Nuclear Reactions

- Radioactive Decay
  - Components of the Nucleus
  - Types of Radioactive Emissions
  - Types of Radioactive Decay
    - Nuclear Equations
  - Nuclear Stability and Mode of Decay
- The Kinetics of Radioactive Decay
  - Rate of Radioactive Decay
  - Radioisotope Dating

Nuclear Reactions

- Nuclear Transmutation: Induced Changes in Nuclei
  - Early transmutation Experiments
    - Nuclear Shorthand Notation
  - Particle Accelerators
- The effects of nuclear Radiation on Matter
  - Effects of Ionizing Radiation on Living Matter
  - Sources of Ionizing Radiation
  - Assessing the Risk from Ionizing Radiation

Nuclear Reactions

- Applications of Radioisotopes
  - Radioactive Tracers
  - Additional Applications of Ionizing Radiation
- The Interconversion of Mass and Energy
  - The Mass Differences Between a Nucleus and its Nucleons
  - Nuclear Binding Energy and Binding Energy per Nucleon
- Applications of Fission and Fusion
  - Nuclear Fission
  - Nuclear Fusion

Nuclear Reactions

- Comparison of Chemical & Nuclear Reactions

Chemical Reactions

1. One substance is converted into another, but atoms never change identity.
2. Orbital electrons are involved as bonds break and form; nuclear particles do not take part.
3. Reactions are accompanied by relatively small changes in energy and no measurable changes in mass.
4. Reaction rates are influenced by temperature, concentration, catalysts, and the compound in which an element occurs.

Nuclear Reactions

1. Atoms of one element typically are converted into atoms of another element.
2. Protons, neutrons, and other particles are involved; orbital electrons take part much less often.
3. Reactions are accompanied by relatively large changes in energy and measurable changes in mass.
4. Reaction rates depend on number of nuclei, but are not affected by temperature, catalysts, or, except on rare occasions, the compound in which an element occurs.

Nuclear Reactions

- Atoms
  - Mass
    - Electrons 9.10939 x 10^{-31} kg
    - Nucleons
      - Protons 1.67262 x 10^{-27} kg
      - Neutrons 1.67493 x 10^{-27} kg
  - Mass of Atom
    - Electrons – 0.03 %
    - Nucleons – 99.97%
  - Volume
    - Nucleus – 10^{-15} the size of an atom
  - Density
    - Nucleus – 10^{14} g/ml

Nuclear Reactions

- Nucleus
  - Nucleons
    - Elementary Particles
      - Protons – Z
      - Neutrons – N
- Isotopes
  - Each unique combination of protons & neutrons represents an "isotope"
  - Most atoms have more than one isotope

Nuclear Reactions

- Isotopes

Isotope Notation $^{A}_{Z}X$

X - Symbol for Particle (H - hydrogen; Cl - Chlorine; Mg - Magnesium, etc.)

Z - Number of Protons

A - Atomic Mass Number (Total Number of Nucleons - Protons + Neutrons)

Note : Atomic Mass(Atomic Weight) - The average of the atomic mass numbers of the naturally occurring isotopes of an element

N - Number of Neutrons (A - Z)

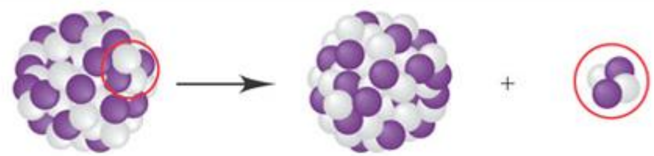
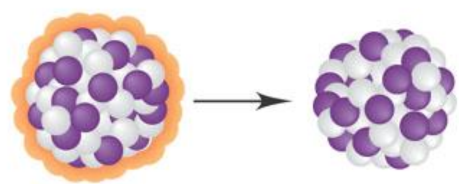
$^{1}_{1}H$ $^{2}_{1}H$ $^{3}_{1}H$ $^{12}_{6}C$ $^{13}_{6}C$ $^{37}_{17}Cl$ $^{35}_{17}Cl$ $^{63}_{29}Cu$ $^{65}_{29}Cu$

