FEEDBACK CONTROL SYSTEMS

LECTURE NOTES-6

Time Response

The output response of a system is the sum of two responses: the forced response and the natural response

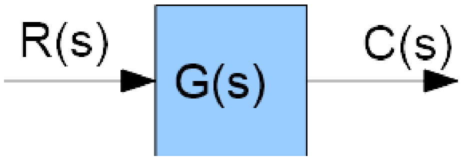
The poles of a transfer function are (1) the values of the Laplace transform variable, s, that cause the transfer function to become infinite or (2) any roots of the denominator of the transfer function that are common to roots of the numerator.

if a factor of the denominator can be canceled by the same factor in the numerator, the root of this factor no longer causes the transfer function to become infinite.

The zeros of a transfer function are (1) the values of the Laplace transform variable, s, that cause the transfer function to become zero, or (2) any roots of the numerator of the transfer function that are common to roots of the denominator.

the roots of the numerator are values of s that make the transfer function zero and are thus zeros

If a factor of the numerator can be canceled by the same factor in the denominator, the root of this factor no longer causes the transfer function to become zero

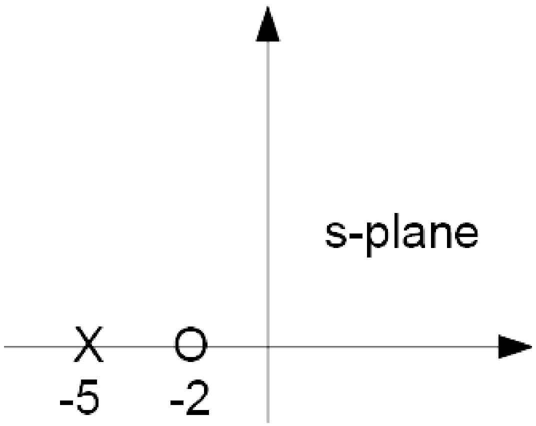


$$R(s) = \frac{1}{s}$$

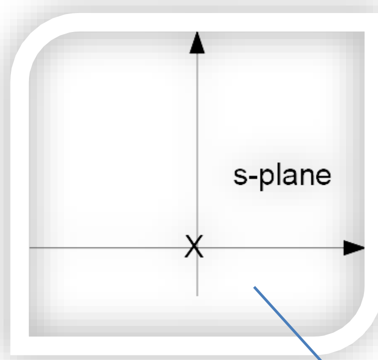
$$G(s) = \frac{s+2}{s+5}$$

$$C(s) = \frac{s+2}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} = \frac{0.4}{s} + \frac{0.6}{s+5}$$

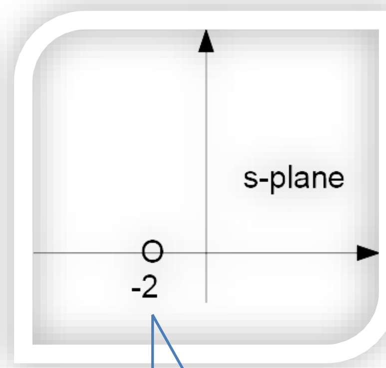
$$c(t) = 0.4 + 0.6e^{-5t}$$



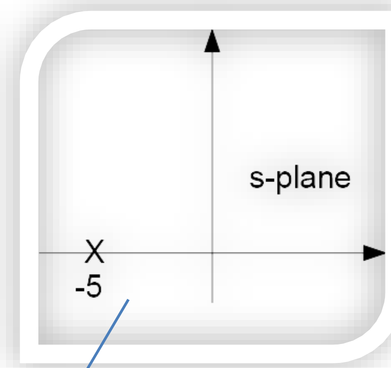
Input Pole



System Zero



System Pole



$$C(s) = \frac{0.4}{s} + \frac{0.6}{s+5}$$

$$c(t) = 0.4 + 0.6e^{-5t}$$

Forced Response

Natural Response

A pole of the input generates the *forced response* (pole at the origin generated from a step function at the output)

