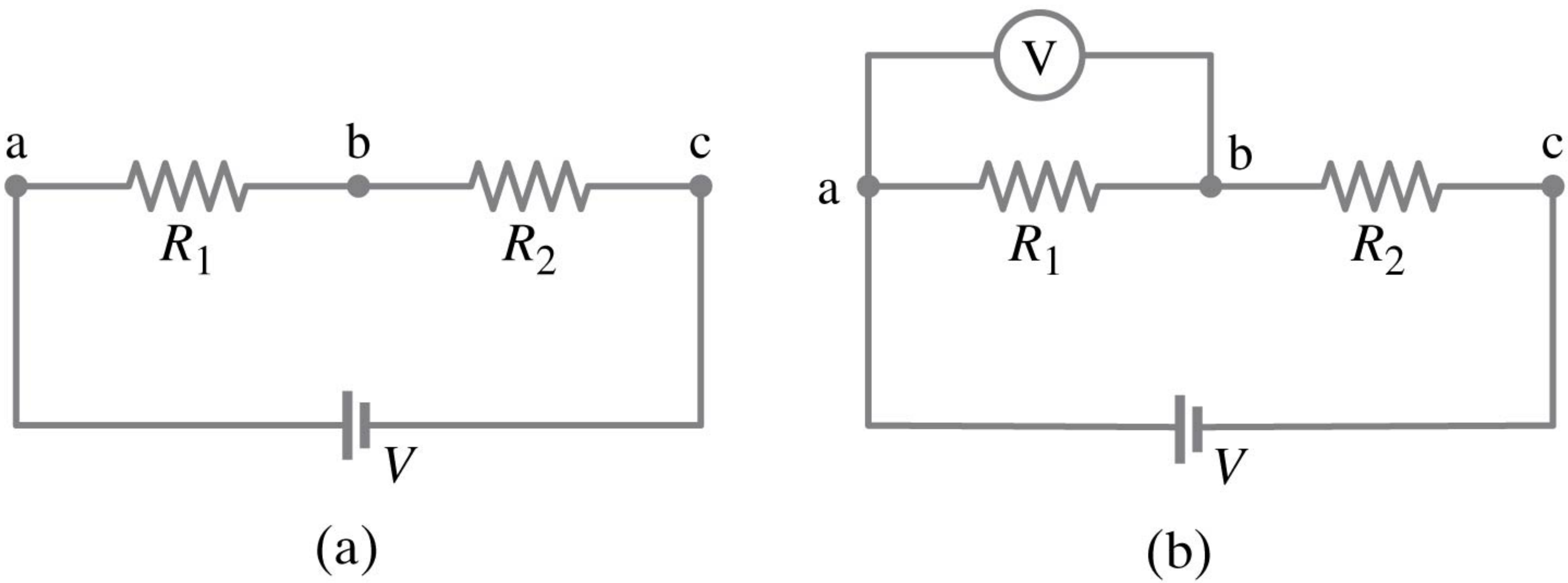
Ammeters and Voltmeters—Measurement Affects the Quantity Being Measured

An ohmmeter measures resistance; it requires a battery to provide a current



Ammeters and Voltmeters—Measurement Affects the Quantity Being Measured

If the meter has too much or (in this case) too little resistance, it can affect the measurement.



RC Circuits

Charging a capacitor: The capacitor of capacitance *C* in the figure is initially uncharged. To charge it, we close switch S on point a. This completes an *RC* series circuit consisting of the capacitor, an ideal battery of *emf* $\mathscr{E}$ and a resistance *R*.
The charge on the capacitor increases according to

$$q = C\mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

in which $C\ = q_0$ is the equilibrium (final) charge and $RC=\tau$ is the capacitive time constant of the circuit. During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC} \quad \text{(charging a capacitor).}$$

And the voltage is:

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

The product *RC* is called the **capacitive time constant** of the circuit and is represented with the symbol *τ.*

$$\tau = RC \quad \text{(time constant).}$$

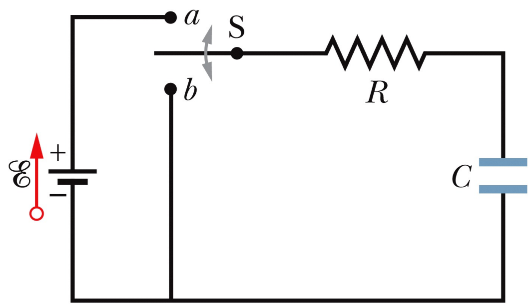
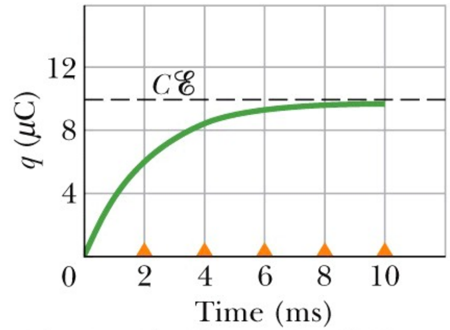


Figure: RC circuit



The plot shows the buildup of charge on the capacitor of the above figure.

