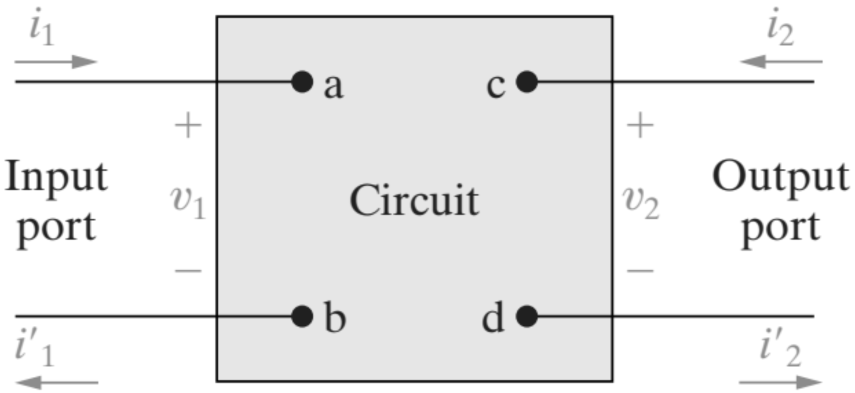
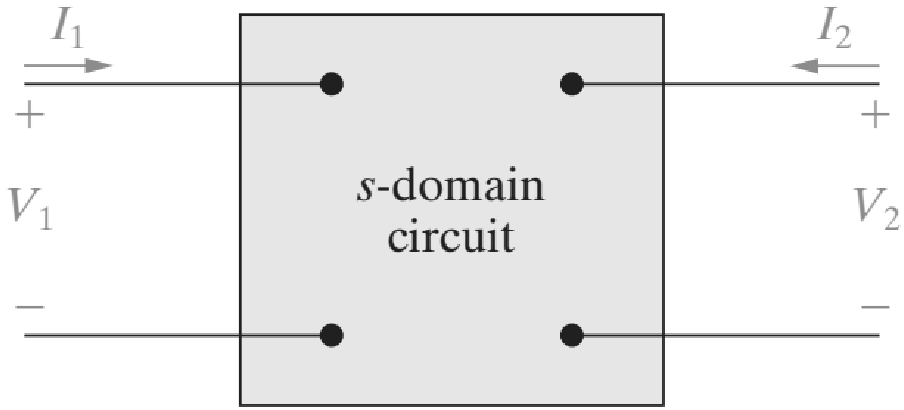
**The Terminal Equations**

In viewing a circuit as a two-port network, we are interested in relating the current and voltage at one port to the current and voltage at the other port.




The references at each port are symmetric with respect to each other; that is, at each port the current is directed into the upper terminal, and each port voltage is a rise from the lower to the upper terminal. This symmetry makes it easier to generalize the analysis of a two-port network and is the reason for its universal use in the literature.

The most general description of the two-port network is carried out in frequency domain jw




Of these four terminal variables, only two are independent. Thus for any circuit, once we specify two of the variables, we can find the two remaining unknowns. For example, knowing and and the circuit within the box, we can determine and Thus we can describe a two port network with just two simultaneous equations. However, there are six different ways in which to combine the four variables:

$$V_1 = z_{11}I_1 + z_{12}I_2,$$
$$V_2 = z_{21}I_1 + z_{22}I_2;$$

$$I_1 = y_{11}V_1 + y_{12}V_2,$$
$$I_2 = y_{21}V_1 + y_{22}V_2;$$

$$V_1 = a_{11}V_2 - a_{12}I_2,$$
$$I_1 = a_{21}V_2 - a_{22}I_2;$$

$$V_2 = b_{11}V_1 - b_{12}I_1,$$
$$I_2 = b_{21}V_1 - b_{22}I_1;$$

$$V_1 = h_{11}I_1 + h_{12}V_2,$$
$$I_2 = h_{21}I_1 + h_{22}V_2;$$

$$I_1 = g_{11}V_1 + g_{12}I_2,$$
$$V_2 = g_{21}V_1 + g_{22}I_2.$$

These six sets of equations may also be considered as three pairs of mutually inverse relations.

The coefficients of the current and/or voltage variables on the right hand side of Eqs. are called the **parameters** of the two-port circuit.

Thus, when using Eqs., we refer to the *z* parameters of the circuit. Similarly, we refer to the *y* parameters, the *a* parameters, the *b* parameters, the *h* parameters, and the *g* parameters of the network.

