PHY404- Solid State Physics II

MAGNETISM-PartIV
Superconductivity

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What is magnetism?

- It has been known since antiquity that “**loadstone**” (magnetite, Fe3O4) and iron attract each other.
- Plato (428/427–348/347 B.C.) and Aristotle mention permanent magnets.
- They are also mentioned in Chinese texts from the 4th century B.C.
- The earliest mention of a magnetic compass used for navigation is from a Chinese text dated 1040–1044 A.D., but it may have been invented there much earlier.
- It was apparently first used for orientation on land, not at sea.

What is magnetism?

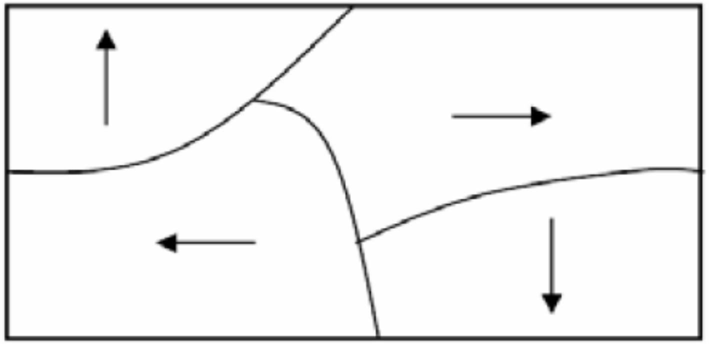
- Oersted (1819) found that a compass needle is deflected by a current-carrying wire in the same way as by a permanent magnet.
- This and later experiments led to the notion that the magnetization of a permanent magnet is somehow due to permanent currents of electrons.
- Biot, Savart, and Ampére established the relationship of the magnetic induction and the current that generates it.
- As we know, Maxwell essentially completed the classical theory of electromagnetism.

The magnetic moment of a free atom has three principal sources:

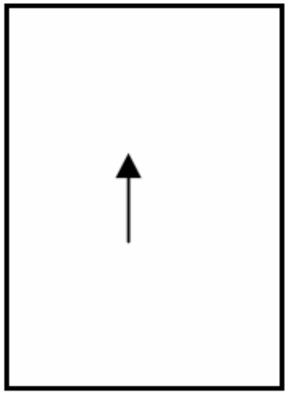
- **The spin** which electrons are endowed
- Their orbital angular momentum about the nucleus
- The change in the orbital moment induced by an applied magnetic field

Magnetic domains

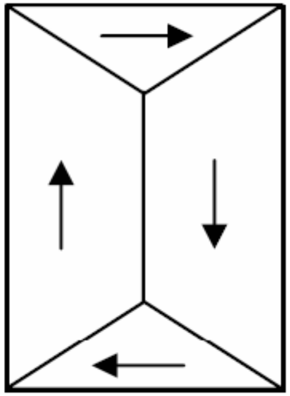
Spin in a material with long range magnetic ordering (ferromagnetic, antiferromagnetic etc.) form domains.



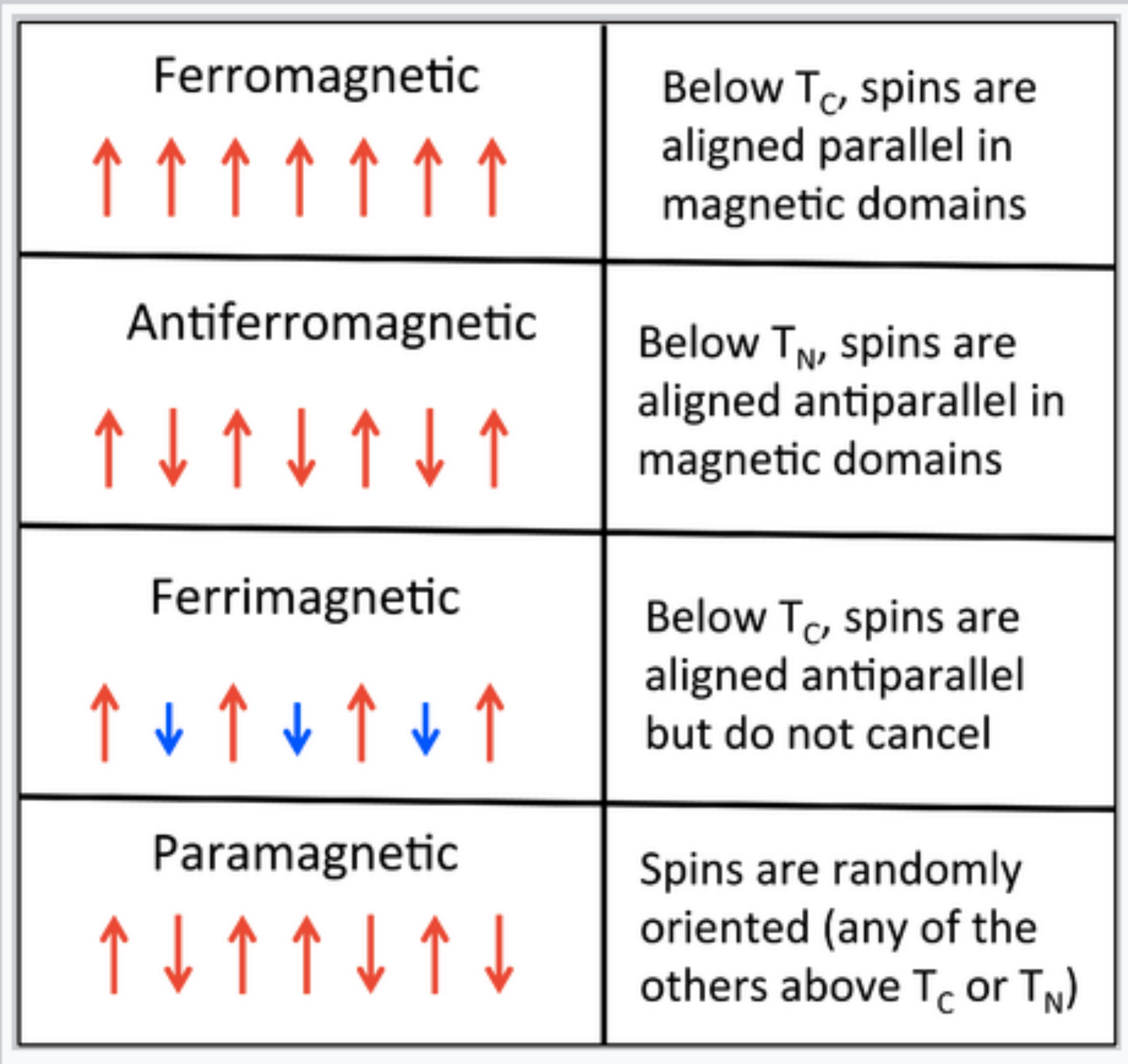
Reason for domain formation:



Higher energy



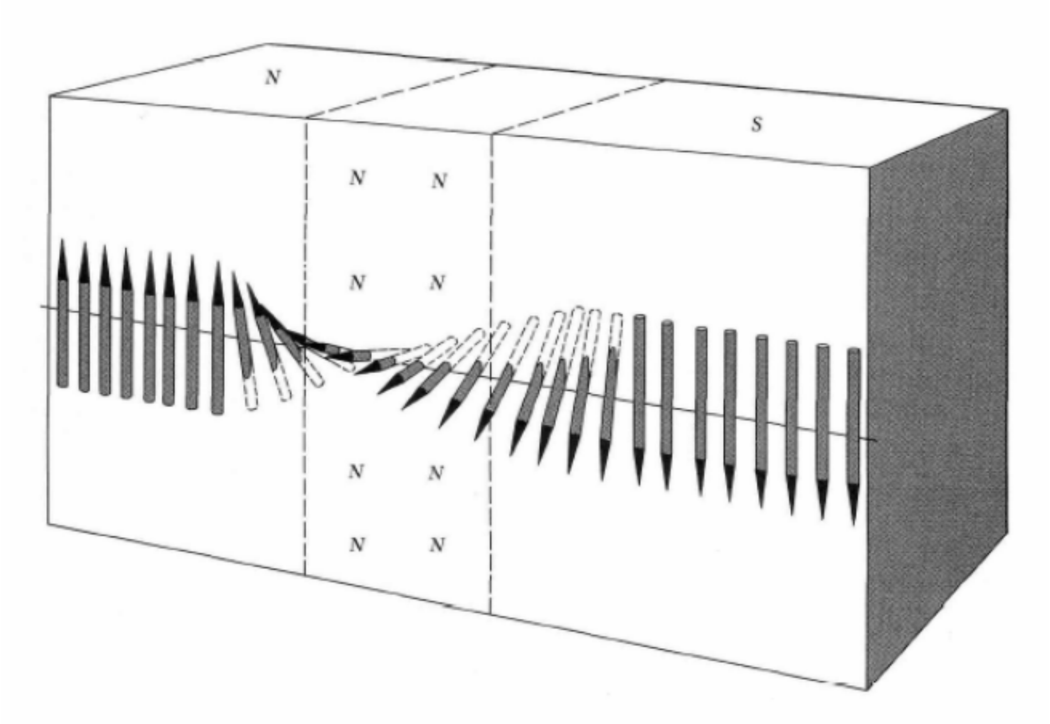
Lower energy



Ferromagnetic ↑ ↑ ↑ ↑ ↑ ↑ ↑	Below T_C, spins are aligned parallel in magnetic domains
Antiferromagnetic ↑ ↓ ↑ ↓ ↑ ↓ ↑	Below T_N, spins are aligned antiparallel in magnetic domains
Ferrimagnetic ↑ ↓ ↑ ↓ ↑ ↓ ↑	Below T_C, spins are aligned antiparallel but do not cancel
Paramagnetic ↑ ↓ ↑ ↑ ↓ ↑ ↓	Spins are randomly oriented (any of the others above T_C or T_N)

Bloch wall

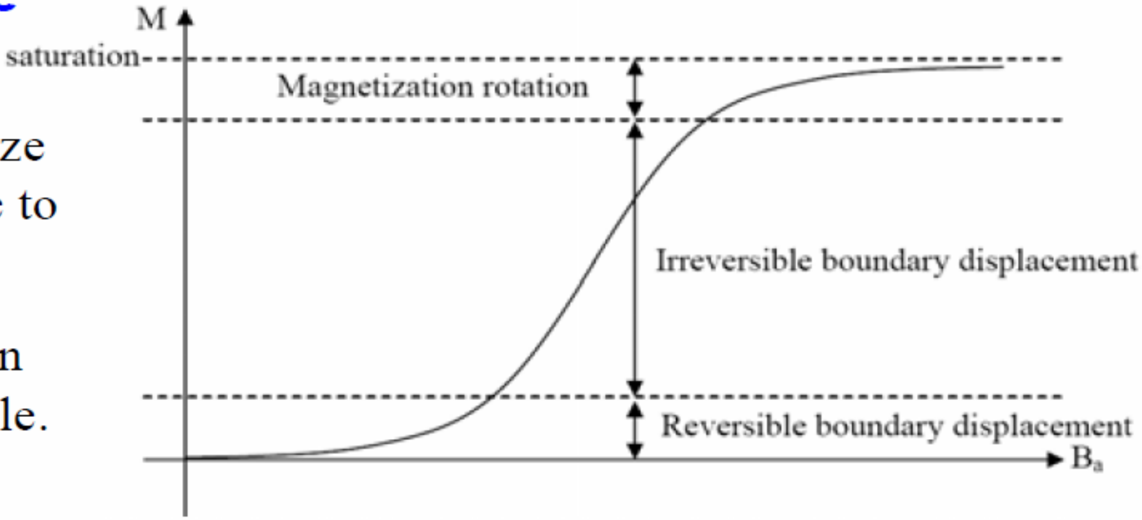
The boundary regions between neighboring domains



The typical size of the domains is 1-100 μm;
the width of the domain walls is much smaller: ~100 nm.

→ the domain structure consists of uniformly magnetized domains separated by narrow boundaries.

