PHY404- Solid State Physics II

MAGNETISM-PartIII

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Ferromagnetism

- *Ferromagnetism* is the phenomenon of spontaneous magnetization – the magnetization exists in a material in the absence of applied magnetic field.
- The best-known examples - transition metals Fe, Co, and Ni.
  Also, other elements and alloys involving transition or rare-earth elements, such as the rare-earth metals Gd, Dy, and the insulating transition metal oxides (CrO_2).
- The phenomenon is restricted to transition and rare-earth elements $\Rightarrow$ it is related to the unfilled $3d$ and $4f$ shells in these substances.
- Ferromagnetism involves the alignment of a significant fraction of the molecular magnetic moments in some favorable direction in the crystal.
- Ferromagnetism appears only below a certain temperature, which is known as the *ferromagnetic transition temperature* or simply as the *Curie temperature* (depends on the substance).

Curie-Weiss Law

The susceptibility of a material, χ, indicates how dramatically a material responds to an applied magnetic field, and is defined as the ratio of the magnetisation of the material, M, and the applied magnetic field, H.

$$\chi = \frac{M}{H} \qquad \text{Equation 1}$$

The magnetisation of a material, M, is defined as the magnetic moment per unit volume or per unit mass of a material and is dependent on the individual magnetic dipole moments of the atoms in the material and on the interactions of these dipoles with each other.

Above the Curie Temperature there will be a change in the susceptibility as the material becomes paramagnetic, therefore giving the equation:

$$\chi = \frac{C}{T - T_c} = \frac{M}{H} \qquad \text{Equation 2}$$

where C is a constant.

