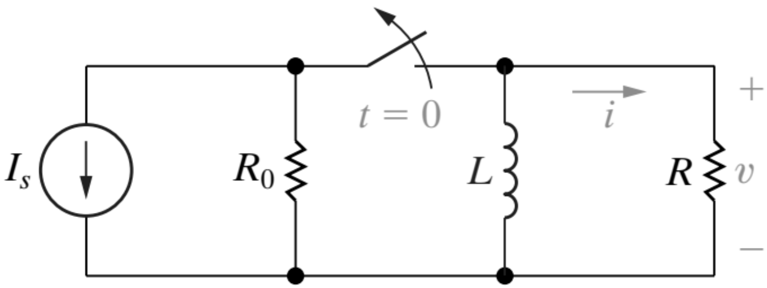
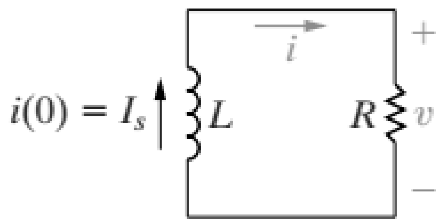
RL and *RC* circuits are also known as **first-order circuits**, because their voltages and currents are described by first-order differential equations





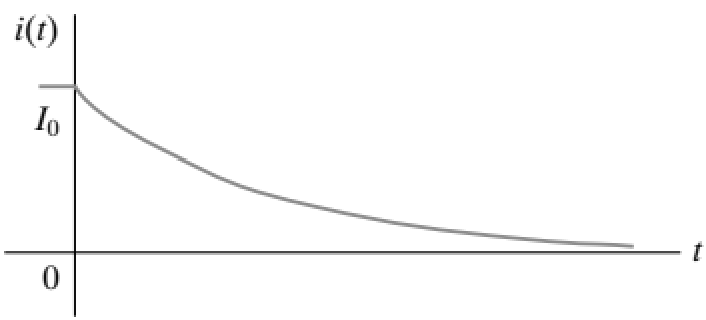
$$L\frac{di}{dt} + Ri = 0,$$

Known as a first-order ordinary differential equation, because it contains terms involving the ordinary derivative of the unknown

$$\frac{di}{dt}dt = -\frac{R}{L}i\,dt. \qquad \frac{di}{i} = -\frac{R}{L}dt. \qquad \int_{i(t_0)}^{i(t)}\frac{dx}{x} = -\frac{R}{L}\int_{t_0}^{t}dy,$$

$$\ln\frac{i(t)}{i(0)} = -\frac{R}{L}t. \qquad i(t) = i(0)e^{-(R/L)t}.$$

$$i(0^-) = i(0^+) = I_0, \quad \rightarrow \quad i(t) = I_0e^{-(R/L)t}, \quad t \geq 0,$$



$$v = iR = I_0Re^{-(R/L)t}, \quad t \geq 0^+.$$

Lets find by using Laplace Transform

$$w = \int_0^t p\,dx = \int_0^t I_0^2 R e^{-2(R/L)x}\,dx$$
$$= \frac{1}{2(R/L)} I_0^2 R(1 - e^{-2(R/L)t})$$
$$= \frac{1}{2} L I_0^2 (1 - e^{-2(R/L)t}), \quad t \geq 0.$$

$$p = vi, \quad p = i^2 R, \quad \text{or} \quad p = \frac{v^2}{R}.$$

$$p = I_0^2 R e^{-2(R/L)t}, \quad t \geq 0^+.$$

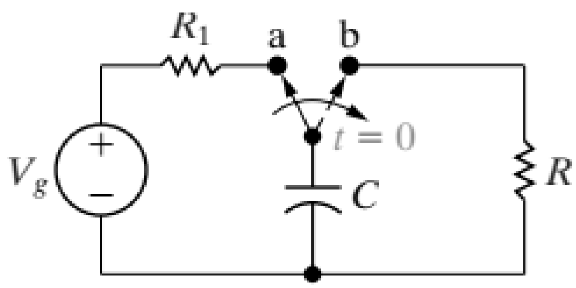
The Significance of the Time Constant

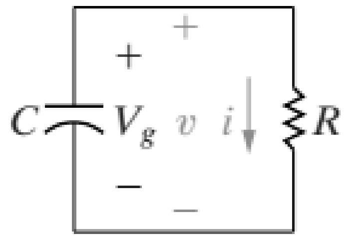
Determines the rate at which the current or voltage approaches zero. The reciprocal of this ratio is the **time constant** of the circuit, denoted