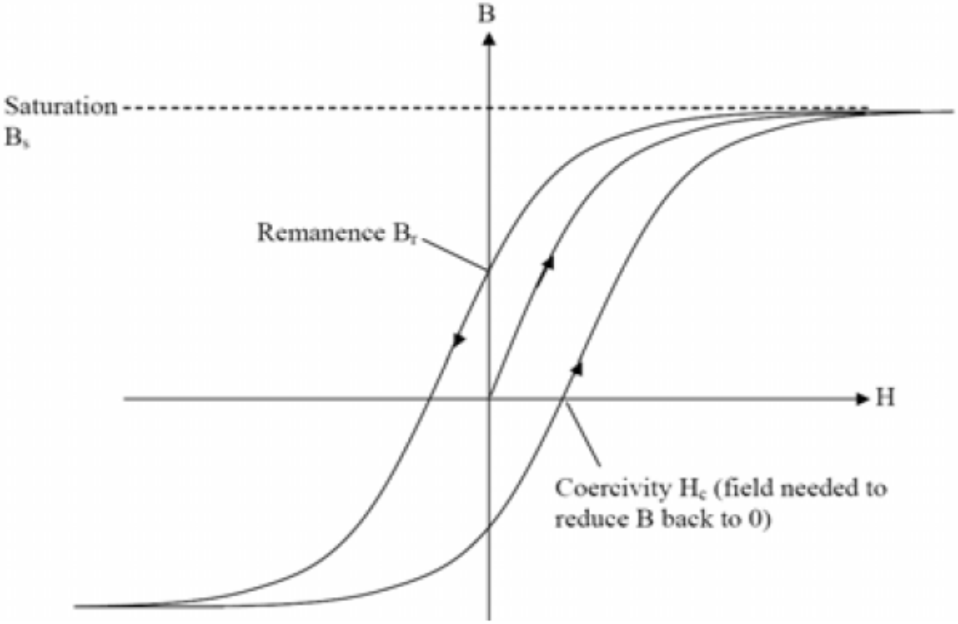
Magnetization curve



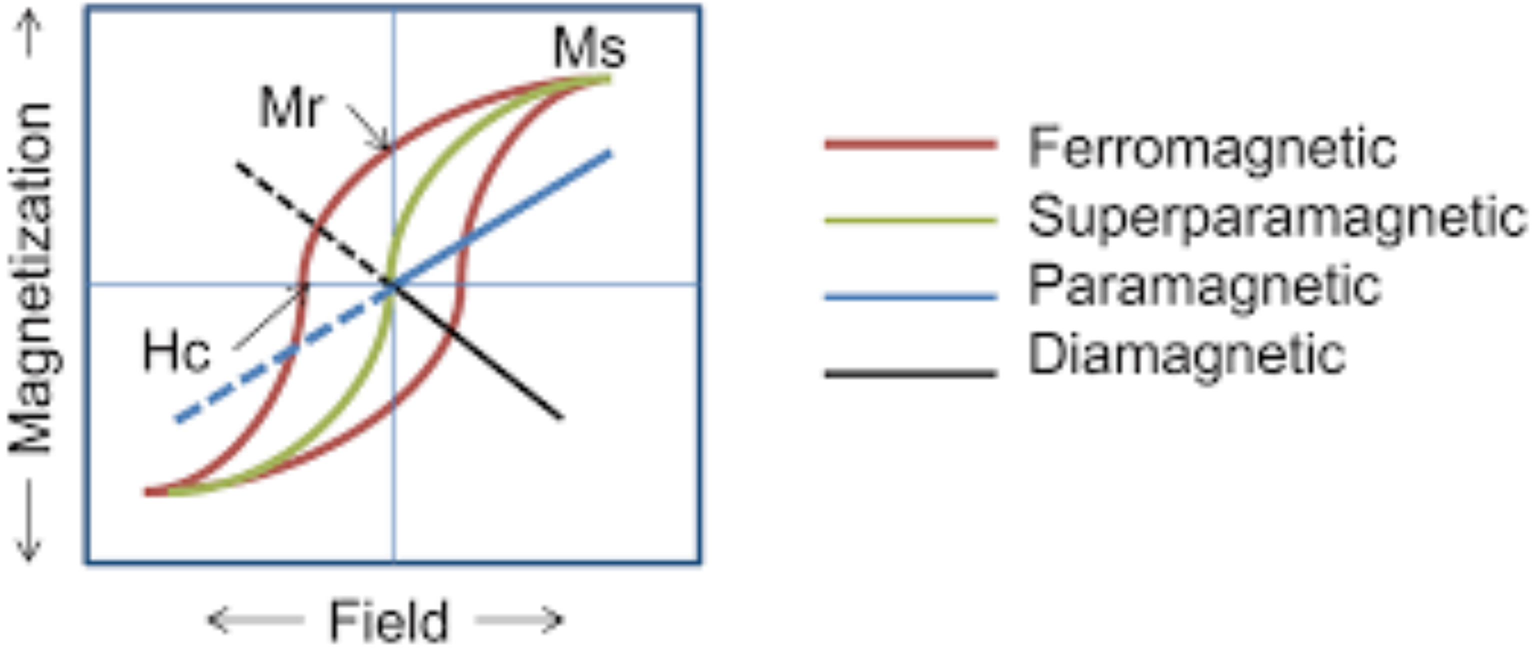
For small field, domain size will change in accordance to the direction of the field.

Change in domain size can be reversible or irreversible.

For large field, domain magnetization will re-align with the external field.



Irreversible boundary displacement and magnetization rotation are the causes of ***hysteresis***

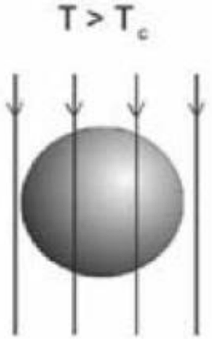

Ms
Mr
Hc
Magnetization
Field
Ferromagnetic
Superparamagnetic
Paramagnetic
Diamagnetic

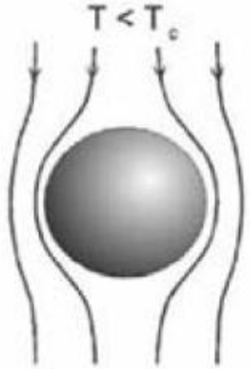
Superconductivity

- In a superconductor, the currents effectively run forever – there are no collisions to slow them down (measurements by File and Mills suggest that the decay time of a supercurrent through a solenoid is no less than 100 000 years)
- Another odd property of superconductors:

<u>The Meissner Effect</u>

- If a superconducting sample is cooled in a small magnetic field, the magnetic field lines will be expelled from the sample (due to the supercurrents forming in a direction to oppose the field, and therefore the field inside the superconductor is zero)

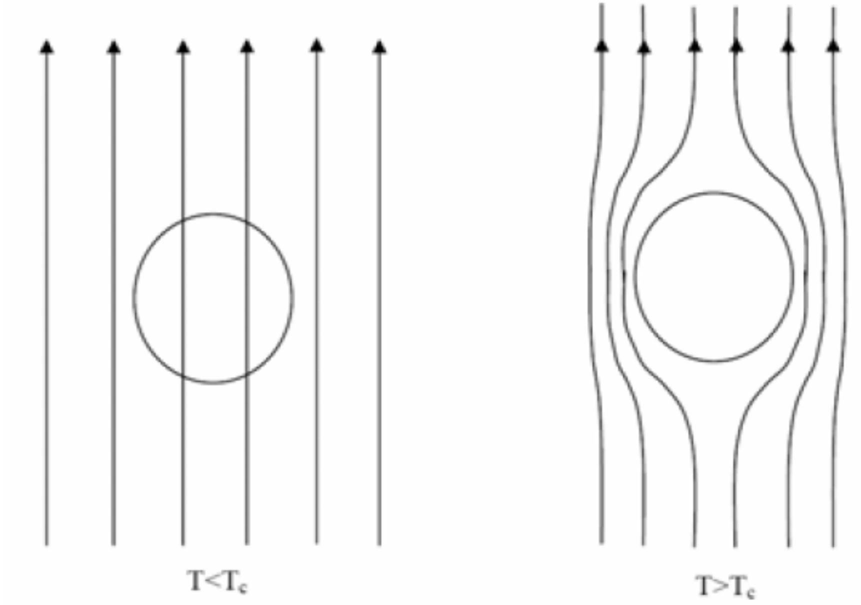




Perfect diamagnetism

Second basic property of superconductors – they expel magnetic field completely when in superconducting phase ($T<T_c$).