## The Two-Port Parameters

We can determine the parameters for any circuit by computation or measurement. The computation or measurement to be made comes directly from the parameter equations. For example, suppose that the problem is to find the *z* parameters for a circuit

$$V_1 = z_{11}I_1 + z_{12}I_2,$$
$$V_2 = z_{21}I_1 + z_{22}I_2;$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega,$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega,$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega,$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega.$$

Each parameter is the ratio of a voltage to a current and therefore is an impedance with the dimension of ohms. We use the same process to determine the remaining port parameters, which are either calculated or measured. A port parameter is obtained by either opening or shorting a port. Moreover, a port parameter is an impedance, an admittance, or a dimensionless ratio. The dimensionless ratio is the ratio of either two voltages or two currents.

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \text{S}, \qquad y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \text{S},$$

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} \text{S}, \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} \text{S}.$$

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0}, \qquad a_{12} = -\frac{V_1}{I_2}\bigg|_{V_2=0} \Omega,$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} \text{S}, \qquad a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0}.$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} \Omega, \qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0},$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}, \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} \text{S}.$$

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0}, \qquad b_{12} = -\frac{V_2}{I_1}\bigg|_{V_1=0} \Omega,$$

$$b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} \text{S}, \qquad b_{22} = -\frac{I_2}{I_1}\bigg|_{V_1=0}.$$

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} \text{S}, \qquad g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0},$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0}, \qquad g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} \Omega.$$

The two-port parameters are also described in relation to the reciprocal sets of equations.

The impedance and admittance parameters are grouped into the immittance parameters.

The term **immittance** denotes a quantity  that is either an impedance or an admittance.

The *a* and *b* parameters are called the **transmission** parameters because they describe the voltage and current at one end of the two-port network in terms of the voltage and current at the other end.

The immittance and transmission parameters are the natural choices for relating the port variables.

In other words, they relate either voltage to current variables or input to output variables.

The *h* and *g* parameters relate cross-variables, that is, an input voltage and output current to an output voltage and input current.

Therefore the *h* and *g* parameters are called **hybrid** parameters.

**Relationships Among the Two-Port Parameters**

Because the six sets of equations relate to the same variables, the parameters associated with any pair of equations must be related to the parameters of all the other pairs. In other words, if we know one set of parameters, we can derive all the other sets from the known set. Because of the amount of algebra involved in these derivations, we merely list the results

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$$

$$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$$

$$y_{12} = -\frac{z_{12}}{\Delta z} = -\frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = -\frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$$

$$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = -\frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$$

$$a_{11} = \frac{z_{11}}{z_{21}} = -\frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = -\frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$$

$$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$$

$$a_{21} = \frac{1}{z_{21}} = -\frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$$

$$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$$

$$b_{11} = \frac{z_{22}}{z_{12}} = -\frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$$

$$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$$

$$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$$

$$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$$

$$\Delta z = z_{11}z_{22} - z_{12}z_{21}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$

$$\Delta a = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21}$$

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$$

$$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$$

$$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$$

$$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{21}}{a_{11}} = \frac{b_{21}}{b_{22}} = \frac{h_{22}}{\Delta h}$$

$$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta a}{a_{11}} = -\frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$$

$$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$$

Although we do not derive all the relationships listed. we do derive those between the *z* and *y* parameters and between the *z* and *a* parameters. These derivations illustrate the general process involved in relating one set of parameters to another. To find the *z* parameters as functions of the *y* parameters, we first solve Eqs. for v1 and v2 We then compare the coefficients of I1 and I2 in the resulting expressions to the coefficients of I1 and I2.

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22}}{\Delta y} I_1 - \frac{y_{12}}{\Delta y} I_2,$$