Nuclear Reactions

- Nuclear Stability
  - Most nuclei are "Unstable"
  - Unstable nuclei exhibit "Radioactivity"
    - Radioactivity – Spontaneous disintegration (decay) by emitting radiation
    - Radiation
      - Alpha – $^{4}_{2}He$ (Helium particles)
      - Beta – $^{0}_{-1}\beta$ (High speed electrons)
      - Gamma – $^{0}_{0}\gamma$ (High energy photons)
      - Half-Life – time required for half the initial number of nuclei to decay
  - Many (most) unstable isotopes appear to be stable because Radioactive Half-Life is very long (>10^9 yrs)

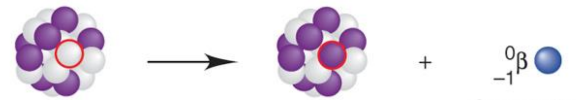
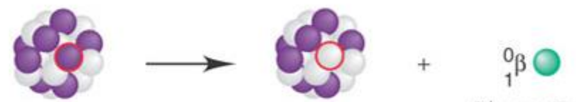
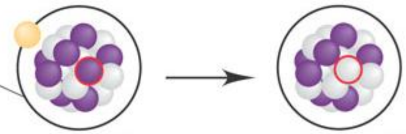
Modes of Radioactive Decay

- Alpha Decay & Gamma Decay

Mode	Emission	Decay Process	Change in		
			A	Z	N
α Decay	α ($^4_2He^{2+}$)	 → + α expelled	−4	−2	−2
Gamma (γ) emission	γ	 excited nucleus → stable nucleus + γ (γ photon radiated)	0	0	0

Modes of Radioactive Decay

- Beta Decay

Mode	Emission	Decay Process	Change in A	Change in Z	Change in N
β^- Decay†	β^- ($^{0}_{-1}\beta$)	 nucleus with xp^+ and yn^0 → nucleus with $(x+1)p^+$ and $(y-1)n^0$ + $^{0}_{-1}\beta$ (β^- expelled) Net: $^{1}_{0}n$ (in nucleus) → $^{1}_{1}p$ (in nucleus) + $^{0}_{-1}\beta$ (β^- expelled)	0	+1	−1
Positron (β^+) emission†	β^+ ($^{0}_{1}\beta$)	 nucleus with xp^+ and yn^0 → nucleus with $(x-1)p^+$ and $(y+1)n^0$ + $^{0}_{1}\beta$ (β^+ expelled) Net: $^{1}_{1}p$ (in nucleus) → $^{1}_{0}n$ (in nucleus) + $^{0}_{1}\beta$ (β^+ expelled)	0	−1	+1
Electron (e^-) capture (EC)†	x-ray	 low-energy orbital; nucleus with xp^+ and yn^0 → nucleus with $(x-1)p^+$ and $(y+1)n^0$ Net: $^{0}_{-1}e$ (absorbed from low-energy orbital) + $^{1}_{1}p$ (in nucleus) → $^{1}_{0}n$ (in nucleus)	0	−1	+1

Nuclear Reactions

- Radioactive Decay
  - When a nuclide decays, it forms a new, more stable nuclide of "lower" energy
  - Excess energy from nuclear decay is removed as emitted radiation and the recoil of the nucleus
  - Product nuclide is called the "daughter" product
  - For a nuclear reaction:

    The total "Z" (# of protons) and the total "A" (sum of protons + neutrons) of the "Reactants" must equal those of the "Products"

Total A / Total Z Reactants = Total A / Total Z Products

Nuclear Reactions

- Alpha (α) decay
  - Loss of alpha (α) particle

    (Helium [$^{4}_{2}He$] nucleus)
  - "A" decreases by 4; "Z" decreases by 2
  - Most common form of radiation by heavy, unstable nuclei to become "more" stable

$$^{226}_{88}\mathrm{Ra} \rightarrow \; ^{222}_{86}\mathrm{Rn} + \; ^{4}_{2}\alpha$$

Nuclear Reactions

- Beta (β) decay
  - 3 types
    - Beta$^-$ (β^-) decay ("negatron" emission)
      - Does not involve expulsion of β^- particle present in nucleus
      - A neutron is converted into a proton, which remains in the nucleus and a β^- particle, which is expelled

$$^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}B^{-}$$

$$^{63}_{28}\mathrm{Ni} \rightarrow {}^{63}_{29}\mathrm{Cu} + {}^{0}_{-1}B^{-}$$

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}B^{-}$$

      - Note: β^- decay results in a product nuclide with the same "A" but a "Z" one "higher" (one more proton – increased nuclear charge) than in the reactant, i.e. element with the next "higher" atomic number is formed