This phenomenon is called ***the Meissner effect***.



$$\boldsymbol{B} = \mu_o(\boldsymbol{H} + \boldsymbol{M}) = 0$$

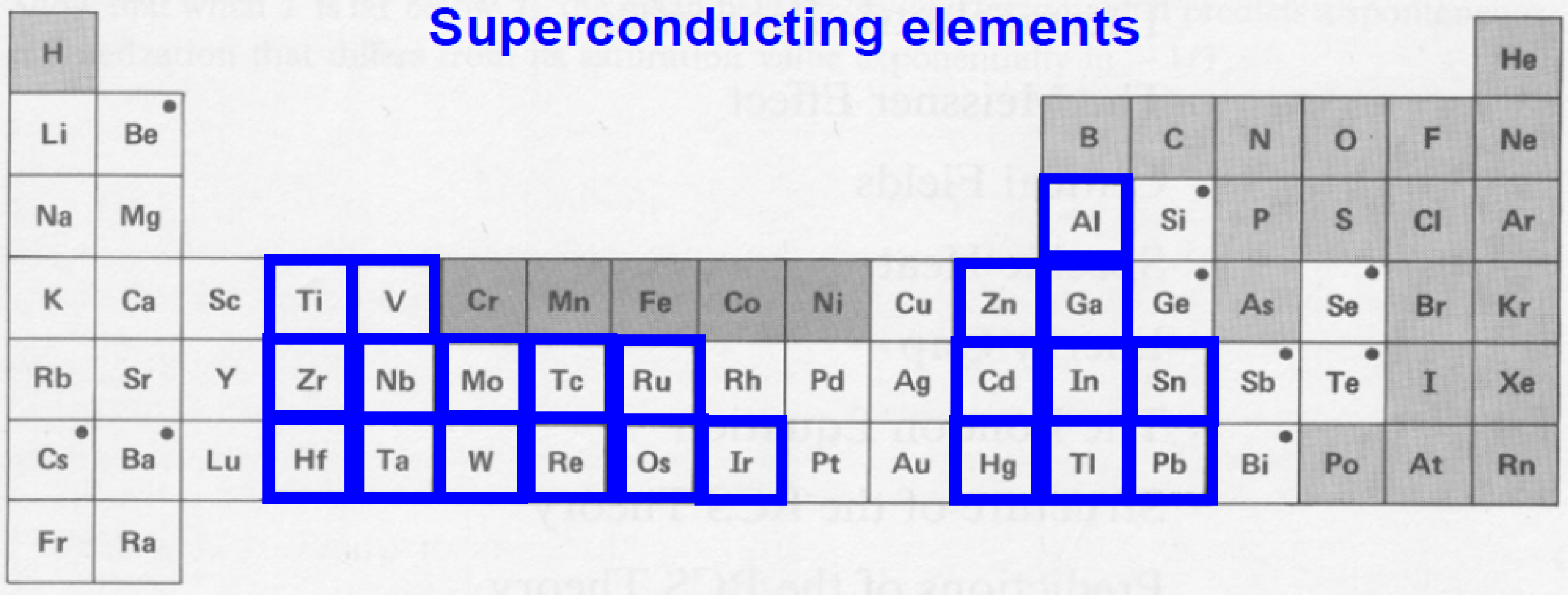
$$\chi = \frac{M}{H} = -1$$

Distinguishes the superconductor from an ideal but normal conductor, for which $dB/dt = 0$

Could explain the expulsion of the flux if a S/C is moved into a magnetic field → motion of metal produces currents, which do not decay because $R = 0$

However, $R = 0$ does not explain the flux expulsion when a S/C is already in a magnetic field and is cooled from its normal to S/C state (through T_c)

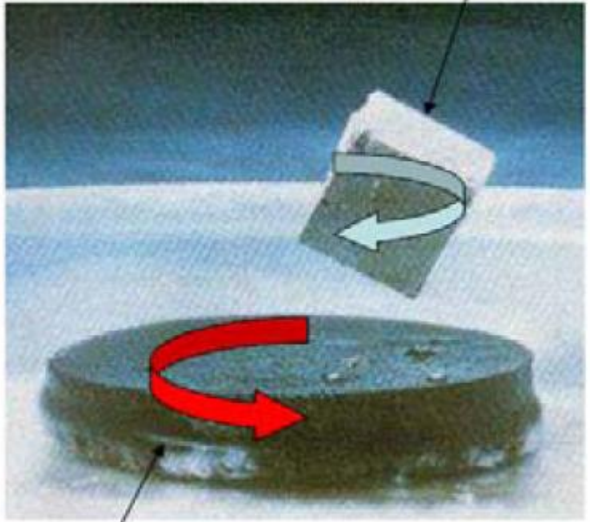
Superconducting elements



- More than 20 metallic elements are superconductors
- Cu, Au, Ag, Na, K and magnetically ordered metals (Fe, Ni, Co) are *not superconductors*
- Certain elements are superconducting at high pressures or as thin films
- Highest T_c of an element is 9.3 K for Nb
- There are thousands of alloys and compounds that exhibit superconductivity
- The highest T_c superconductors tend to be poor conductors in the normal state
- Record T_c is currently ~ 138 K (a ceramic consisting of Tl, Hg, Cu, Ba, Ca, Sr, and O)

Magnetic Levitation

- This is what causes the levitation of magnets above superconducting samples (the supercurrents form to counterbalance the magnetic force, and when the forces are equal and opposite, the magnet floats)
- Potential application: levitation of magnetic trains (no friction)



Electrons in magnet, which create a fixed magnetic field

Superconducting electrons in sample
(in a direction which counters the magnet to expel the magnetic field)

Maglev Train

The China's new maglev train

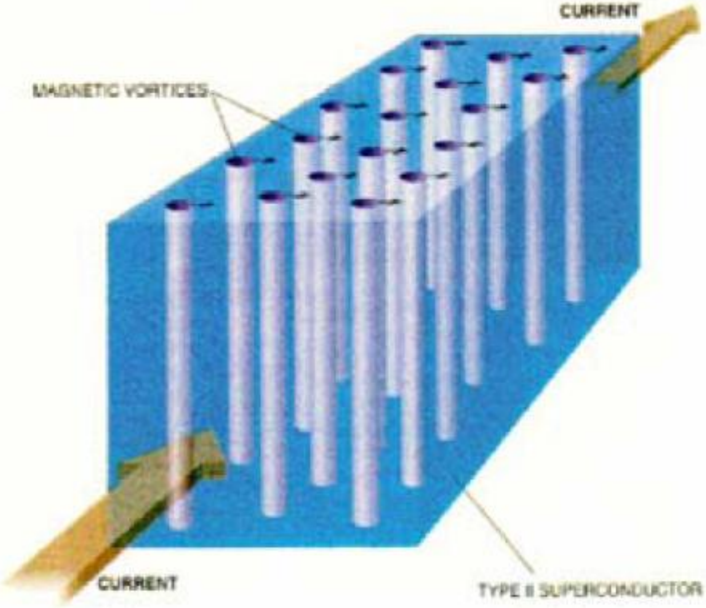
High speed (600km/h)

Maglev train, also called magnetic levitation train, a floating vehicle for land transportation that is supported by either electromagnetic attraction or repulsion.