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\Delta y} = -\frac{y_{21}}{\Delta y} I_1 + \frac{y_{11}}{\Delta y} I_2.$$

$$z_{11} = \frac{y_{22}}{\Delta y},$$

$$z_{12} = -\frac{y_{12}}{\Delta y},$$

$$z_{21} = -\frac{y_{21}}{\Delta y},$$

$$z_{22} = \frac{y_{11}}{\Delta y}.$$

**Reciprocal Two-Port Circuits**

If a two-port circuit is **reciprocal**, the following relationships exist among the port parameters:

$$z_{12} = z_{21},$$

$$y_{12} = y_{21},$$

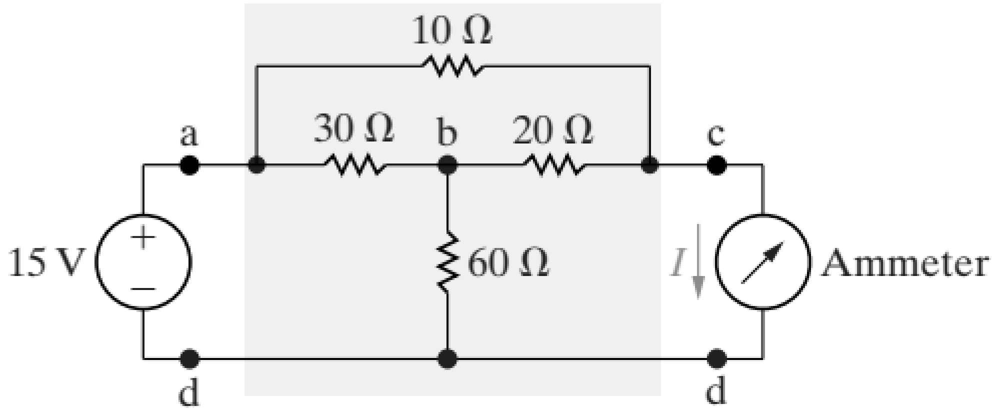
$$a_{11}a_{22} - a_{12}a_{21} = \Delta a = 1,$$

$$b_{11}b_{22} - b_{12}b_{21} = \Delta b = 1,$$

$$h_{12} = -h_{21},$$

$$g_{12} = -g_{21}.$$

A two-port circuit is reciprocal if the interchange of an ideal voltage source at one port with an ideal ammeter at the other port produces the same ammeter reading. Consider, for example, the resistive circuit shown in Fig



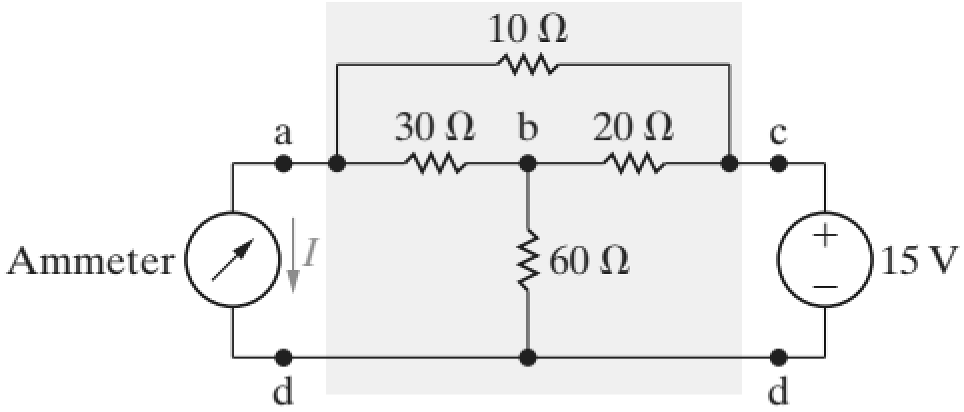
When a voltage source of 15 V is applied to port ad, it produces a current of 1.75 A in the ammeter at port cd. The ammeter current is easily determined once we know the voltage Vbd. Thus

$$\frac{V_{bd}}{60} + \frac{V_{bd} - 15}{30} + \frac{V_{bd}}{20} = 0,$$

and $V_{bd} = 5$ V. Therefore

$$I = \frac{5}{20} + \frac{15}{10} = 1.75 \text{ A}.$$

If the voltage source and ammeter are interchanged, the ammeter will still read 1.75 A. We verify this by solving the circuit shown in