$$i(t) = I_0 e^{-t/\tau}, \quad t \geq 0,$$
$$v(t) = I_0 R e^{-t/\tau}, \quad t \geq 0^+,$$
$$p = I_0^2 R e^{-2t/\tau}, \quad t \geq 0^+,$$
$$w = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}), \quad t \geq 0.$$

$$\tau = \text{time constant} = \frac{L}{R}.$$

The natural response of an *RC* circuit is developed from the circuit shown in Fig





We begin by assuming that the switch has been in position a for a long time, allowing the loop made up of the dc voltage source the resistor and the capacitor *C* to reach a steady-state condition.

we will discuss how the capacitor voltage actually builds to the steady-state value of the dc voltage source, but for now the important point is that when the switch is moved from position a to position b (at t=0), the voltage on the capacitor is. Because there can be no instantaneous change in the voltage at the terminals of a capacitor, the problem reduces to solving the circuit shown in Fig.

Deriving the Expression for the Voltage

$$C\frac{dv}{dt} + \frac{v}{R} = 0.$$

$$v(t) = v(0)e^{-t/RC}, \quad t \geq 0.$$

$$v(0^-) = v(0) = v(0^+) = V_g = V_0,$$

$$\tau = RC.$$

$$v(t) = V_0 e^{-t/\tau}, \quad t \geq 0,$$

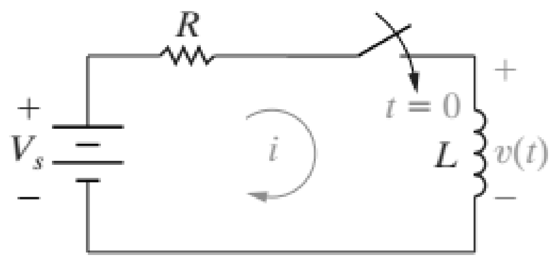
$$i(t) = \frac{v(t)}{R} = \frac{V_0}{R}e^{-t/\tau}, \quad t \geq 0^+,$$

$$p = vi = \frac{V_0^2}{R}e^{-2t/\tau}, \quad t \geq 0^+,$$

$$w = \int_0^t p\,dx = \int_0^t \frac{V_0^2}{R}e^{-2x/\tau}\,dx$$

$$= \frac{1}{2}CV_0^2(1 - e^{-2t/\tau}), \quad t \geq 0.$$

We are now ready to discuss the problem of finding the currents and voltages generated in first-order *RL* or *RC* circuits when either dc voltage or current sources are suddenly applied.



Energy stored in the inductor at the time the switch is closed is given in terms of a nonzero initial current The task is to find the expressions for the current in the circuit and for the voltage across the inductor after the switch has been closed.

$$V_s = Ri + L\frac{di}{dt},$$

$$\frac{di}{dt} = \frac{-Ri + V_s}{L} = \frac{-R}{L}\left(i - \frac{V_s}{R}\right).$$