Type I and Type II Superconductors

- Type I superconductor: A field can be applied to some maximum value before it becomes "normal". The field does not penetrate the superconductor (Meissner effect). Most metals belong to this class (eg. Zn, V, Ti)
- Type II superconductor: A field can be applied up to a critical value, H_{C1}, where the field lines penetrate the sample. This is known as the vortex state. After the field is increased to H_{C2}, the material is no longer superconducting. These are the "high-T_c" superconductors, like $YBa_2Cu_3O_7$.

Penetration of magnetic field lines in a type II superconductor



Magnetic properties: type I superconductors

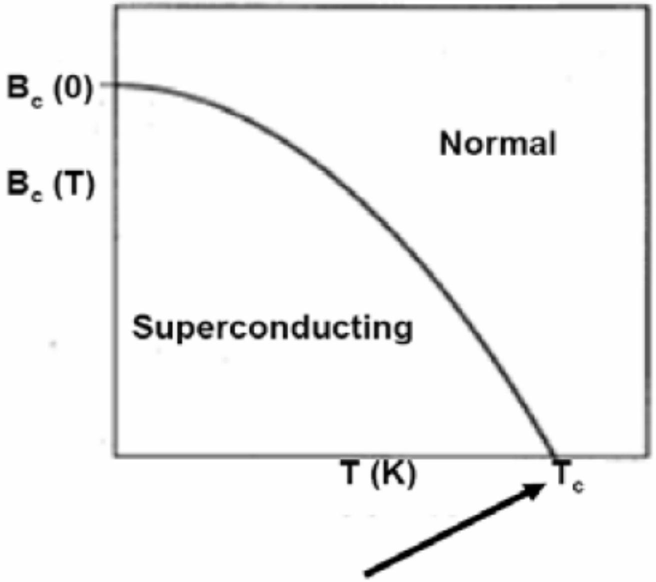
Superconductors are divided into 2 types, depending on their behavior in a magnetic field.

All superconducting elements are type I, except Nb

Superconductivity is destroyed by the presence of some magnetic field, ***the critical field***, B_c.   Typically B_c ~ tens of mT for type I

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

B-T phase diagram



Critical field goes to zero as $T \to T_c$

Field B_c does not need to be external:

Critical current I_c causes a magnetic field B_c, which destroys superconductivity

Penetration depth

Surface currents expel magnetic flux from type I superconductors

Currents actually penetrate the sample slightly (~ 100 nm)

Magnetic field decreases exponentially inside sample:

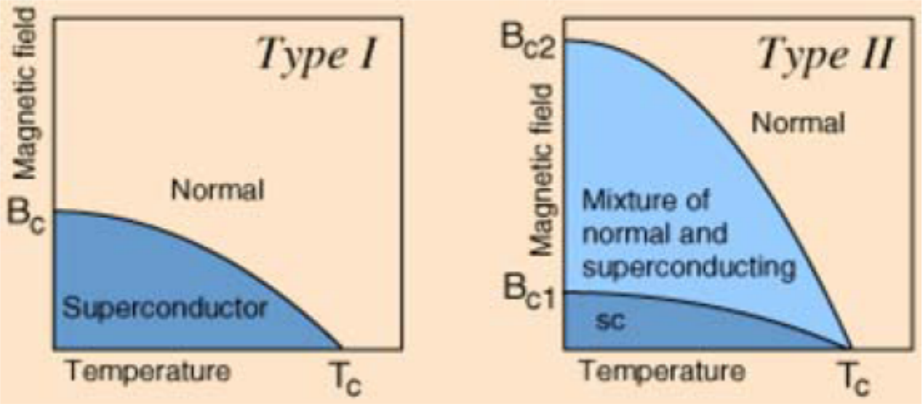
$$B(x) = B_o\, e^{-x/\lambda} \qquad \lambda(T) = \frac{\lambda(0)}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}}$$

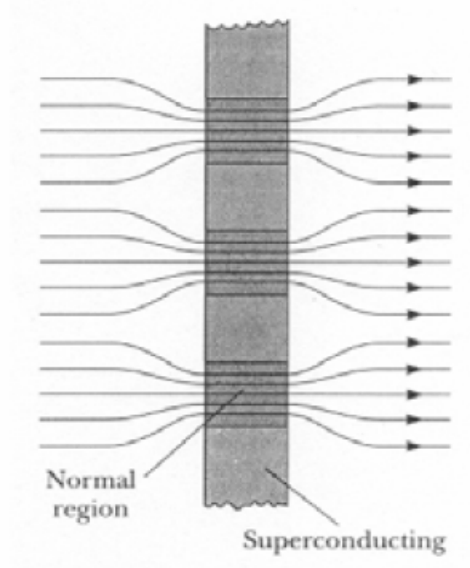
As $T \rightarrow T_c$, λ increases and B penetrates deeper into the sample.

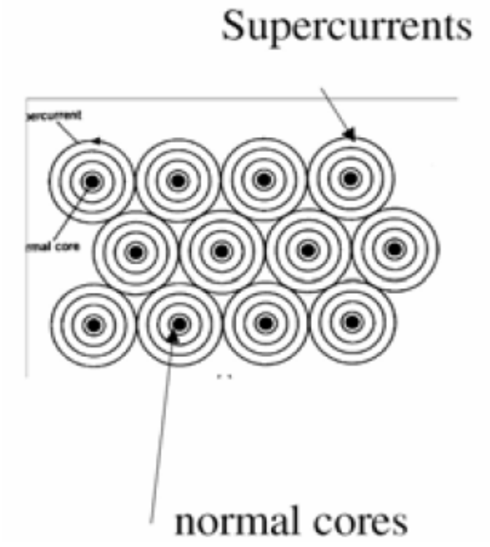
At T_c, $\lambda \rightarrow \infty$, and the sample goes normal

Type II superconductors

- Alloys (and Nb and V)
- Higher T_C, B_C, I_C than type I
- More applications e.g.Nb$_3$Sn magnets
- 2 critical fields:
  $B < B_{C1}$ - just like a type I
  $B > B_{C2}$ - normal
  Intermediate fields – sample has both superconducting and normal regions