Curie-Weiss Law

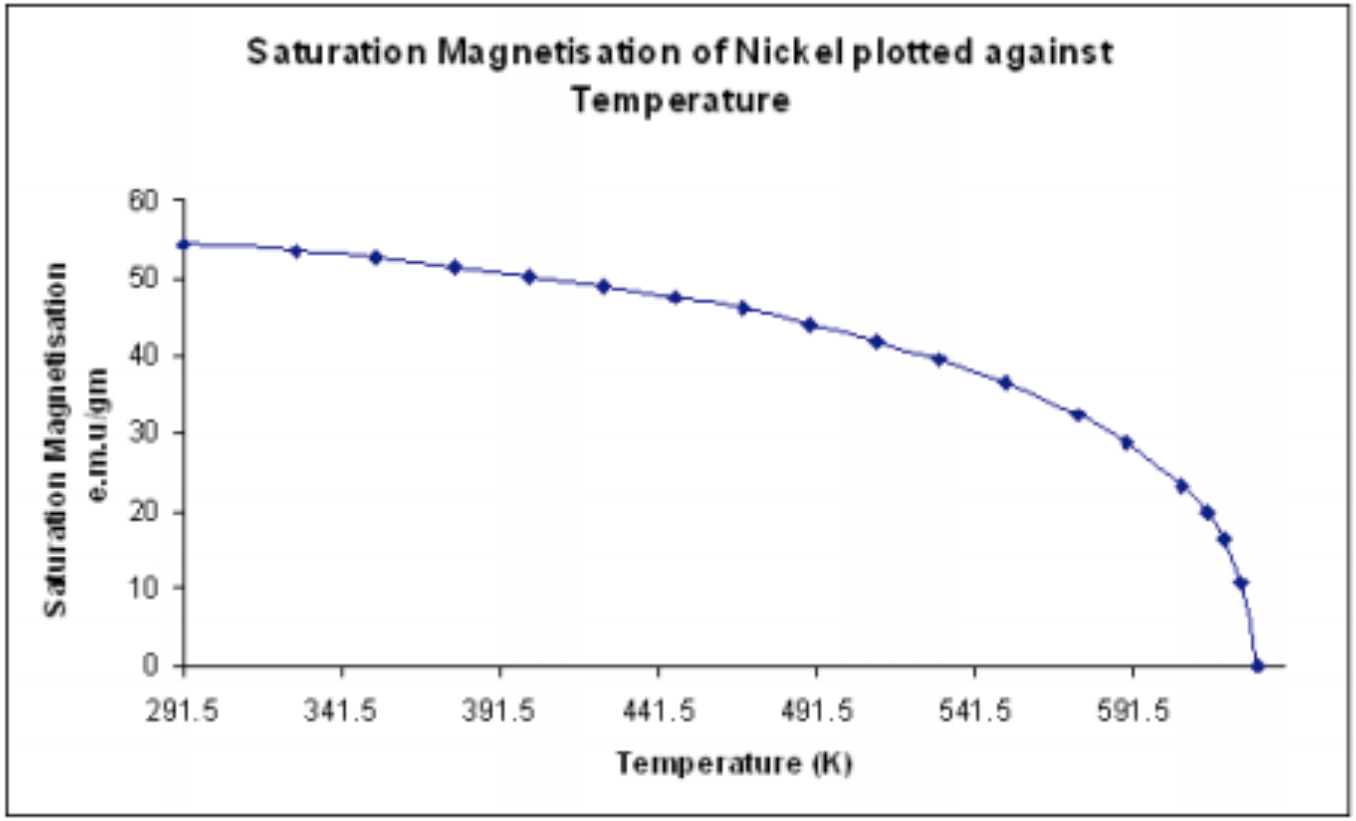


Figure J. Variation of saturation magnetisation with temperature for Nickel. (Data from Weiss and Forrer, 1926)

Curie-Weiss Law

In this region the substance is paramagnetic, and its susceptibility is given by

$$\chi = \frac{C}{T - T_C}$$

- Curie-Weiss law.

C ***- Curie constant***

T_C ***- Curie temperature***

The Curie-Weiss law can be derived using arguments (Weiss):

In the ferromagnetics the moments are magnetized spontaneously, which implies the presence of an internal field to produce this magnetization. The field is assumed that this field is proportional to the magnetization:

$$H_E = \lambda M \qquad \lambda \text{ is the } \textit{Weiss constant}$$

Weiss called this field the molecular field.

In reality, the origin of this field is the ***exchange interaction***.

Exchange interaction

- Consider for example the system of two electrons. Two possible arrangements for the spins of the electrons: either parallel or antiparallel.

If they are parallel - electrons remain far apart (exclusion principle).

If spins are antiparallel, the electrons may come closer together and their wave functions overlap considerably.

- These two arrangements have different energies because, when the electrons are close together, the energy rises as a result of the large Coulomb repulsion (an explanation of the first Hund rule).

$\rightarrow$ the electrostatic energy of an electron system depends on the relative orientation of the spins: the difference in energy defines the exchange energy.

- The exchange interaction is short-ranged $\Rightarrow$ only nearest neighbor atoms are responsible for producing the exchange field.

The magnitude of the exchange field is very large – of the order of 10^3 T

Consider the paramagnetic phase: an applied magnetic field H_0 causes a finite magnetization. This in turn causes a finite exchange field H_E.

If χ_P is the paramagnetic susceptibility, the induced magnetization is given by

$$M = \chi_p (H_0 + H_E) = \chi_p (H_0 + \lambda M)$$

Note that $M = \chi H$, where χ - constant holds only if the fractional alignment of magnetic moments is small: this is where the assumption is used that the sample is in the paramagnetic phase.

Solve the above equation for the magnitude of the magnetization M:

$$M = \frac{\chi_p H_0}{1 - \chi_p \lambda}$$

The paramagnetic susceptibility χ_P is given by the Curie law $\chi_P = C/T$, where C is the Curie constant.

Then the susceptibility of the ferromagnetic material is $\chi = \frac{M}{H_0} = \frac{C}{T - C\lambda} = \frac{C}{T - T_C}$