$$\frac{di}{dt}dt = \frac{-R}{L}\left(i - \frac{V_s}{R}\right)dt,$$

$$di = \frac{-R}{L}\left(i - \frac{V_s}{R}\right)dt.$$

$$\frac{di}{i - (V_s/R)} = \frac{-R}{L}dt,$$

$$\int_{I_0}^{i(t)} \frac{dx}{x - (V_s/R)} = \frac{-R}{L}\int_0^t dy,$$

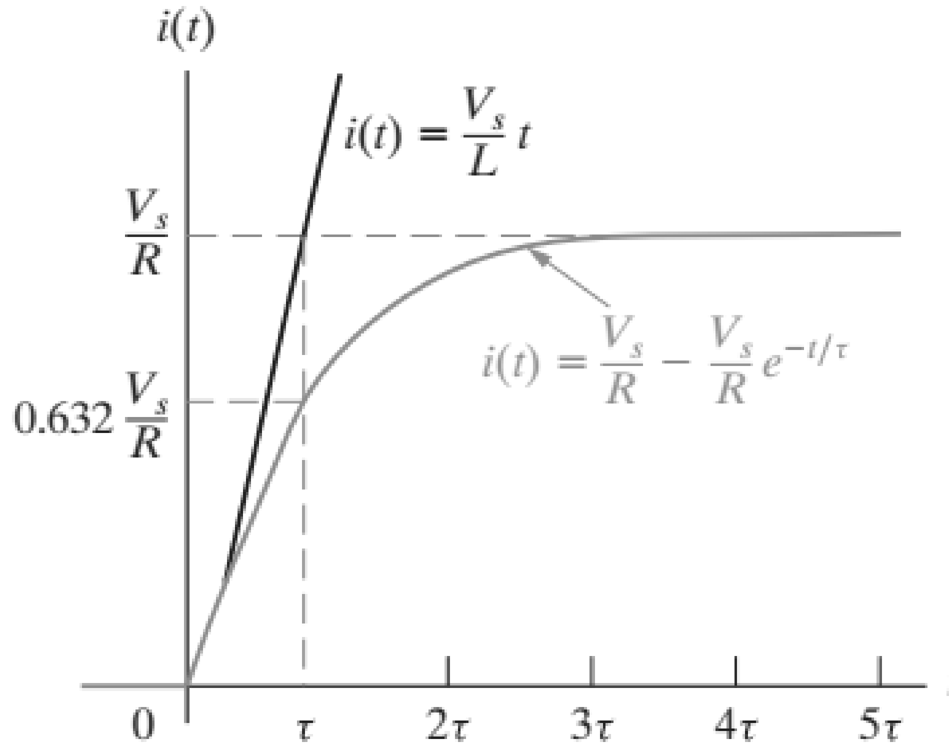
$$\ln\frac{i(t) - (V_s/R)}{I_0 - (V_s/R)} = \frac{-R}{L}t,$$

$$\frac{i(t) - (V_s/R)}{I_0 - (V_s/R)} = e^{-(R/L)t},$$

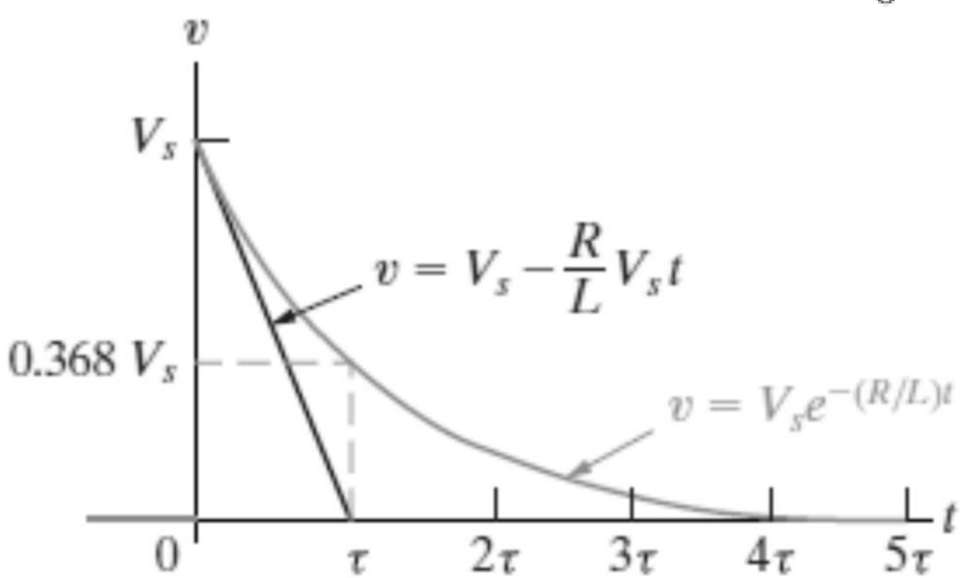
$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-(R/L)t}.$$