Nuclear Reactions

- Beta (β) decay (Con't)
  - 3 types (Con't)
    - Positron Emission
      - Emission of β^+ particle from the nucleus
      - β^+ particle $({}^{0}_{1}e)$ is called a "Positron"
      - The Positron is the "antiparticle" of the electron
      - A key tenant in Modern Physics is that most "fundamental" particles have corresponding "antiparticles"
      - Positron emission – Proton in nucleus is converted into a neutron, and a positron is expelled – Opposite effect of β^- emission
        Product has same "A", but Z is one "lower" (1 less proton – element with next lower atomic number is formed)

$$ {}^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{5}\mathrm{B} + {}^{0}_{1}e \quad \textit{(positron)} $$

Nuclear Reactions

- Beta (β) decay (Con't)
  - 3 types (Con't)
    - Electron (e^-) Capture
      - The interaction of the nucleus with an electron from a low atomic energy orbital
      - A "proton" is converted to a "neutron"

$$ {}^{1}_{1}\mathbf{p} \quad + \quad {}^{1}_{0}\boldsymbol{e} \quad \rightarrow \quad {}^{1}_{0}\mathbf{n} \quad + \quad \mathbf{x\text{-}rays} $$

      - The orbital "vacancy" is filled by an electron from a "Higher" energy level with the energy difference being carried off by x-ray photons and neutrinos

$$ {}^{55}_{26}\mathbf{Fe} \quad + \quad {}^{0}_{-1}\mathbf{e} \quad \rightarrow \quad {}^{55}_{25}\mathbf{Mn} \quad + \quad \mathbf{h\nu} \;\; \mathbf{(x\text{-}rays\ \&\ neutrinos)} $$

      - Electron capture has the same effect as positron emission: "Z" lower by "1", "A" unchanged

Nuclear Reactions

- Gamma (γ) Emission
  - Emission (radiation) of high-energy γ photons from an "excited" nucleus
  - An "excited" nucleus is the product of a nuclear process
  - The nucleus returns to a lower energy (more stable) by emitting the gamma photons of varying energies
  - Gamma emissions accompany many (mostly β) types of decay

$$^{215}_{84}\mathrm{Po} \rightarrow {}^{211}_{82}\mathrm{Pb} + {}^{4}_{2}\alpha + \text{gamma rays}$$

$$^{99}_{43}\mathrm{Tc} \rightarrow {}^{99}_{44}\mathrm{Ru} + {}^{0}_{-1}B + \text{several gamma}$$

  - Gamma rays have no mass or charge
  - Gamma emission does not change A or Z
  - Positrons & electrons annihilate each other with the release of energy as γ rays

$$^{0}_{1}B + {}^{0}_{-1}\mathrm{e} \rightarrow 2\,{}^{0}_{0}\gamma$$

Practice Problem

- Naturally occurring Thorium (Th) under goes α decay

$$^{232}_{90}\text{Th} \rightarrow \;^{A}_{Z}\text{X} + \;^{4}_{2}\alpha$$

$$^{232}_{90}\text{Th} \rightarrow \;^{228}_{88}\text{Ra} + \;^{4}_{2}\alpha$$

- Zirconium-86 undergoes electron capture

$$^{86}_{40}\text{Zr} + \;^{0}_{-1}\text{e} \rightarrow \;^{A}_{Z}\text{X}$$

$$^{86}_{40}\text{Zr} + \;^{0}_{-1}\text{e} \rightarrow \;^{86}_{39}\text{Y} \;\; \text{(Yttrium)}$$

Note: Proton is converted to a neutron

Nuclear Reactions

- Nuclear Stability and the Mode of Radioactive Decay
  - Unstable nuclides can decay in several ways
  - Whether a nuclide will or not decay can be predicted
  - The mode of decay can be predicted
- Band of Stability
  - Stability is a function of 2 factors
    - The number of neutrons (N), the number of protons (z), and their ratio (N/Z)
      - Related primarily to nuclides that undergo one of the 3 modes of β decay
    - The total mass of the nuclide
      - Related to nuclides that undergo α decay