$$\frac{V_{bd}}{60} + \frac{V_{bd}}{30} + \frac{V_{bd} - 15}{20} = 0.$$

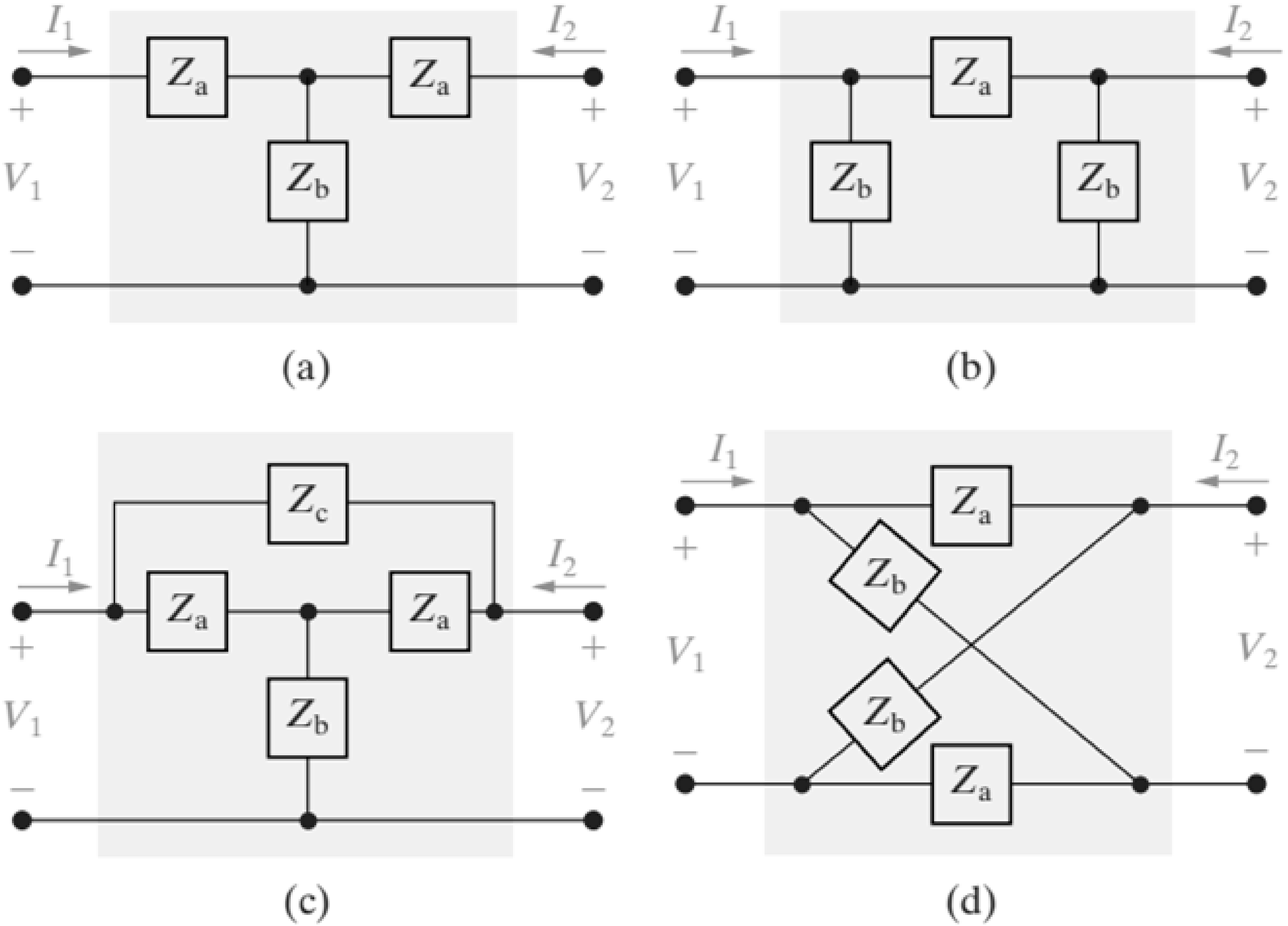
$V_{bd} = 7.5$ V. The current $I_{ad}$ equals

$$I_{ad} = \frac{7.5}{30} + \frac{15}{10} = 1.75 \text{ A}.$$

A two-port circuit is also reciprocal if the interchange of an ideal current source at one port with an ideal voltmeter at the other port produces the same voltmeter reading.

For a reciprocal two-port circuit, only three calculations or measurements are needed to determine a set of parameters. A reciprocal two-port circuit is **symmetric** if its ports can be interchanged without disturbing the values of the terminal currents and voltages.

Figure shows four examples of symmetric two-port circuits. In such circuits, the following additional relationships exist among the port parameters



$I_1$
$I_2$
$Z_a$
$Z_b$
$Z_c$
$V_1$
$V_2$
+
−
(a)
(b)
(c)
(d)