When the initial energy in the inductor is zero, is zero.

$$i(t) = \frac{V_s}{R} - \frac{V_s}{R}e^{-(R/L)t}.$$



i(t)
i(t) = V_s/L t
V_s/R
0.632 V_s/R
i(t) = V_s/R − V_s/R e^{−t/τ}
0
τ
2τ
3τ
4τ
5τ
t



v
V_s
v = V_s − R/L V_s t
0.368 V_s
v = V_s e^{−(R/L)t}
0
τ
2τ
3τ
4τ
5τ
t

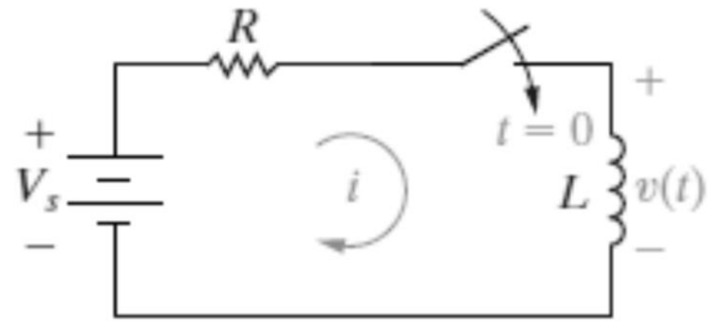
$$i(t) = \frac{V_s}{R} - \frac{v(t)}{R},$$

$$\frac{di}{dt} = -\frac{1}{R}\frac{dv}{dt}.$$

multiply by *L*

$$v = -\frac{L}{R}\frac{dv}{dt}.$$

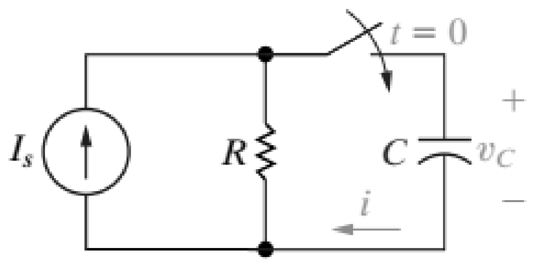
$$\frac{dv}{dt} + \frac{R}{L}v = 0.$$



$$V_s = Ri + L\frac{di}{dt},$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V_s}{L}.$$

The Step Response of an *RC* Circuit



We can find the step response of a first order *RC* circuit by analyzing the circuit shown in Fig.

$$C\frac{dv_C}{dt} + \frac{v_C}{R} = I_s.$$

$$v_C = I_sR + (V_0 - I_sR)e^{-t/RC}, \quad t \geq 0.$$

$$\frac{di}{dt} + \frac{1}{RC}i = 0.$$

$$i = \left(I_s - \frac{V_0}{R}\right)e^{-t/RC}, \quad t \geq 0^+,$$

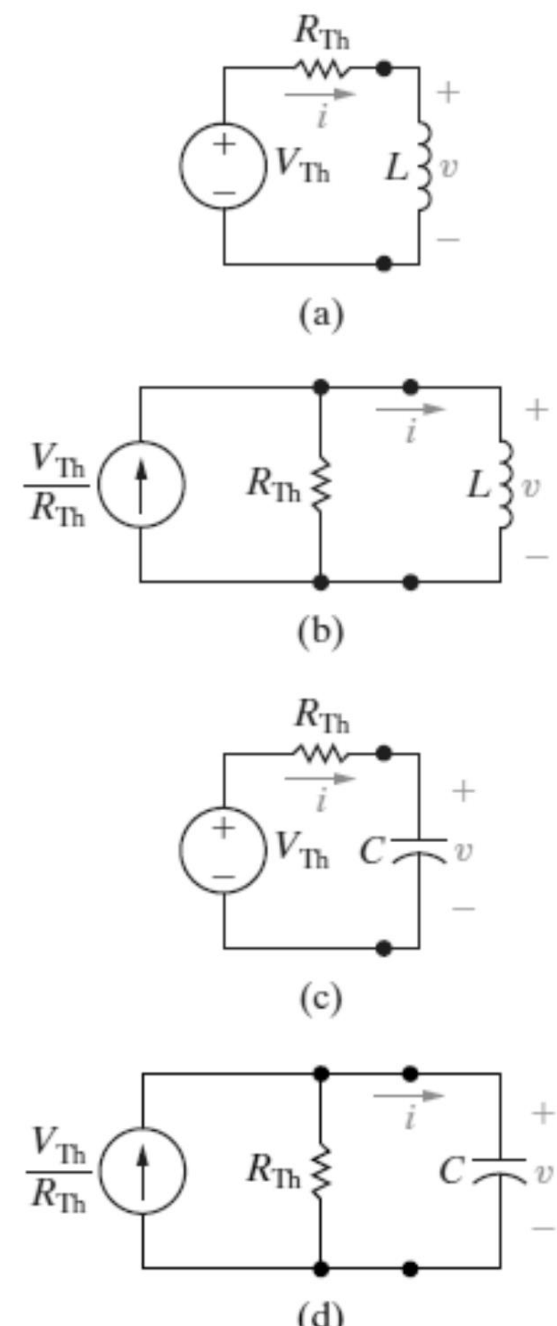
$$\frac{dx}{dt} + \frac{x}{\tau} = K, \qquad x_f = K\tau,$$