Nuclear Reactions

- Band of Stability (Con't)
  - Plot of number of Neutrons vs number of Protons
    - No. Neutrons (N) = Mass number (A) – No. Protons (Z)
    - Stable Nuclides (N/Z < 1) (Very few exist)

$$^{1}_{1}\mathrm{H} \quad N = 1 - 1 = 0; \quad Z = 1; \quad N/Z < 1$$

$$^{3}_{2}\mathrm{He} \quad N = 3 - 2 = 1; \quad Z = 2; \quad N/Z < 1$$

    - Stable Nuclides (N = Z; N/Z =1)

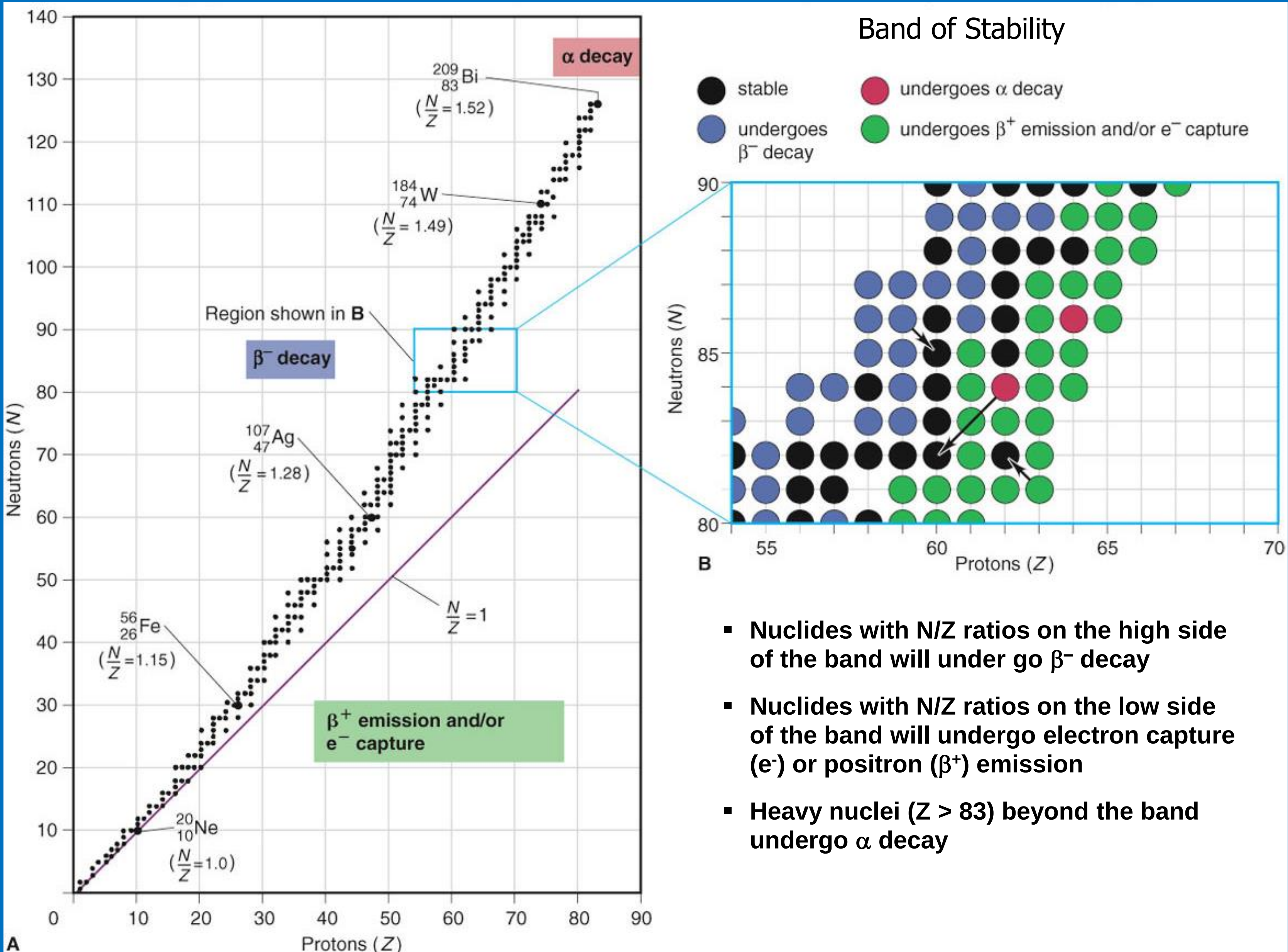
$$^{4}_{2}\mathrm{He} \qquad ^{12}_{6}\mathrm{C} \qquad ^{16}_{8}\mathrm{O} \qquad ^{20}_{10}\mathrm{Ne}$$