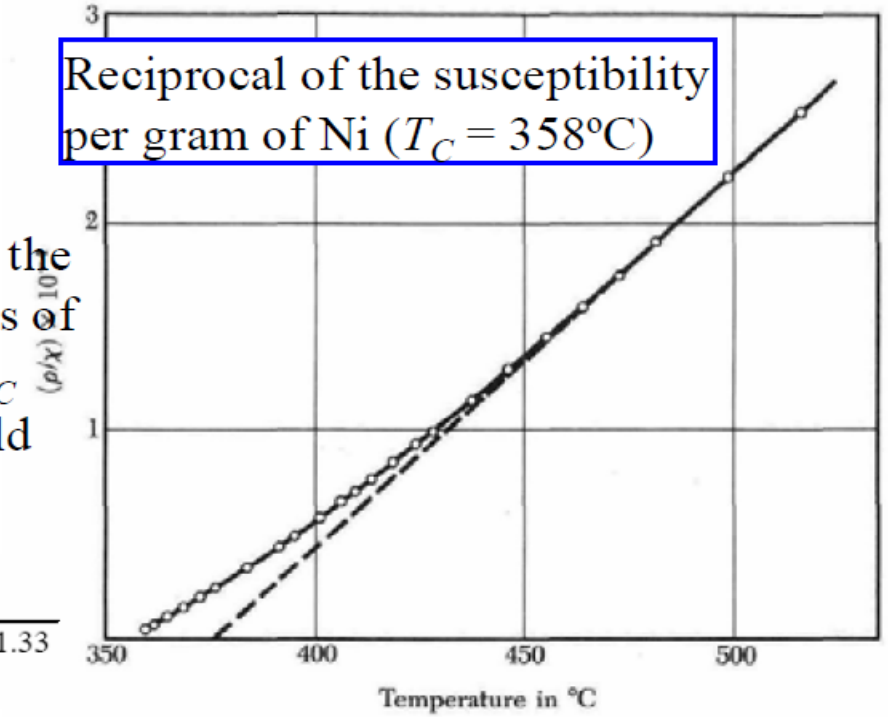
- Curie-Weiss law; $T_C = C\lambda$

The susceptibility (5) has a singularity at T_C.
At this temperature (and below) → a spontaneous magnetization
(χ is infinite ⇒ can have a finite M for zero B_0)

Last time, we obtained for C: $$C = \frac{\mu_0 N p^2 \mu_B{}^2}{3k_B}$$ where $p = g[J(J+1)]^{1/2}$

Then the Curie temperature is given by

$$T_C = \frac{\mu_0 N p^2 \mu_B{}^2 \lambda}{3k_B}$$



Deviations from Curie-Weiis only in the vicinity of the T_C - strong fluctuations of the magnetic moments close to the T_C can not be described by the mean field theory.

Accurate calculations: $$\chi \propto \frac{C}{(T - T_C)^{1.33}}$$

One can also use the mean field approximation below the T_C to find the magnetization as a function of temperature.

Proceed as before but instead of the Curie law for paramagnetics (valid for not too high magnetic fields and not too low temperatures) can use the complete Brillouin function.

Omit the applied magnetic field and replace H by the exchange field $E_E = \lambda M$

Get $$M = NgJ\mu_B B_J\left(\frac{\mu_0 gJ\mu_B \lambda M}{k_B T}\right)$$ where $B_J(x)$ is the Brillouin function.

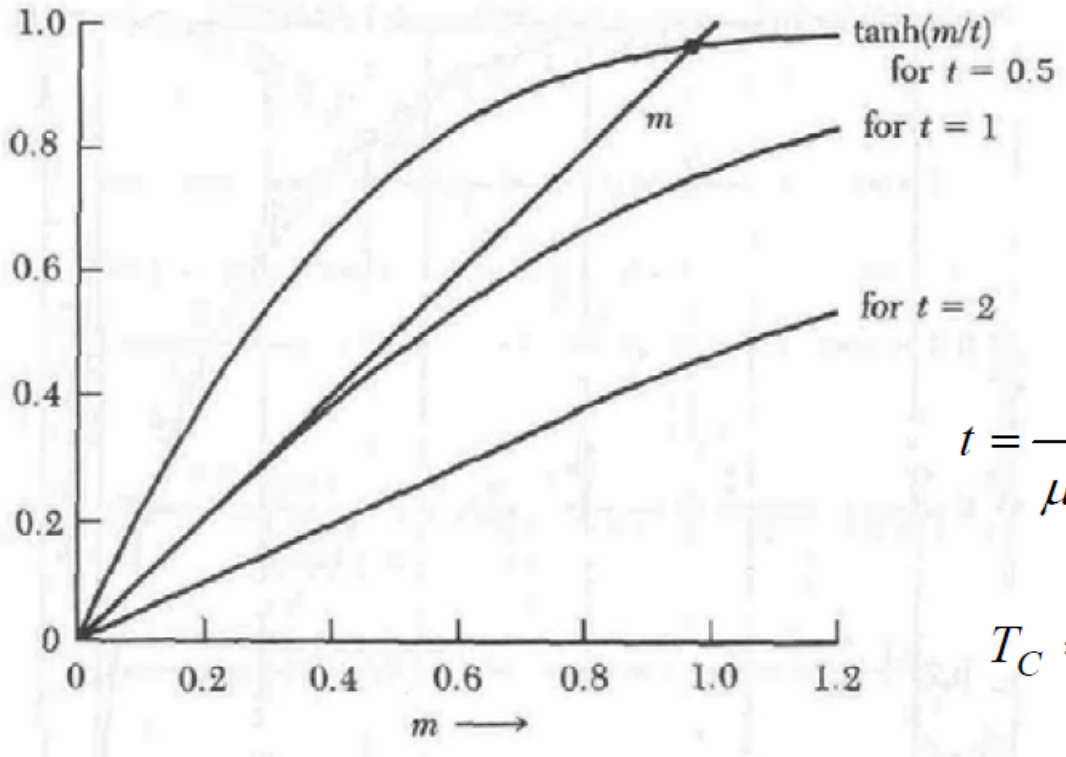
For J = s=1/2, it has the form $$M = Ng\mu_B \tanh\left(\frac{\mu_0 g\mu_B \lambda M}{k_B T}\right)$$

This is a transcendental equation in M, which can be solved numerically.