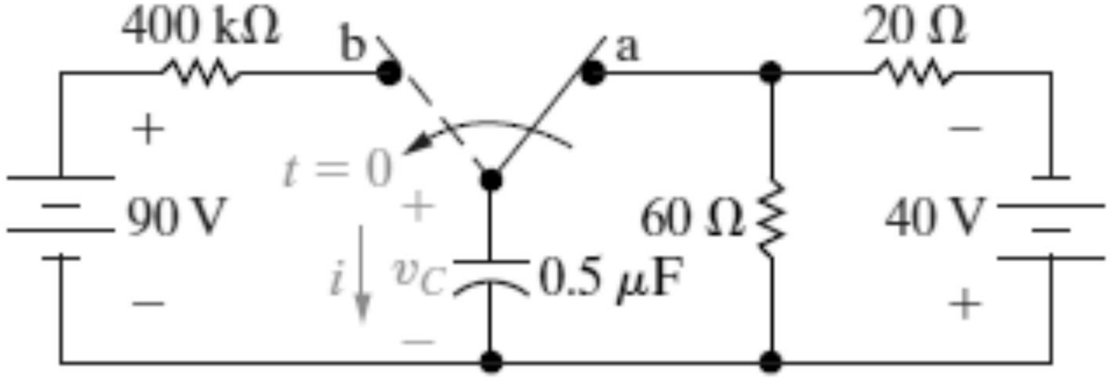
$$\frac{dx}{dt} = \frac{-x}{\tau} + K = \frac{-(x - K\tau)}{\tau} = \frac{-(x - x_f)}{\tau}.$$

$$\frac{dx}{x - x_f} = \frac{-1}{\tau}dt.$$

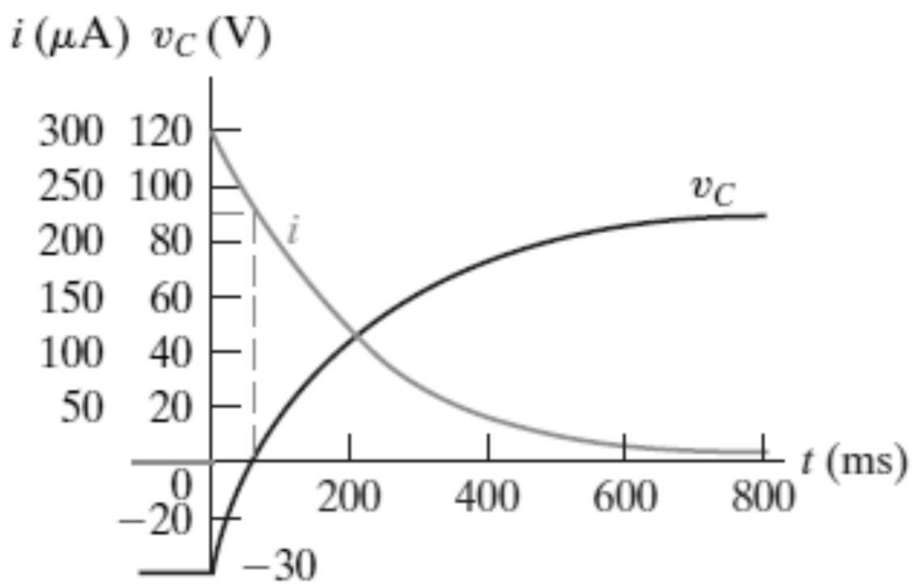
$$\int_{x(t_0)}^{x(t)} \frac{du}{u - x_f} = -\frac{1}{\tau}\int_{t_0}^{t} dv.$$