    - The ratio N/Z (neutrons/protons) gradually increases (N/Z >1) beyond Z = 10
    - As the N/Z ratio increases, stable nuclides will tend to be confined to a narrow band
    - Nuclides with N/Z ratios above or below the "band of stability" will undergo nuclear decay

Nuclear Reactions

- Band of Stability (Con't)
  - Nuclides with N/Z ratios on the high side of the band will under go β^- decay
  - Nuclides with N/Z ratios on the low side of the band will undergo electron capture (e^-) or positron (β^+) emission
  - Heavy nuclei beyond the band undergo α decay
  - All nuclei with $Z > 83$ are "unstable" and radioactive
    - Includes the largest "down group" members of the groups 1A(1) through 8A(18), actinium and the actinides (Z = 89 -103), and the other elements of the 4th (6d) transition series (Z = 104 – 112)

Nuclear Reactions



- **Nuclides with N/Z ratios on the high side of the band will under go β^- decay**
- **Nuclides with N/Z ratios on the low side of the band will undergo electron capture (e^-) or positron (β^+) emission**
- **Heavy nuclei (Z > 83) beyond the band undergo α decay**

Nuclear Reactions

- Stability and Nuclear Structure
  - Protons have a Positive Charge & Neutrons are Neutral

What Holds the Nucleus Together??

  - Electrostatic Repulsive forces between protons that should break the nucleus apart are balanced by the presence of the "Attractive Strong Force"
  - The Strong Force exists between ALL nucleons
  - The Strong Force is "137" times as strong as the "Repulsive Forces" operating, but it operates over very short distances within the nucleus
  - Competition between the "strong force" and the "repulsive forces" determines the stability of a nucleus

Nuclear Reactions

- Stability and Nuclear Structure (Con't)
  - Oddness vs Evenness of N (neutrons) & Z (protons)
    - Elements with an "even" no. of protons (Z) usually have a larger number of stable nuclides
    - Well over half of the "stable" elements have both even N and even Z
    - Only 4 elements with "odd" N and "odd" Z are stable:

$$^{2}_{1}\mathrm{H} \qquad ^{6}_{3}\mathrm{Li} \qquad ^{10}_{5}\mathrm{B} \qquad ^{14}_{7}\mathrm{N}$$

  - Nucleon Energy Levels
    - Similar to the pairing of electron spins (+½ & -½), nucleons also exhibit spin properties
    - Nuclear stability is increased when both "N" & "Z" have "like" spin

Nuclear Reactions

- Stability and Nuclear Reactions

Number of Stable Nuclides
For Elements 48 - 54

Element	Atomic No. (*Z*)	No. of Nuclides
Cd	**48**	**8**
In	49	2
Sn	**50**	**10**
Sb	51	2
Te	**52**	**8**
I	53	1
Xe	**54**	**9**

*Even *Z* shown in boldface.

Even – Odd breakdown
of Stable Nuclides

Z	*N*	No. of Nuclides
Even	Even	157
Even	Odd	53
Odd	Even	50
Odd	Odd	4
	TOTAL	264

Nuclear Reactions

- The noble gases with Z = 2 (He), 10 (Ne), 18 (Ar), etc. are exceptionally stable (completed outer electron shells
- Nuclides with N or Z values of 2, 8, 20, 28, 50, 82 (and N = 126, designated "magic numbers," are also very stable because it is believed they correspond to the number of neutrons or protons in "filled nucleon" levels

$$^{50}_{22}\mathbf{Ti}\ (\mathbf{N = 28}) \qquad ^{88}_{38}\mathbf{Sr}\ (\mathbf{N = 50}) \qquad \mathbf{Sn}\ (\mathbf{10\ nuclides}\ [\mathbf{Z = 50}])$$