Write it the form $M = Ng\mu_B \tanh x$ where $x = \frac{\mu_0 g\mu_B \lambda}{k_B T} M$

or $m = \tanh\frac{m}{t}$ where $m = \frac{M}{Ng\mu_B}$ $t = \frac{k_B T}{\mu_0 N g^2 \mu_B{}^2 \lambda}$

Graphical solution of $m = \tanh\frac{m}{t}$



$$m = \frac{M}{Ng\mu_B}$$

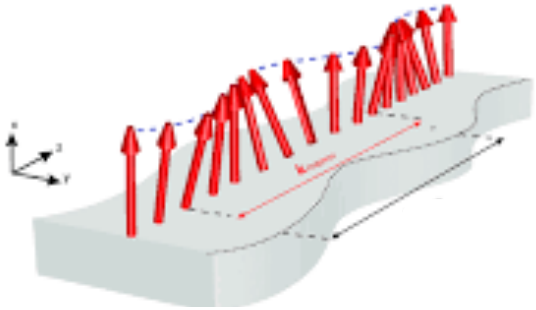
$$t = \frac{k_B T}{\mu_0 N g^2 \mu_B{}^2 \lambda} = \frac{T}{T_C}$$

$$T_C = \frac{\mu_0 N (g\mu_B)^2 \lambda}{k_B}$$

The curve for $t = 1$ (or $T = T_C$) is tangent to the straight line m at the origin; this temperature marks the onset of ferromagnetism.

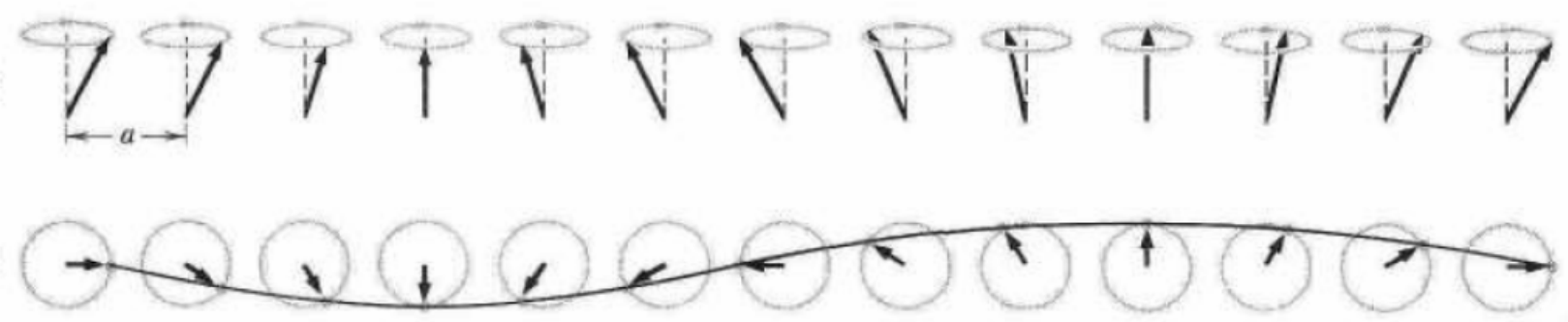
As $t \to 0$ the intercept moves up to $m = 1 \Rightarrow$ all magnetic moments are aligned at $T = 0$. Maximum magnetization: $m = 1 \Rightarrow M = Ng\mu_B$

Spin waves



When one of the spins is tilted or disturbed, however, it begins to precess – due to the field from the other spins.

Due to the exchange interaction between nearest neighbors the disturbance propagates as a wave through the system:



Lattice waves – phonons - atoms oscillate around their equilibrium positions, their displacements are correlated through lattice forces.

In spin waves – ***magnons*** - the spins precess around the equilibrium magnetization and their precessions are correlated through exchange forces.

