Instantaneous Power

$$p = vi.$$ instantaneous power.

$$v = V_m \cos(\omega t + \theta_v),$$

$$i = I_m \cos(\omega t + \theta_i),$$

$$v = V_m \cos(\omega t + \theta_v - \theta_i),$$

$$i = I_m \cos \omega t.$$

$$p = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t.$$

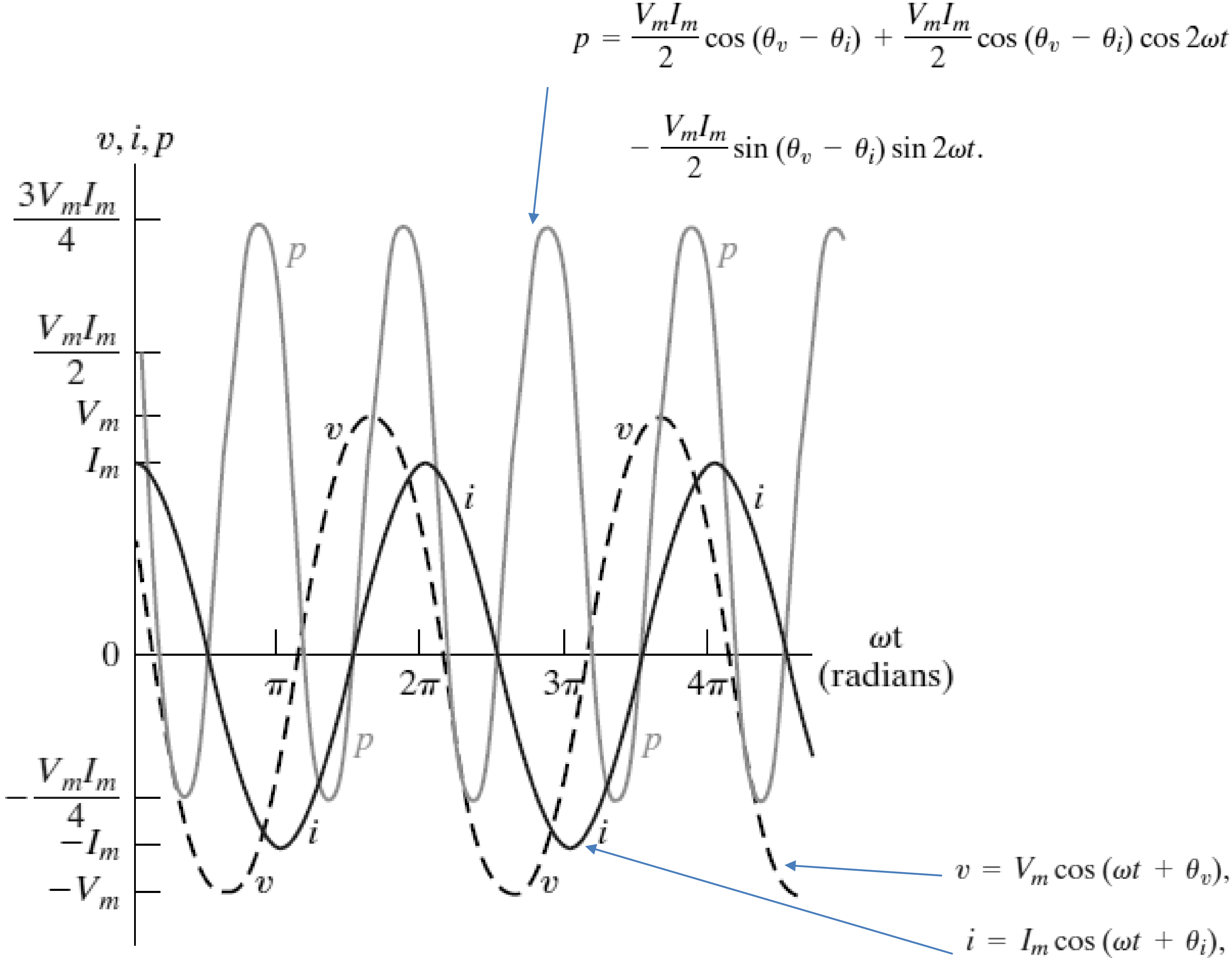
$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i).$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$$