Nuclear Reactions

- Predicting the Mode of Radioactive Decay
  - An unstable nuclide 'generally' decays in a 'mode' that shifts its N/Z ratio toward the "Band of Stability"
    - Neutron-rich Nuclides
      - Mass No. (A) > Atomic Mass of Element
      - High N/Z lie above band of stability
      - Undergo β^- (negatron emission), which converts a neutron into a proton, reducing the N/Z ratio
    - Proton-Rich Nuclides
      - Mass No. (A) < Atomic Mass of Element
      - Low N/Z lie below band of stability
      - Lighter elements undergo β^+ (positron emission) and heavier elements undergo e^- capture, both of which convert a proton into a neutron, increasing the N/Z ratio

Nuclear Reactions

- Predicting the Mode of Radioactive Decay (Con't)
  - Heavy Nuclides
    - Nuclides with Z > 83
    - Too heavy to be stable
    - Undergo α decay, which reduces both the Z and N values by two units per emission

Practice Problem

Which of the following would be expected to be "stable" and which would be expected to be "unstable?"

(a) $^{18}_{10}\text{Ne}$ $\quad$ $N = 8\ (18 - 10) \quad Z = 10 \quad N/Z = 8/10 = 0.8 \quad N < Z$

Unstable - Too few neutrons; nuclide below band of stability

B^{+} **(Positron Decay)**

(b) $^{32}_{16}\text{S}$ $\quad$ $N = Z = 16 \quad N/Z = 1.0$

Stable - With Z < 20, Even Z and Even N

(c) $^{236}_{90}\text{Th}$ $\quad$ $N = 146\ (236 - 90) \quad Z = 90 \quad Z > 83$

Unstable - Radioactive

(d) $^{123}_{56}\text{Ba}$ $\quad$ $N = 67; \quad Z = 56; \quad N/Z = 1.20 < 1.3$ **(from band of stability)**

Stable

For Z values of 55 - 60, band of stability has N / Z of about 1.3

Nuclide has too few Neutrons

Practice Problem

Predict the Mode of Decay for the following

(a) $^{12}_{5}B$ $Z = 5$ (boron); $N / Z = 12 / 5 = 2.4$; $A = 12 >$ Atomic Mass (10.81)

Above Band of Stability $\therefore$ Neutron Rich $\Rightarrow$ B^- Decay

(b) $^{234}_{92}U$ $Z = 92$ (Uranium); $Z > 83$ $\therefore$ Alpha (α) decay

(c) $^{81}_{33}As$ $Z = 33$ (Arsenic) $A = 81 >$ Atomic Mass (74.92)

$N = A - Z = 81 - 33 = 48$ $N / Z = 48 / 33 = 1.45$

Above Band of Stability $\therefore$ Neutron Rich $\Rightarrow$ B^- Decay

(d) $^{127}_{57}La$ $Z = 57$ (Lanthanum)

$A = 127 <$ Atomic Mass (138.9) $\therefore$ Proton Rich

$N = A - Z = 127 - 57 = 70$ $N / Z = 70 / 57 = 1.23$

Below Band of Stability $\therefore$ proton rich $\Rightarrow$ B^+ emission and / or e^- capture