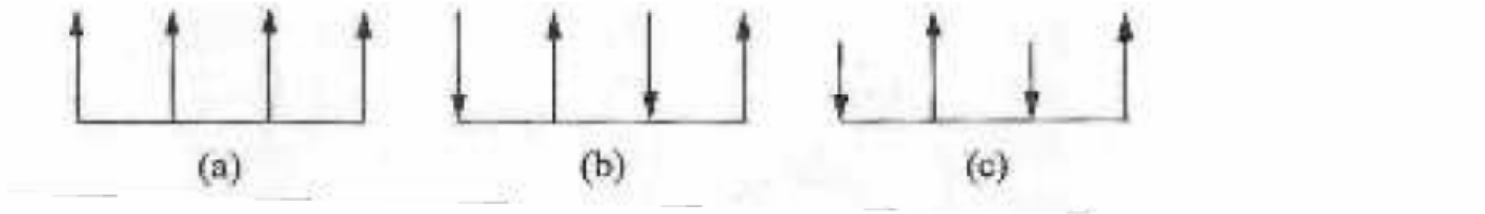
Ferrimagnetism

The exchange interaction model leads to ferromagnetism, if the constant J is positive: the parallel-aligned state has a lower energy then the antiparallel.

The negative constant J leads to antiferromagnetism or ferrimagnetism.

Antiferromagnetic arrangement: the dipoles have equal moments, but adjacent dipoles point in opposite directions ⇒ zero net magnetization.

Ferrimagnetic arrangement: pattern: neighboring dipoles point in opposite directions, but the moments are unequal ⇒ a finite net magnetization.



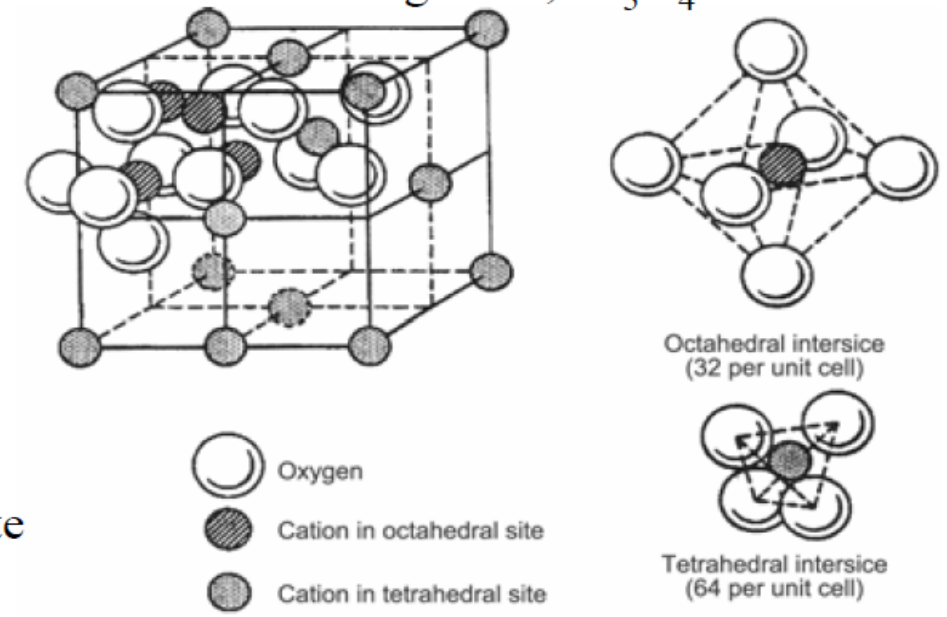
Magnetic arrangements: (a) ferromagnetic, (b) antiferromagnetic, (c) ferrimagnetic.

A typical example of a ferrimagnetic material is magnetite, Fe_3O_4.

More explicitly, $FeO \cdot Fe_2O_3 \rightarrow$
two types of iron ions:
Fe^{2+} (ferrous) and Fe^{3+} (ferric).

Spinel structure (AB_2O_4):

The unit cell contains 56 ions,
24 – iron; the rest - oxygen.



Fe ions - two different coordinate environments: a tetrahedral and octahedral (surrounded by 4 and 6 oxygen ions)

8 of the 16 Fe^{3+} ions in the unit cell are in each type of position

The tetrahedral structure has moments oriented opposite to those of the octahedral one $\rightarrow$ complete cancellation of the contribution of the Fe^{3+} ions.

$\Rightarrow$ the net moment is entirely due to the 8 Fe^{2+} ions (octahedral sites)

Each of these ions has six 3d electrons, whose spin orientations are $\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow$.
$\Rightarrow$ each ion carries a moment equal to $4\mu_B$.