$$- \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t.$$



$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$
$- \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t.$
v, i, p
$\frac{3V_m I_m}{4}$
$\frac{V_m I_m}{2}$
V_m
I_m
0
$-\frac{V_m I_m}{4}$
$-I_m$
$-V_m$
π
2π
3π
4π
ωt
(radians)
p
v
i
$v = V_m \cos(\omega t + \theta_v),$
$i = I_m \cos(\omega t + \theta_i),$

Average and Reactive Power

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t.$$

$$p = P + P\cos 2\omega t - Q\sin 2\omega t,$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i),$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i).$$

P is called the ***average power, and Q is called the reactive power. Average*** power is sometimes called **real power, because it describes the power in a** circuit that is transformed from electric to nonelectric energy.

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p\,dt,$$

$$p = P + P\cos 2\omega t - Q\sin 2\omega t,$$

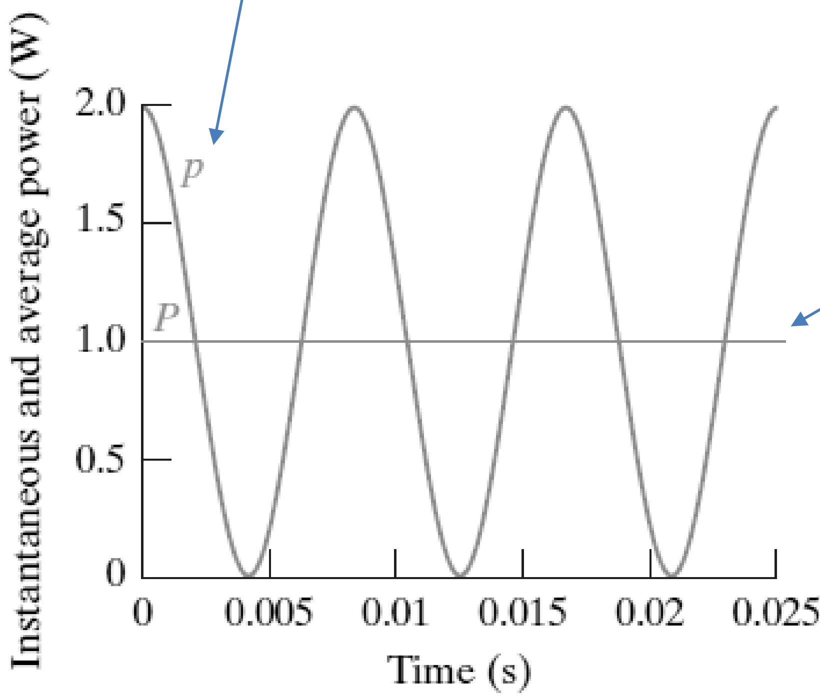
Power for Purely Resistive Circuits → $\theta_v = \theta_i.$

$$p = P + P\cos 2\omega t - Q\sin 2\omega t,$$

=0

$$p = P + P\cos 2\omega t.$$

instantaneous real power



$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i),$$

instantaneous real power can **never be negative**. in other words, **power cannot be extracted from a purely resistive network**. Rather, all the electric energy is dissipated in the form of thermal energy.

Power for Purely Inductive Circuits

$$\theta_v - \theta_i = +90^\circ$$

$$p = -Q \sin 2\omega t.$$

$$p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t.$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i).$$