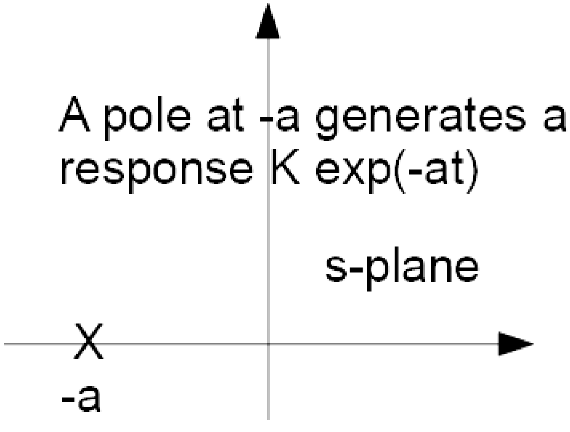
A pole at the transfer function generated natural response (the pole at -5 generated exp(-5t))

A pole on the real axis generated an exponential response, where a pole location is at real axis. Thusi the farther to the left a polse is on the negative real axis, the faster the exponential transient response will decay to zero

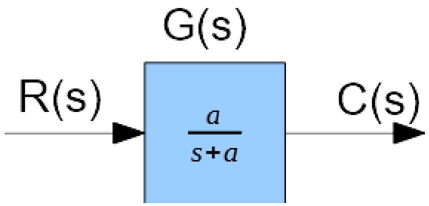
The zeros and poles generate amplitudes for both forced and natural responses (A and B)

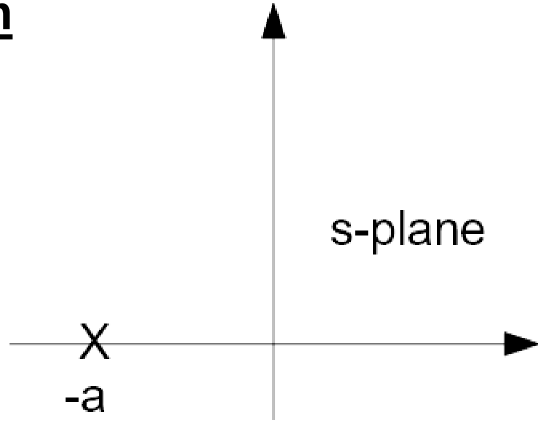
Each poles of the transfer function on real axis generates exponential response that is a component of natural response