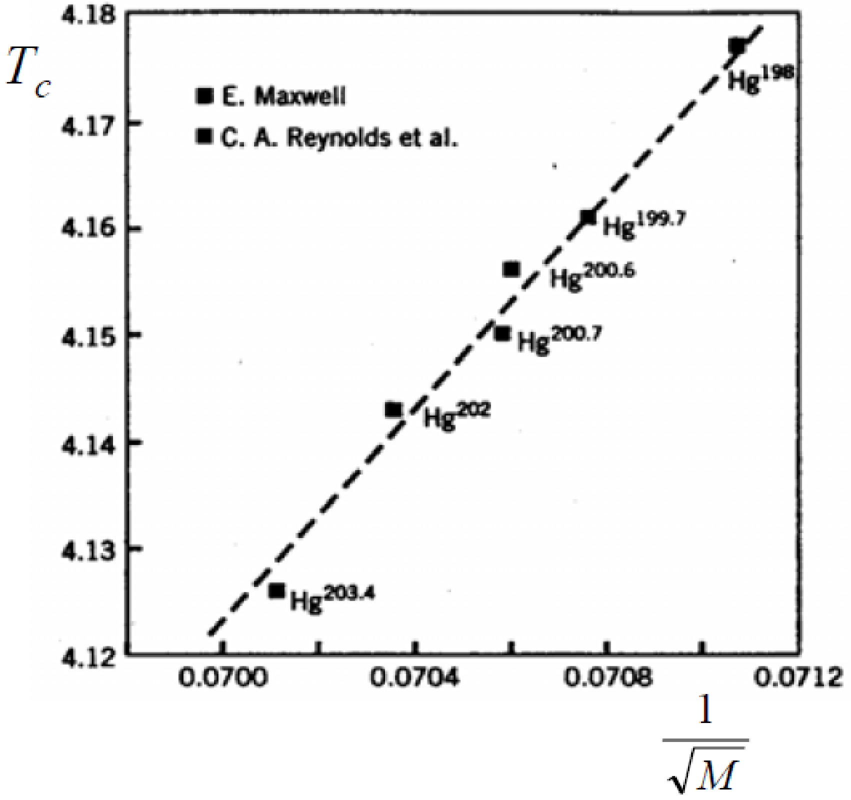




Isotope effect

- T_C depends on atomic mass: $$T_c \propto \frac{1}{\sqrt{M}}$$
- More general: $T_c M^a = constant$
- Suggests superconductivity is not just an electronic effect

Hg - Maxwell, Reynolds et al. (1950)



Isotope effect: 1st direct evidence of interaction of electrons and the lattice

Suggests that lattice vibrations play a part in the superconducting process

Experimental evidence for an energy gap – heat capacity

Specific heat of normal metal has form:

$$AT + BT^3$$

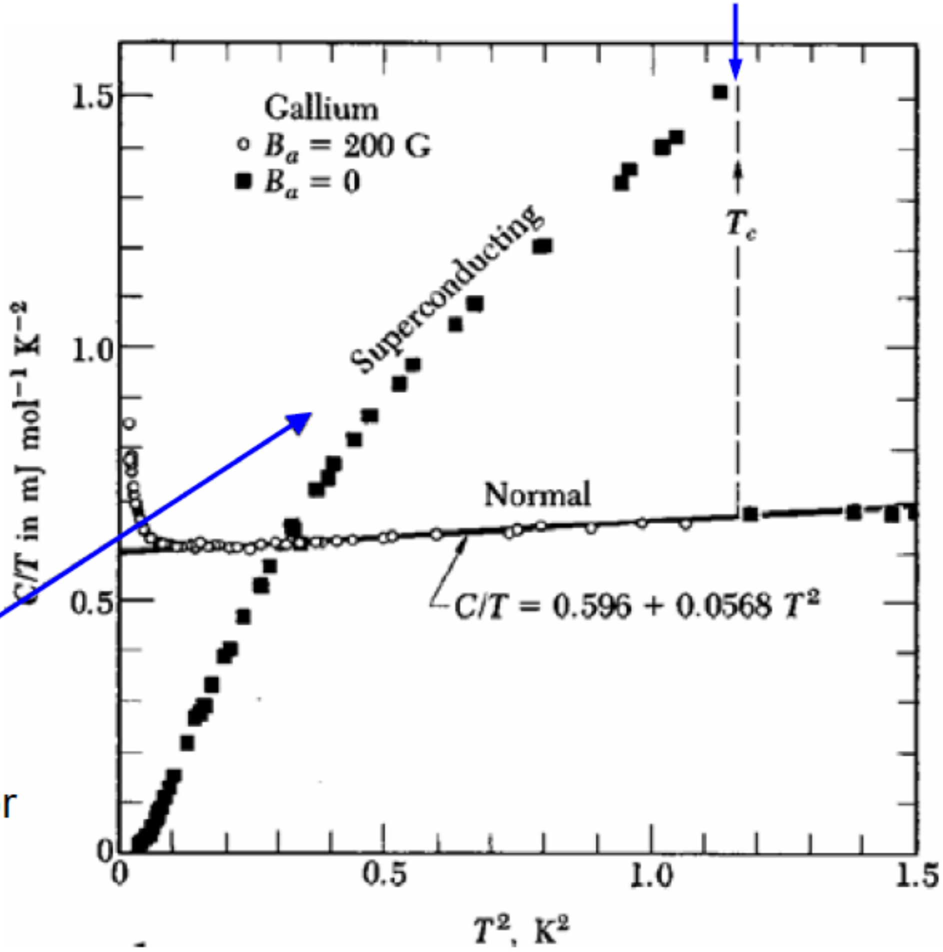
Electronic vibrations ***Phonons***

Exponential T-dependence

$$C \propto e^{-b/kT}$$

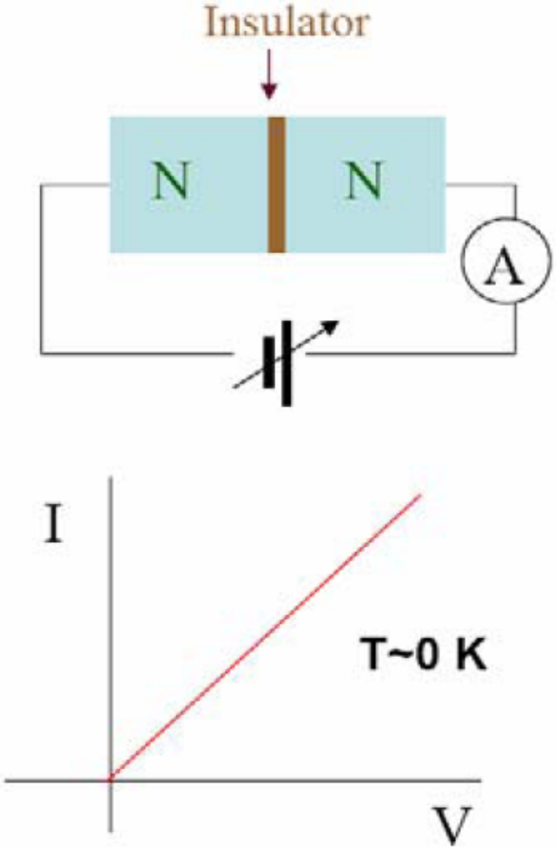
Characteristic of thermal behavior of a system whose excited levels are separated from the ground state by an energy 2Δ