RC Circuits

Discharging a capacitor: Assume now that the capacitor of the figure is fully charged to a potential V_0 equal to the emf $\mathscr{E}$ of the battery. At a new time *t=0*, switch S is thrown from a to b so that the capacitor can discharge through resistance *R*.
When a capacitor discharges through a resistance R, the charge on the capacitor decays according to

$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor),}$$

where q_0 ($=CV_0$) is the initial charge on the capacitor.

During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \quad \text{(discharging a capacitor).}$$

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

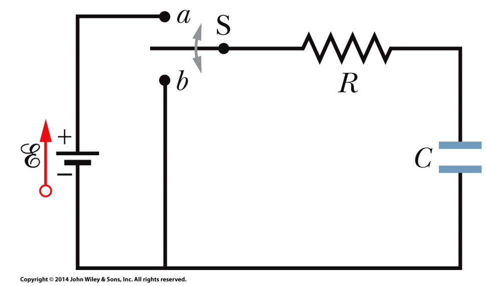
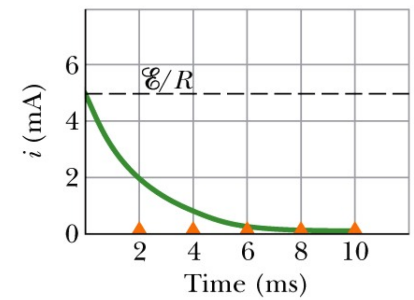


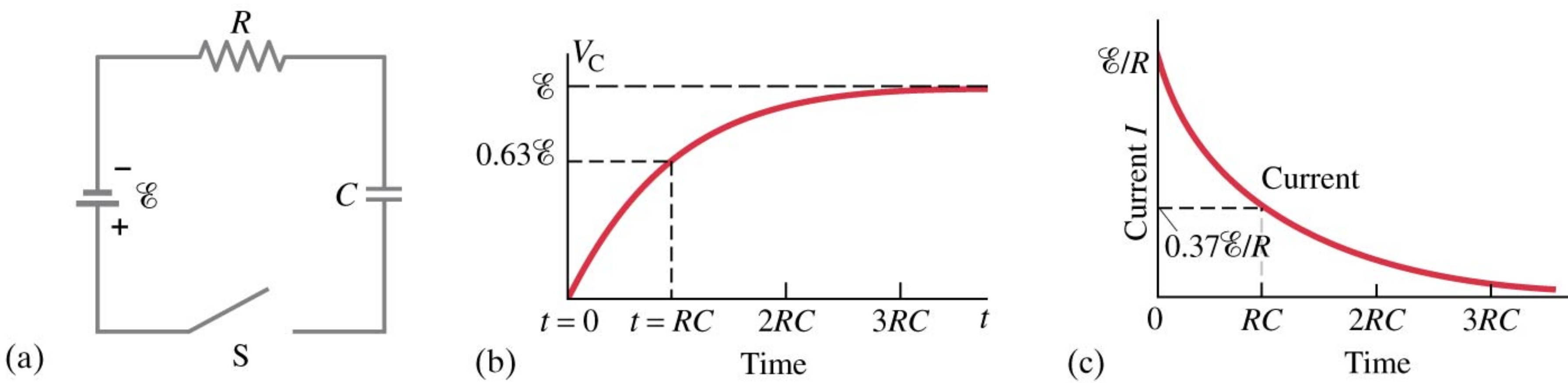
Figure: RC circuit



A plot shows the decline of the charging current in the circuit of the above figure.

RC Circuits—Resistor and Capacitor in Series

When the switch is closed, the capacitor will begin to charge.



RC Circuits—Resistor and Capacitor in Series

The voltage across the capacitor increases with time:

$$V_C = \mathscr{E}\left(1 - e^{-t/RC}\right)$$

This is a type of exponential.

RC Circuits—Resistor and Capacitor in Series

The charge follows a similar curve:

$$Q = Q_0\left(1 - e^{-t/RC}\right)$$

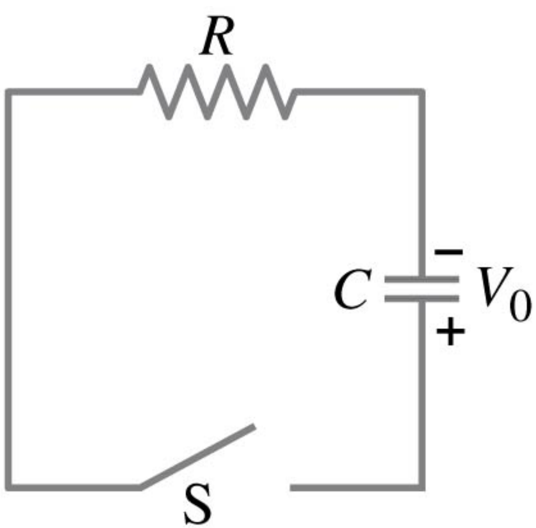
This curve has a characteristic time constant:

$$\tau = RC$$

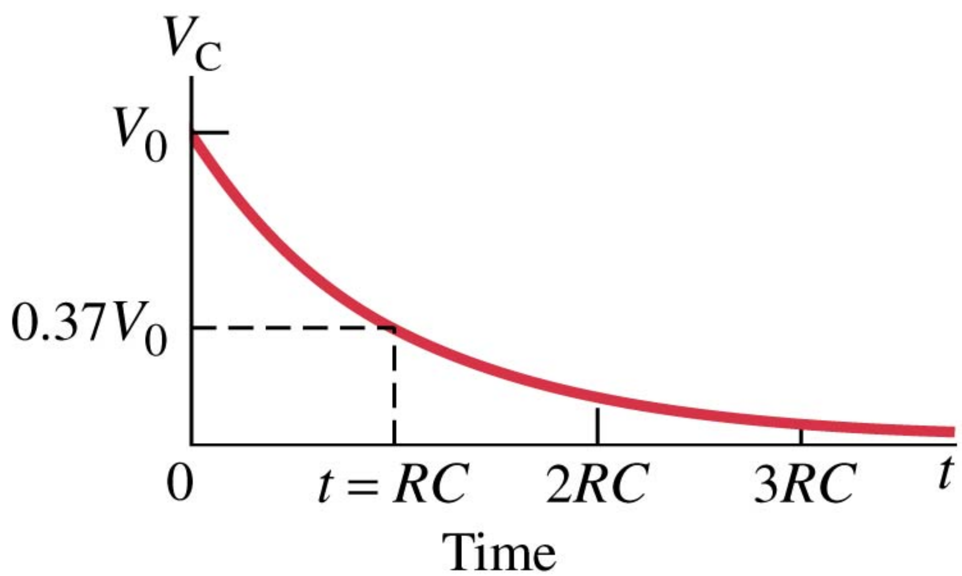
RC Circuits—Resistor and Capacitor in Series

If an isolated charged capacitor is connected across a resistor, it discharges:

$$Q = Q_0 e^{-t/RC}$$



(a)



(b)

SUMMARY

Emf

- The **emf** (work per unit charge) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad \text{(definition of } \mathscr{E}\text{).}$$

Single-Loop Circuits

- Current in a single-loop circuit:

$$i = \frac{\mathscr{E}}{R + r},$$

Power

- The rate P of energy transfer to the charge carriers is

$$P = iV,$$

- The rate P_r at which energy is dissipated as thermal energy in the battery is $$P_r = i^2 r.$$

- The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathscr{E}.$$

Series Resistance

- When resistances are in series

$$R_{\text{eq}} = \sum_{j=1}^{n} R_j$$

Parallel Resistance

- When resistances are in parallel

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j}$$

RC Circuits

- The charge on the capacitor increases according to

$$q = C\mathscr{E}(1 - e^{-t/RC})$$

- During the charging, the current is

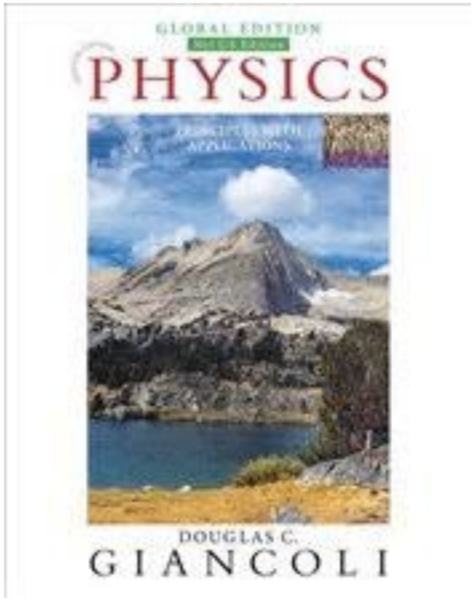
$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC}$$

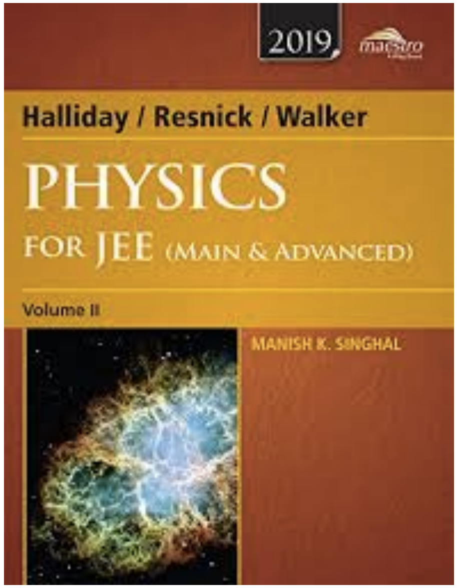
- During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$

References

- Giancoli's book
- Halliday&Resnick's book





Next Lecture

Question solutions on the topics discussed today will be done next week.