Input pole generates forced response



First Order System



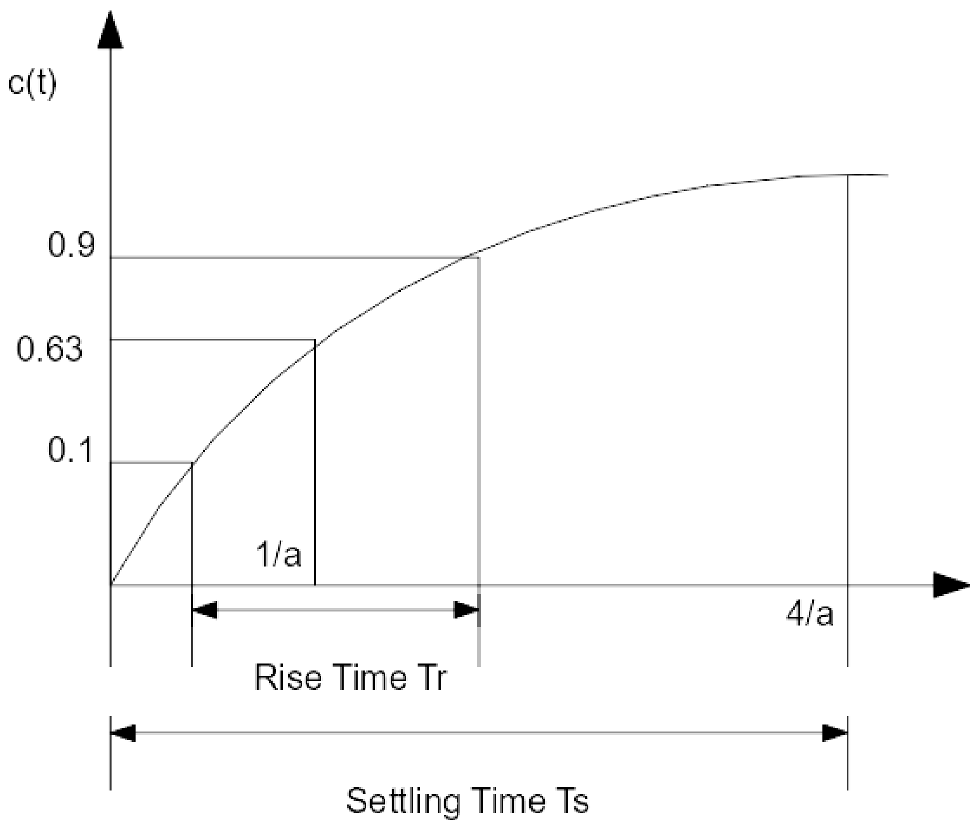


-First order system without zero

$$R(s)=\frac{1}{s}$$

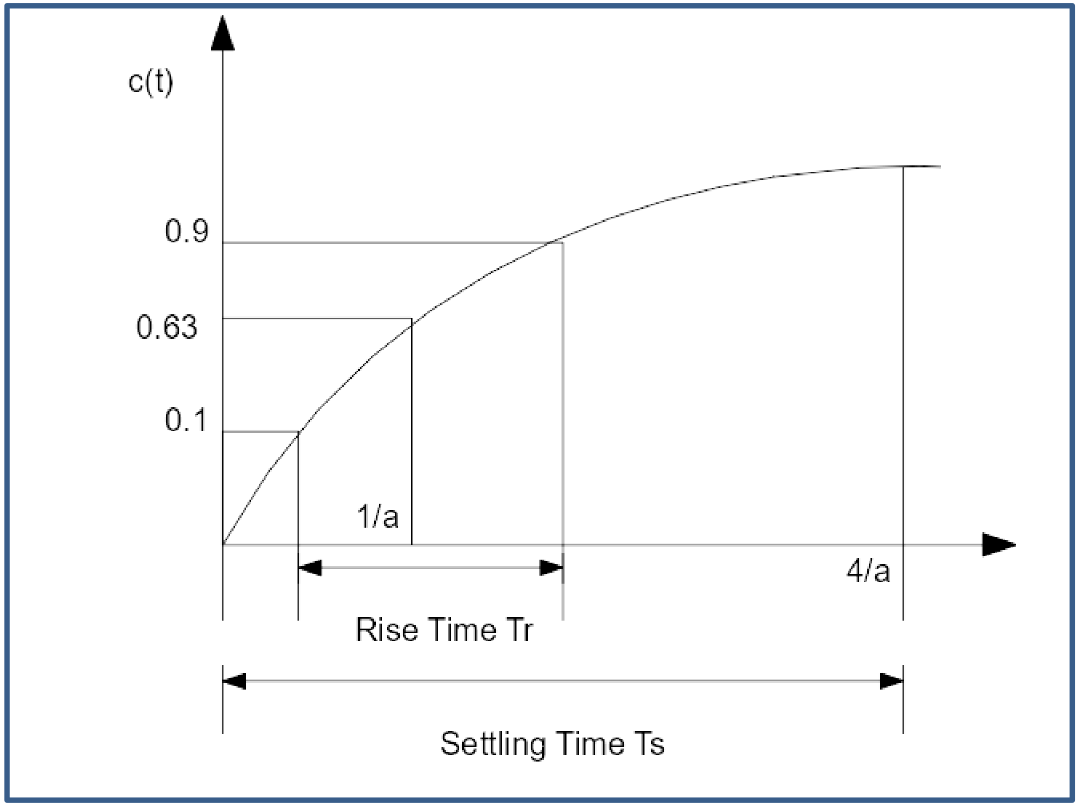
$$C(s)=R(s)G(s)=\frac{a}{s(s+a)}$$

$$c(t)=c_f(t)-c_n(t)=1-e^{-at}$$



-1/a is called time constant or exponential frequency

- Time to decay 37% of initial value



Rise Time: Tr $$Tr = \frac{2.2}{a}$$

Settling Time, Ts $$Ts = \frac{4}{a}$$

General Second Order System

$$G(s) = \frac{w_n^{\ 2}}{s^2 + 2\zeta w_n s + w_n^{\ 2}}$$

Damping Ratio, ζ

Natural Frequency, w_n

$$\zeta = \frac{\text{Exponential Decay Frequency}}{\text{Natural Frequency}} = \frac{1}{2\pi}\frac{\text{Natural Period}}{\text{Exponential Time Constant}}$$