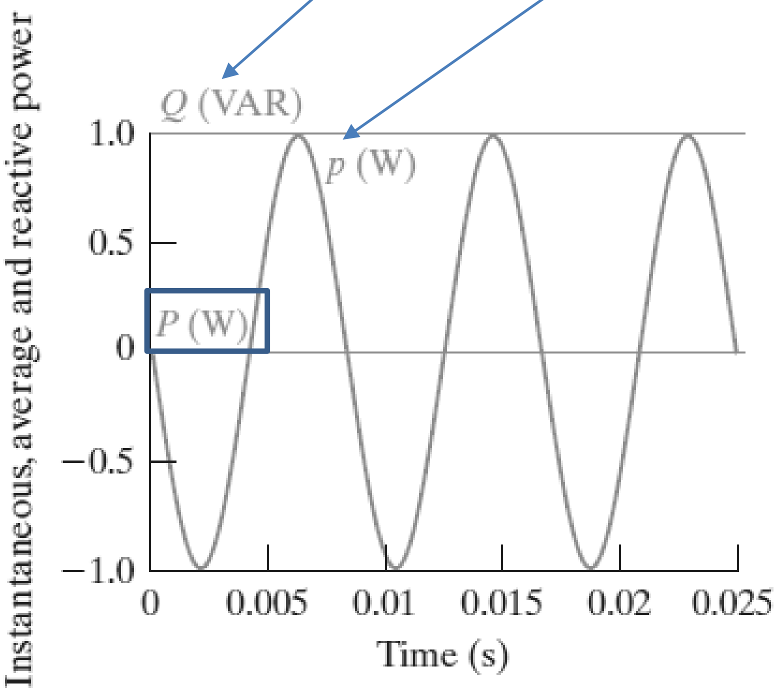


Figure 10.4 ▲ Instantaneous real power, average power, and reactive power for a purely inductive circuit.

A measure of the power associated with purely inductive circuits is the reactive power *Q. The name reactive power comes from the characterization* of an inductor as a reactive element; its impedance is purely reactive. Note that average power *P and reactive power Q carry the same* dimension. To distinguish between average and reactive power, we use the units *watt (W) for average power and* ***var (volt-amp reactive, or VAR) for*** reactive power.

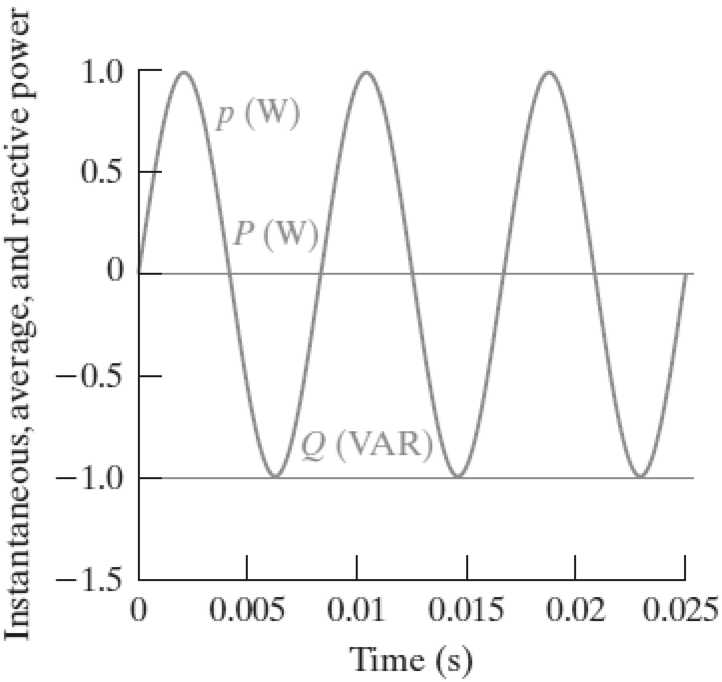
Power for Purely Capacitive Circuits

$$\theta_v - \theta_i = -90^\circ.$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$$
$$- \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t.$$

$$p = -Q\sin 2\omega t.$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i).$$



Average power is zero, so there is no transformation of energy from electric to nonelectric form. In a purely capacitive circuit, the power is continually exchanged between the source driving the circuit and the electric field associated with the capacitive elements.

