FEEDBACK CONTROL SYSTEMS

LECTURE NOTES-2

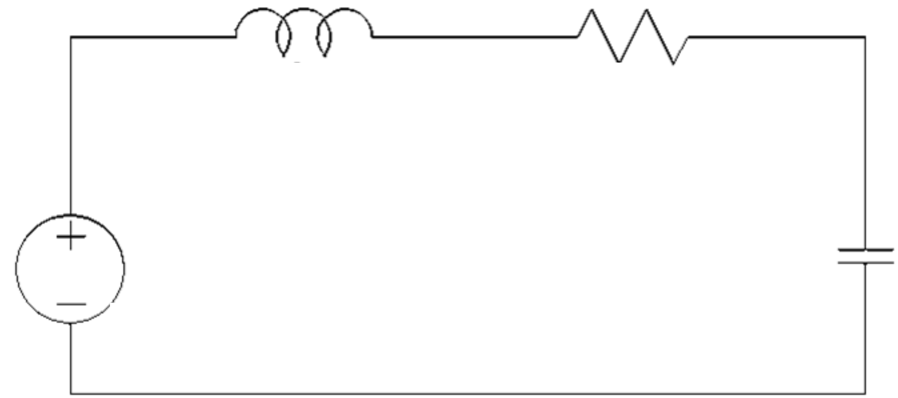
Electrical Network Modeling

Electric Circuits

Capacitor	**Resistor**	**Inductor**
$v(t) = \frac{1}{C}\int i(t)dt$	$v(t) = Ri(t)$	$v(t) = L\frac{di(t)}{dt}$
$i(t) = C\frac{dv(t)}{dt}$	$i(t) = \frac{1}{R}v(t)$	$i(t) = \frac{1}{L}\int v(t)dt$
$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{Cs}$	$Z(s) = \frac{V(s)}{I(s)} = R$	$Z(s) = \frac{V(s)}{I(s)} = Ls$

Example:

Obtain the transfer function Vc(s)/V(s)