Discontinuity at T_C



From Kittel-Phillips

Experimental evidence for an energy gap – tunneling

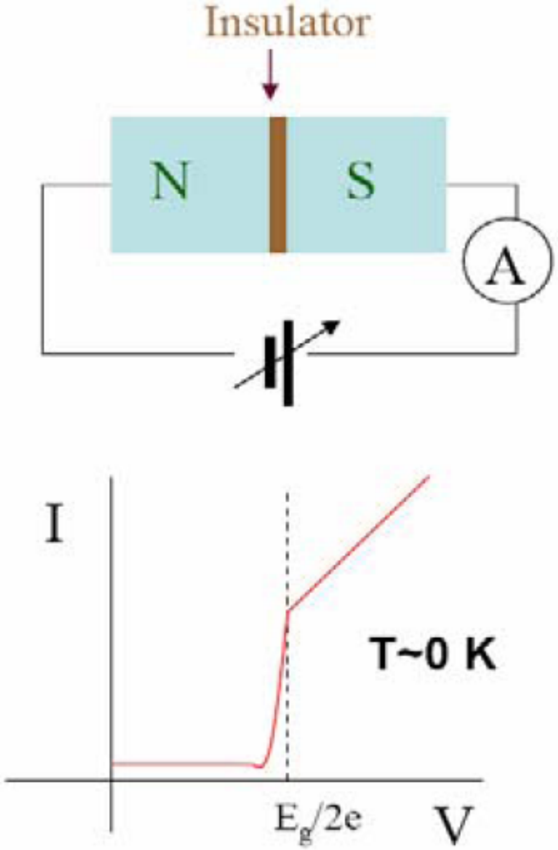


Ohm's law obeyed

Thin insulator → electrons tunnel

As T increases towards T_c, threshold voltage decreases

⇒ *energy gap decreases with increasing T*



No current flows below $eV=\Delta$

Theory

Phenomenological:

- F & H. London (1935)
- Ginzburg& Landau (1950)

Quantum:

- Fröhlich (1950)
- Bardeen, Cooper & Schrieffer, BCS (1957)

London model

- Using two fluid model of Gorter and Casimir:
  Assume only a fraction of electrons $n_s(T)/n$ participate in supercurrent
- $n_s(T)$ is the ‘density of superconducting electrons:
  $n_s \sim n$ at $T<<Tc$, $n_s \rightarrow 0$ at $T \rightarrow T_c$
- $n - n_s$ electrons exhibit normal dissipation
- Current and supercurrent flow in parallel ⇒ superconducting electrons carry all current, normal current is inert and can be ignored

London equations

In an electric field $\mathbf{E}$, S/C electrons will accelerate without dissipation, so we can relate the mean velocity v_s to the current density $\mathbf{j}$:

$$m\frac{d\mathbf{v}_\mathbf{s}}{dt} = -e\mathbf{E} \quad \text{using} \quad \mathbf{j} = -e\mathbf{v}_\mathbf{s} n_s \quad \text{get} \quad \boxed{\frac{d}{dt}\mathbf{j} = \frac{n_s e^2}{m}\mathbf{E}}$$

1st London equation

In a steady state, $\mathbf{j} = \mathrm{const} \Rightarrow \mathbf{E} = 0$ Electric field inside a S/C vanishes

Maxwell's equation: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = 0 \Rightarrow \mathbf{B} = \mathrm{const}$

These equations describe the magnetic fields and current densities within a perfect conductor, but they are incompatible with the Meissner effect.

From the above, have

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{m}{n_s e^2}\nabla \times \frac{\partial \mathbf{j}}{\partial t}$$

London assumed that

(2nd London equation)

$$\boxed{\mathbf{B} = -\frac{m}{n_s e^2}\nabla \times \mathbf{j}}$$

i.e. to successfully predict the Meissner effect the constant of integration must be chosen to be zero

Combining equation $\mathbf{B} = -\frac{m}{n_s e^2} \nabla \times \mathbf{j}$ and $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

and using $\nabla \times \nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - (\nabla \cdot \nabla)\mathbf{B} = -\nabla^2 \mathbf{B}$

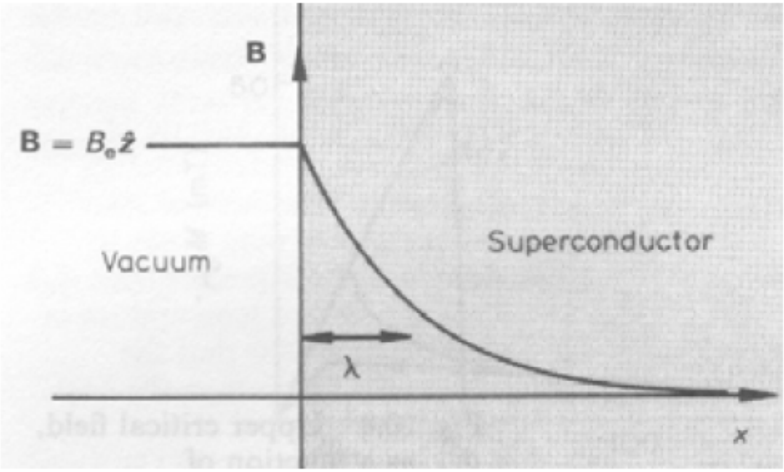
get $\nabla^2 \mathbf{B} = \frac{\mu_0 n_s e^2}{m} \mathbf{B}$ and $\nabla^2 \mathbf{j} = \frac{\mu_0 n_s e^2}{m} \mathbf{j}$