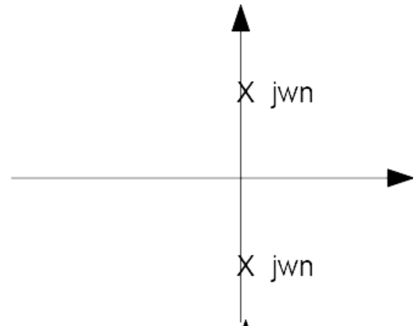
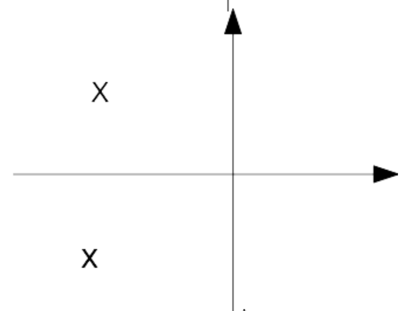
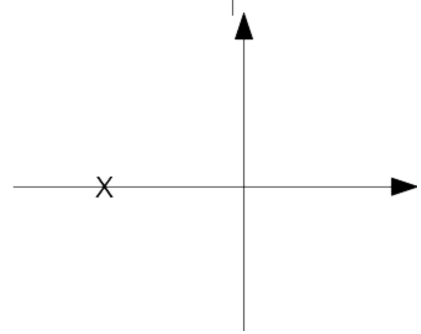
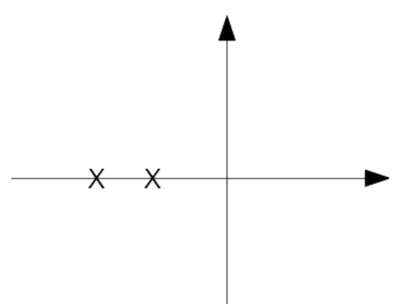
$$\zeta = \frac{\text{Exponential Decay Frequency}}{\text{Natural Frequency}} = \frac{|\sigma|}{w_n} = \frac{a}{2w_n}$$

$$G(s) = \frac{b}{s^2 + as + b}$$

$$w_n = \sqrt{b}$$

$$a = 2\zeta w_n$$

$$s_{1,2} = -\zeta w_n \pm \sqrt{\zeta^2 - 1}$$

ζ	Poles	Step Response
0		Undamped
$0 < \zeta < 1$		Underdamped
$\zeta = 1$		Critically Damped
$\zeta > 1$		Over Damped

Underdamped Second Order Systems

$$C(s) = \frac{w_n{}^2}{s\left(s^2 + 2\zeta w_n s + w_n{}^2\right)} = \frac{K_1}{s} + \frac{K_2}{\left(s^2 + 2\zeta w_n s + w_n{}^2\right)}$$

$$c(t) = 1 - e^{-\zeta w_n t}\left(\cos w_n \sqrt{1-\zeta^2}\, t + \frac{\zeta}{\sqrt{1-\zeta^2}\,\sin w_n \sqrt{1-\zeta^2}\, t}\right)$$

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}}\, e^{-\zeta w_n t} \cos\left(w_n \sqrt{1-\zeta^2}\, t - \phi\right)$$

$$\phi = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$OS = e - \left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

$$\zeta = \frac{-\ln(OS)}{\sqrt{\pi^2 + \ln^2(OS)}}$$

$$Ts = \frac{4}{\zeta w_n} = \frac{\pi}{\sigma_d}$$

$$Tp = \frac{\pi}{w_n\sqrt{1-\zeta^2}} = \frac{\pi}{w_d}$$

w_d = damped frequency of oscillation

σ_d = exponential damping frequency

Pole-Zero Cancellation:

For three pole system, a zero term cancel out

$$T(s) = \frac{K(s+z)}{(s+p_3)(s^2+as+b)}$$

If zero –z very close to –p3 pole

$$C_1(s) = \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)} = \frac{1}{s} - \frac{3.5}{s+5} + \frac{3.5}{s+6} - \frac{1}{s+3.5}$$

$$C_1(s) = \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)} = \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} - \frac{0.033}{s+3.5}$$

The pole at -4.01 is very close to the zero at -4

$$c_2(t) \approx 0.87 - 5.3e^{-5t} + 4.4e^{-6t}$$