I

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t)$$

$$LsI(s) + RI(s) + \frac{1}{C}\frac{1}{s}I(s) = V(s)$$

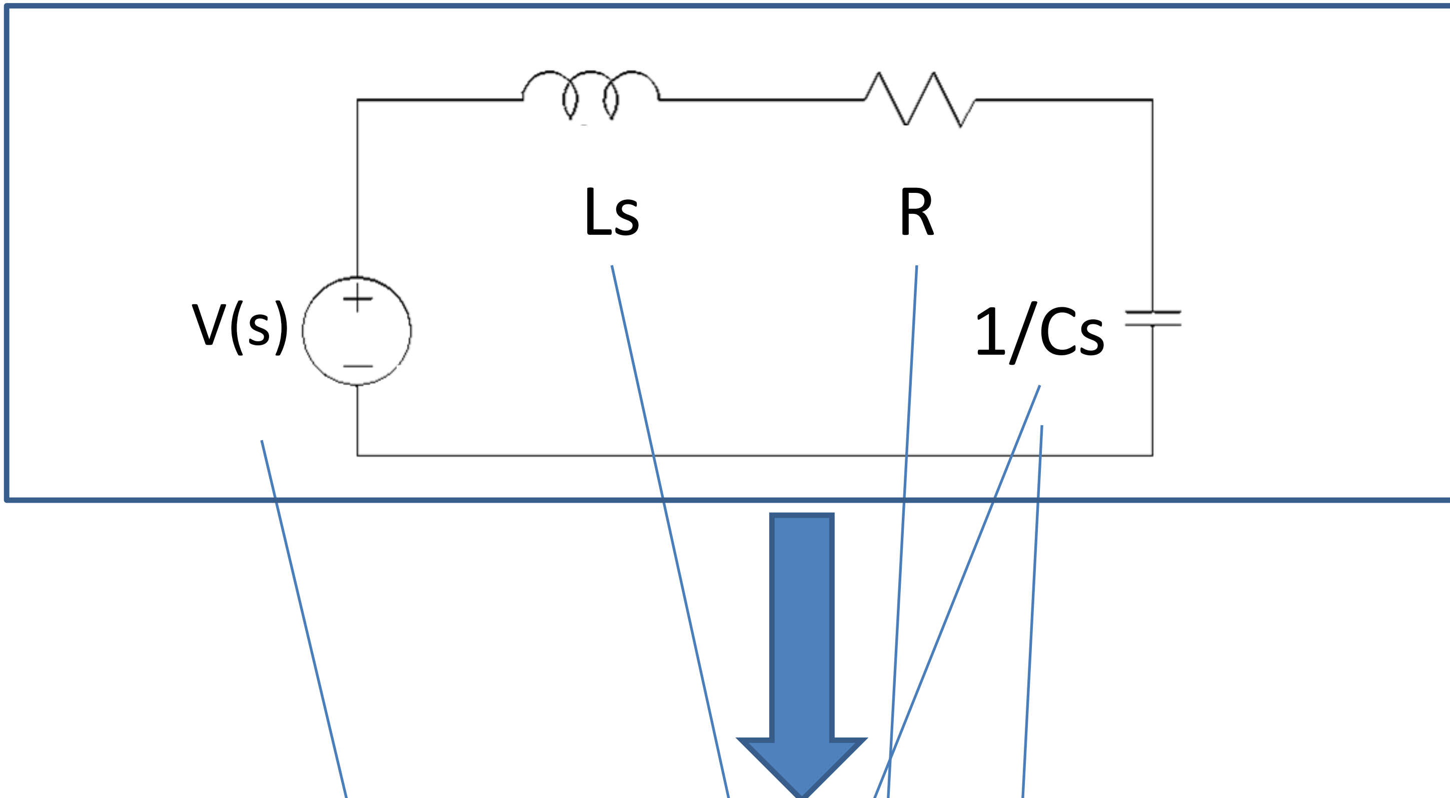
II

$$\frac{1}{C}\int_0^t \mathrm{i}(\tau)\mathrm{d}\tau = v_c(t)$$

$$\frac{1}{C}\frac{1}{s}I(s) = Vc(s)$$

$$\frac{Vc(s)}{V(s)} = \frac{\frac{1}{C}\frac{1}{s}I(s)}{I(s)\left(Ls + R + \frac{1}{C}\frac{1}{s}\right)}$$

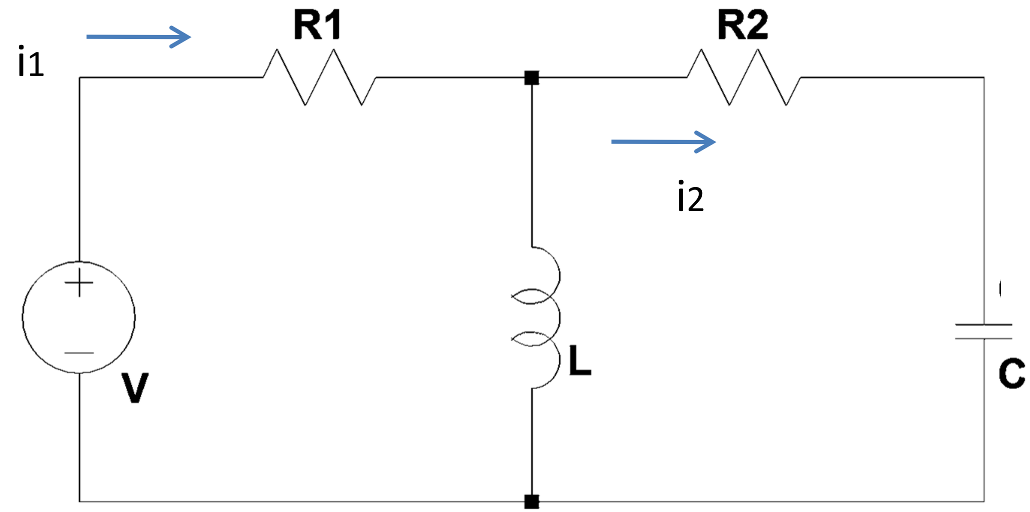
$$\frac{Vc(s)}{V(s)} = \frac{1}{(CLs^2 + RCs + 1)}$$