Solution (one-dimensional case): $B(x) = B_o \, e^{-x/\lambda}$

$$\lambda = \left(\frac{m}{\mu_0 n_s e^2} \right)^{1/2}$$

- the London penetration depth

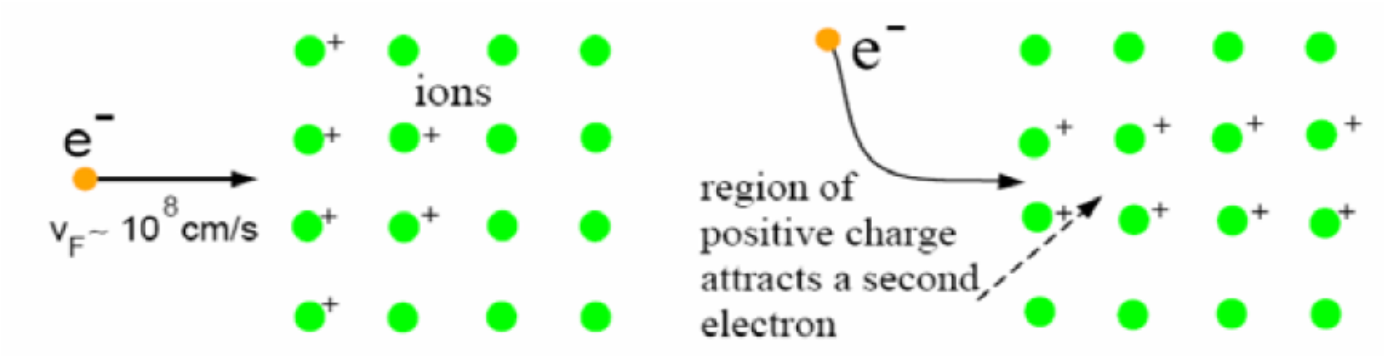
i.e. the Meissner effect is predicted



Solution for $\mathbf{j}$ gives a *surface current* – exponentially decaying into a S/C

BCS theory

- Fröhlich (1950): e-e attraction via phonons (…. Isotope effect)
- Cooper (1956): electrons just above the Fermi surface form bound pairs
- Most stable when center of mass is at rest and total spin = 0, So, $+\boldsymbol{k}\uparrow$ and $-\boldsymbol{k}\downarrow$
- Attractive interaction is provided by lattice vibrations – phonons
- First electron deforms the lattice and second electron is then attracted by the deformation (i.e. the changed positive charge distribution)



- Time-scales: electron motion ~ 10^{-16} s; lattice deformed for ~10^{-13} s
  In this time, first electron has traveled $\sim v_F \tau \sim 10^6 ms^{-1} \times 10^{-13} s \sim 1000\text{Å}$

Lattice deformation attracts 2nd electron without it feeling the Coulomb repulsion of the 1st

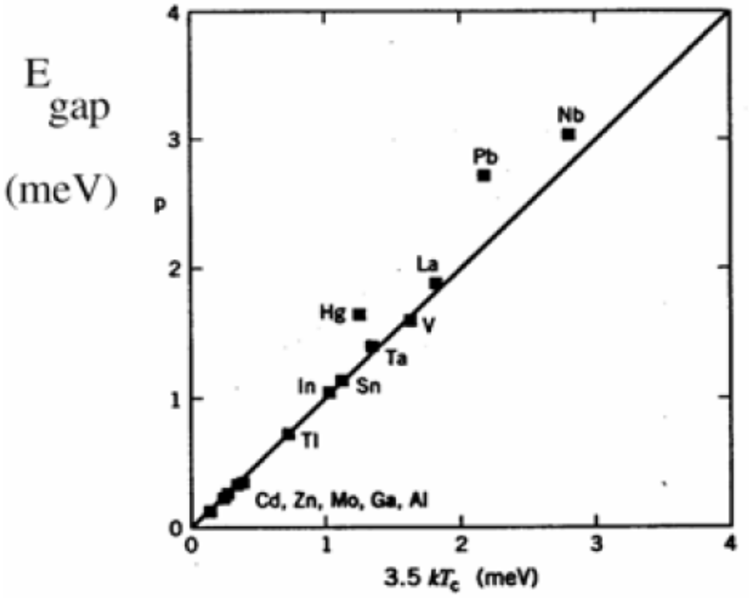
- Cooper calculation: solve Schrödinger eq.for 2 interacting electrons in the presence of a Fermi sphere of non-interacting electrons. Only effect of *N-2* electrons - restrict $\boldsymbol{k}$ values of e-e pair to be $> \boldsymbol{k}_F$, i.e. outside the Fermi sphere
- Cooper pair – boson.
- A single, coherent wave function extending over entire system. Can't change momentum of a pair without changing all pairs
- Bardeen, Cooper and Schrieffer (BCS) → extend Cooper's theory, construct a ground state where all electrons form bound pairs
- Each electron now has 2 roles:
  - Provide restriction on allowed wavevectors via Pauli principle
  - Participate in bound pair (called a Cooper pair)
- Electron-phonon interactions: responsible for resistance of metals *and* superconductivity
- Superconductors are generally poor conductors in normal state

Energy gap: a Cooper pair has a lower energy than 2 individual electrons. BCS gives

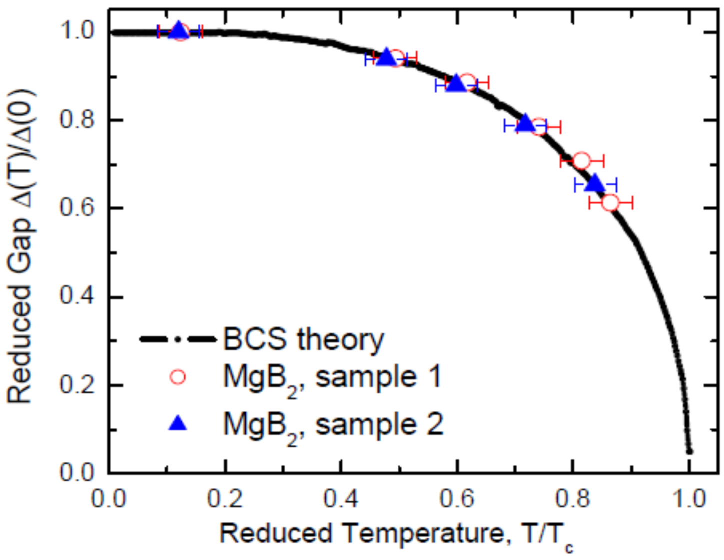
$$E_{gap}(T=0)=3.53k_BT_c$$

$$E_{gap}\sim 10^{-4}E_F$$

BCS calculated gap



T-dependence of energy gap



Summary

- When a superconductor is cooled below the critical temperature (T_C), it enters a new state, in which its resistance vanishes.
- Superconductors expel magnetic field completely when in superconducting phase – the Meissner effect
- When a magnetic field higher than a certain value called *the critical field* (B_c) is applied to a superconductor, it reverts to a normal state
- Type I and type II superconductors are distinguished by their behavior in a magnetic field. In a type II S/C there are 2 critical fields. At intermediate fields, the material has both superconducting and normal regions
- Electrodynamics of superconductors is described by phenomenological London equations
- BCS theory – microscopic mechanism for superconductivity through the formation of *e-e* Cooper pairs via electron-phonon interaction.

  A Cooper pair has a lower energy than 2 individual electrons. The energy difference is 2Δ - energy gap.