$$x(t) = x_f + [x(t_0) - x_f]e^{-(t-t_0)/\tau}.$$

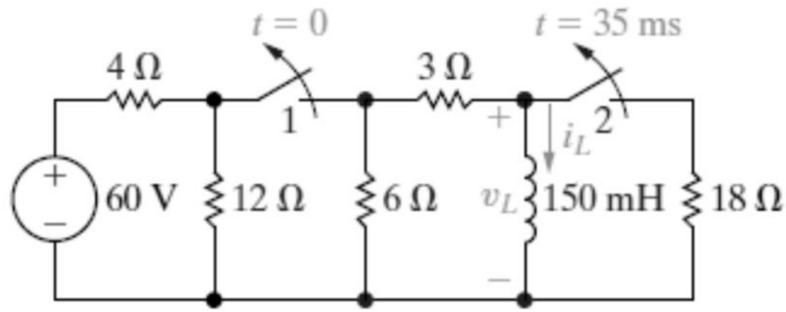




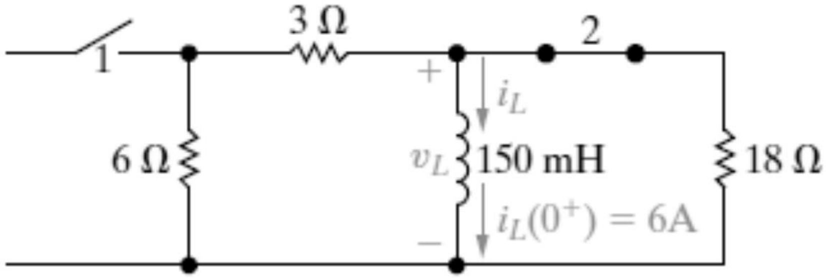
400 kΩ
b
a
20 Ω
+
−
t = 0
+
90 V
60 Ω
40 V
i
v_C
0.5 μF
−
−
+



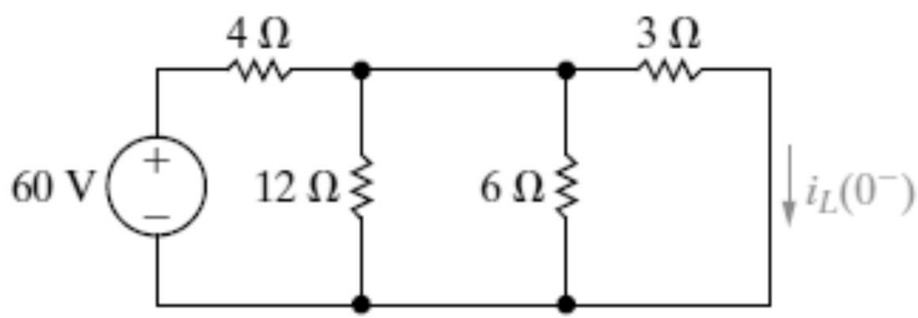
i (μA) v_C (V)
300 120
250 100
200 80
150 60
100 40
50 20
0
−20
−30
v_C
i
200
400
600
800
t (ms)



t = 0
t = 35 ms
4 Ω
3 Ω
1
2
60 V
12 Ω
6 Ω
150 mH
18 Ω



3 Ω
2
1
6 Ω
150 mH
18 Ω
$i_L(0^+) = 6A$



4 Ω
3 Ω
60 V
12 Ω
6 Ω
$i_L(0^-)$