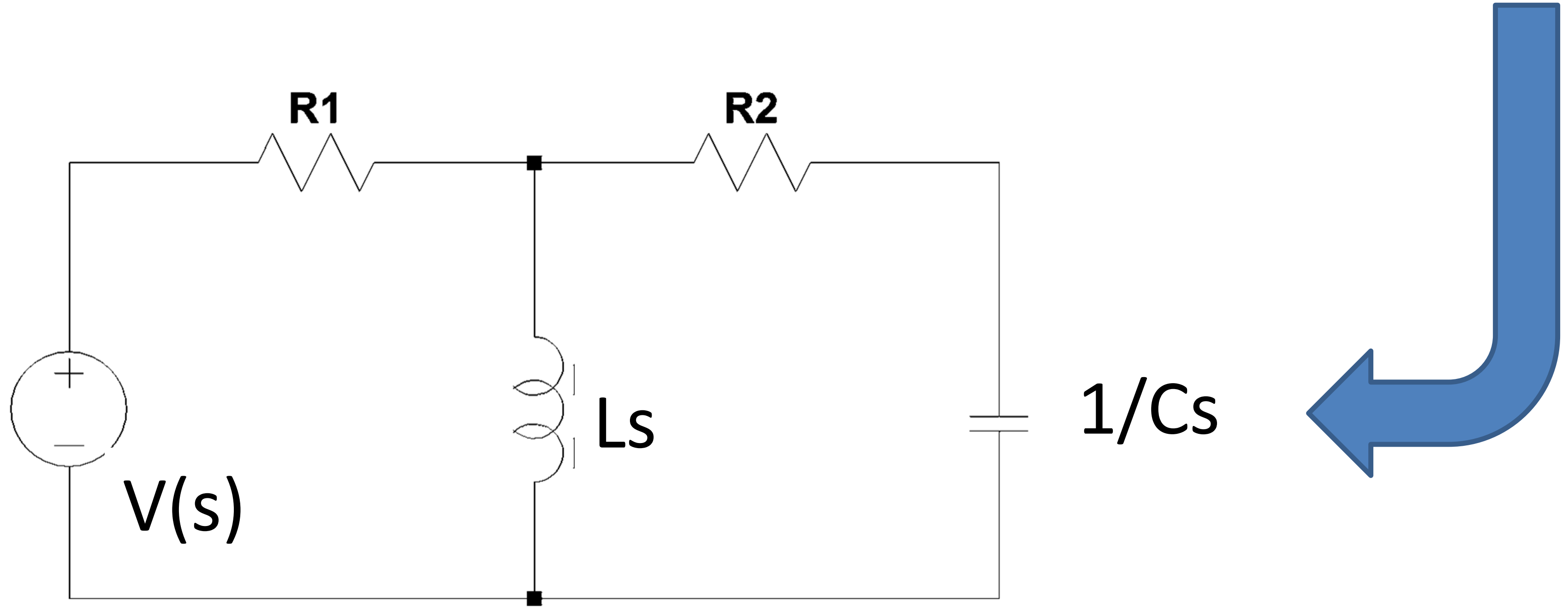


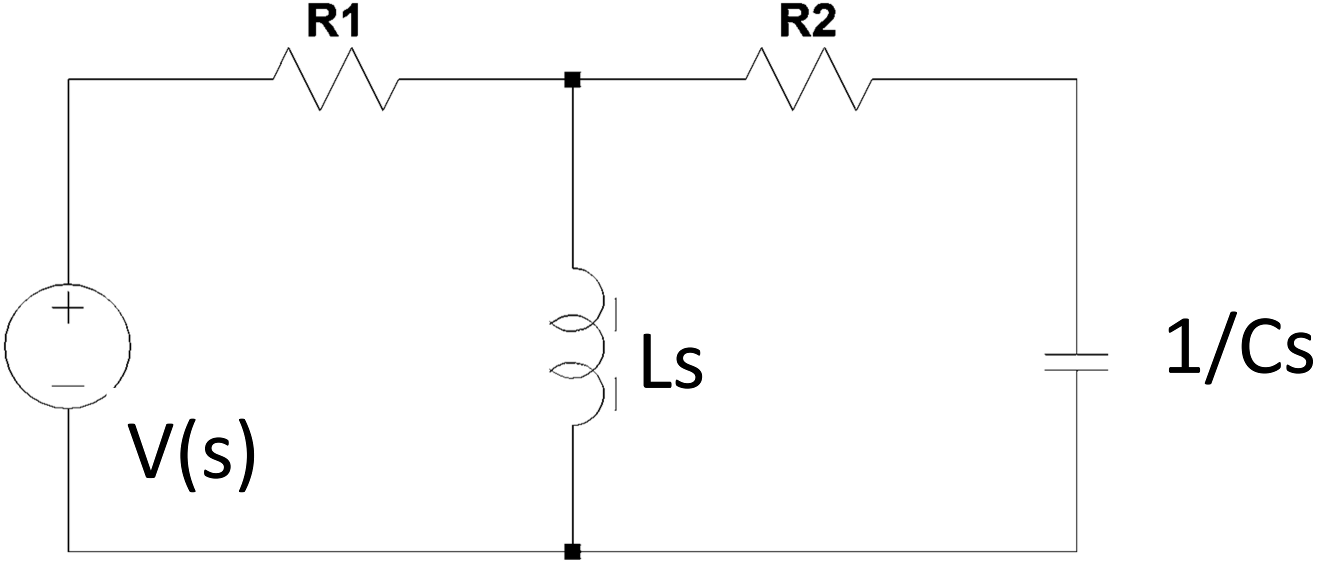
$$\frac{Vc(s)}{V(s)} = \frac{\frac{1}{C}\frac{1}{s}I(s)}{I(s)\left(Ls + R + \frac{1}{C}\frac{1}{s}\right)}$$

Example:

Obtain the transfer function $I_2(s)/V(s)$







$$(R_1 + Ls)I_1(s) - LsI_2(s) = V(s)$$

$$-LsI_1(s) + \left(Ls + R_2 + \frac{1}{Cs}\right)I_2(s) = 0$$

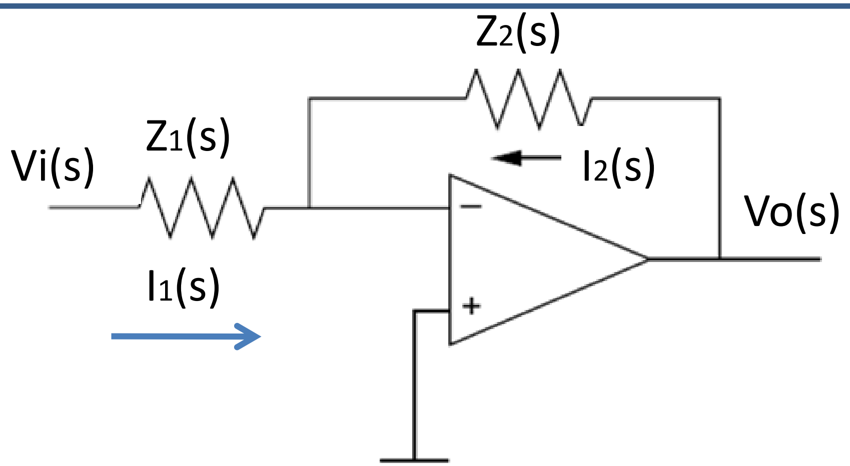
$$G(s) = \frac{I_2(s)}{V(s)} = \frac{LCs^2}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

Operational Amplifiers (OpAmp)

1. Differential input, V2(t)-V1(t)
2. High input impedance, Z_i=inf
3. Low output impedance Z_o=0
4. High Constant gain amplification A=inf

$$v_o(t) = A(v_2(t) - v_1(t))$$

Inverting OpAmp



$$I_1(s) = -I_2(s)$$

$$I_1(s) = \frac{V_i(s)}{Z_1(s)}$$

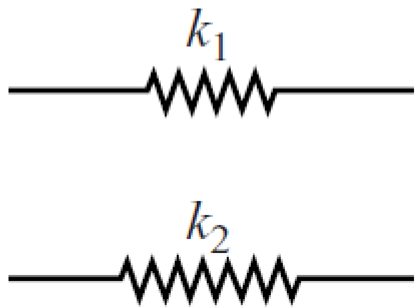
$$-I_2(s) = -\frac{V_0(s)}{Z_2(s)}$$

$$\frac{V_0(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Mechanical System Modeling

Spring	Viscous Damper	Mass
$f(t) = K\int v(t)dt$ $f(t) = Kx(t)$ $Z(s) = \frac{F(s)}{X(s)} = K$	$f(t) = f_v v(t)$ $f(t) = f_v \frac{dx(t)}{dt}$ $Z(s) = \frac{F(s)}{X(s)} = f_v s$	$f(t) = M\frac{dv(t)}{dt}$ $f(t) = M\frac{d^2x(t)}{dt^2}$ $Z(s) = \frac{F(s)}{X(s)} = Ms^2$

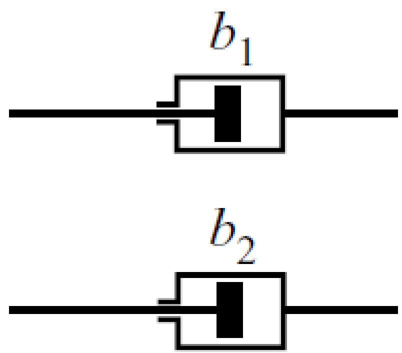
Spring



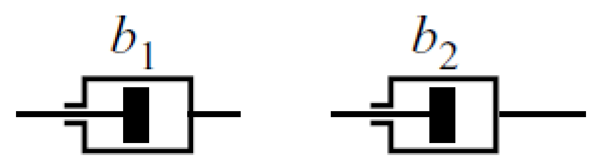
$$k_1 + k_2$$

$$\frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$$

Damper



$$b_1 + b_2$$



$$\frac{1}{\frac{1}{b_1} + \frac{1}{b_2}}$$

Rotational Mechanical System Modeling

Spring	Damper	Inertia
$T(t) = K\int w(t)dt$ $T(t) = K\theta(t)$ $Z(s) = \frac{T(s)}{\theta(s)} = K$	$T(t) = Dw(t)$ $T(t) = D\frac{d\theta(t)}{dt}$ $Z(s) = \frac{T(s)}{\theta(s)} = Ds$	$T(t) = J\frac{dw(t)}{dt}$ $T(t) = J\frac{d^2\theta(t)}{dt^2}$ $Z(s) = \frac{T(s)}{\theta(s)} = Js^2$