The Power Factor

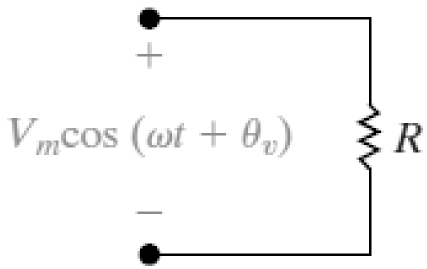
$\theta_v - \theta_i$ plays a role in the computation of both average and reactive power and is referred to as the **power factor angle. The cosine of** this angle is called the **power factor, abbreviated pf, and the sine of this** angle is called the **reactive factor, abbreviated rf. Thus**

$$\text{pf} = \cos(\theta_v - \theta_i),$$

$$\text{rf} = \sin(\theta_v - \theta_i).$$

To completely describe this angle, we use the descriptive phrases **lagging power factor and leading power factor. Lagging power factor implies that current lags voltage—** hence an inductive load. Leading power factor implies that current leads voltage—hence a capacitive load. Both the power factor and the reactive factor are convenient quantities to use in describing electrical loads.

The rms Value and Power Calculations



$$P = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_m^2 \cos^2(\omega t + \phi_v)}{R} dt$$
$$= \frac{1}{R}\left[\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi_v) dt\right].$$

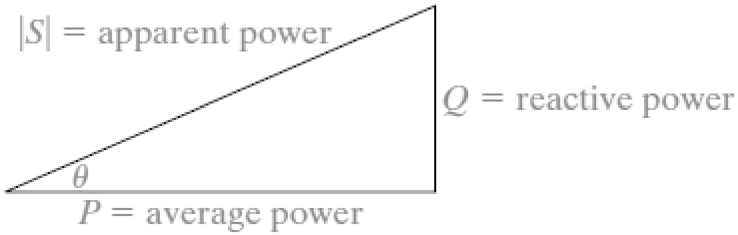
$$P = \frac{V_{\text{rms}}^2}{R}. \qquad P = I_{\text{rms}}^2 R.$$

The rms value is also referred to as the **effective value of the sinusoidal** voltage (or current). The rms value has an interesting property: Given an equivalent resistive load, *R, and an equivalent time period, T,* the rms value of a sinusoidal source delivers the same energy to *R as does* a dc source of the same value

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$
$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$
$$= V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i);$$

and, by similar manipulation,

$$Q = V_{\text{eff}} I_{\text{eff}} \sin(\theta_v - \theta_i).$$

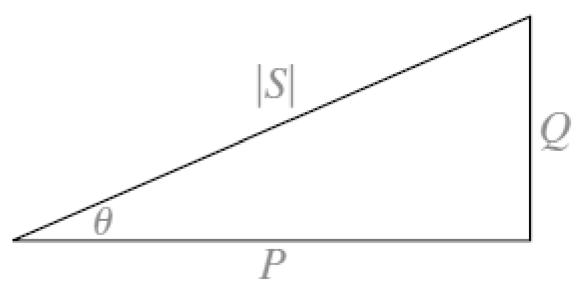


$$\tan\theta = \frac{Q}{P}.$$

$$\frac{Q}{P} = \frac{(V_m I_m/2)\sin(\theta_v - \theta_i)}{(V_m I_m/2)\cos(\theta_v - \theta_i)} = \tan(\theta_v - \theta_i).$$

The magnitude of complex power is referred to as **apparent power**

$$|S| = 2\ \overline{P^2 + Q^2}.$$



Power Calculations

We are now ready to develop additional equations that can be used to calculate real, reactive, and complex power

$$S = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + j\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2}[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

$$= \frac{V_m I_m}{2}e^{j(\theta_v - \theta_i)} = \frac{1}{2}V_m I_m \underline{/(\theta_v - \theta_i)}.$$

$$S = V_{\text{eff}} I_{\text{eff}} \underline{/(\theta_v - \theta_i)}$$

$$= V_{\text{eff}} I_{\text{eff}} e^{j(\theta_v - \theta_i)}$$

$$= V_{\text{eff}} e^{j\theta_v} I_{\text{eff}} e^{-j\theta_i}$$

$$= \mathbf{V}_{\text{eff}} \mathbf{I}_{\text{eff}}^{*}.$$