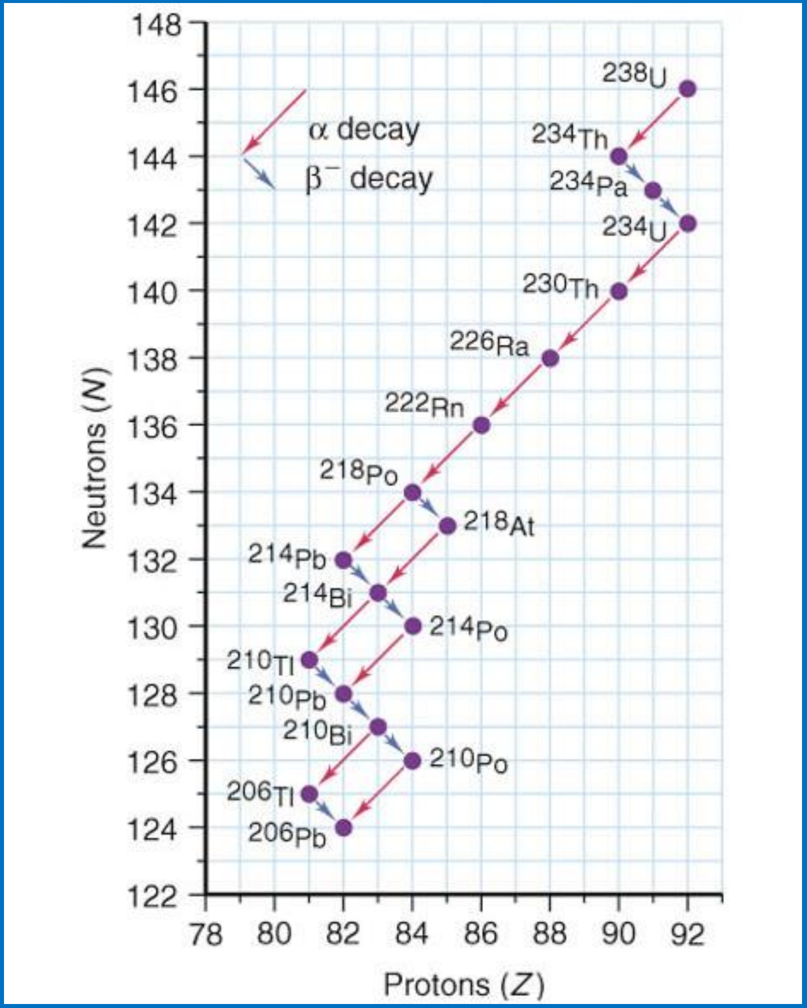
Nuclear Reactions

- Decay Series
  - Parent nuclide may undergo a "series" of decay steps before a "stable daughter" nuclide is formed

The Uranium–238 Decay Series

U-238 undergoes a series of 14 steps involving both α and β decay until Pb-206 forms

Gamma (γ) emission accompanies many steps



Nuclear Reactions

- Kinetics of Radioactive Decay
  - Chemical and Nuclear systems both tend toward maximum stability (equilibrium)
  - The type and number of nucleons in an unstable nucleus change in a predictable direction to give a stable N/Z ratio
  - The tendency of a reaction (chemical or nuclear) toward equilibrium does not relate to the amount of time it takes to reach that equilibrium
  - Radioactive (unstable) nuclei decay at a characteristic rate, regardless of the chemical substance in which they occur

Nuclear Reactions

- Decay Rate (Activity) is the change in the number of nuclei divided by the change in time

$$\textbf{Decay Rate (A)} = -\frac{\Delta N}{\Delta t}$$

- The “metric system” unit of radioactivity is the “Becquerel (Bq) – 1 disintegration per second (d/s)
- The “Curie” (Ci), originally defined as the number of disintegrations per second in “1 gram radium-226,” is now fixed at:

$$1\,\text{Ci} = 3.70 \times 10^{10}\ \text{d/s}$$

- Millicuries (mCi) & microcuries (μCi) are commonly used

Nuclear Reactions

- Decay Rate per unit time is proportional to the number of radioactive nuclei present

$$\textbf{Decay Rate } (A) \propto N$$

$$A = kN$$

where k is called the Decay Constant

$$\textbf{Recall: } A = -\frac{\Delta N}{\Delta t}$$

$$A = -\frac{\Delta N}{\Delta t} = kN$$

Activity depends only on N raised to 1^{st} power and the value of k

$\therefore$ Radioactivity is a 1^{st} order reaction

Nuclear Reactions

- Radioactive Half-Life
  - The half-life of a nuclide is the time it takes for half the nuclei present to decay
  - The number nuclei remaining is halved after each half-life

$$ {}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}B \qquad t_{1/2} = 5,730\ \mathrm{yr} $$

  - Derivation of half-life expression

$$ A = -\frac{\Delta N}{\Delta t} = kN $$

$$ \frac{\Delta \mathrm{N}}{\mathrm{N}} = -\mathrm{k}\Delta \mathrm{t} $$

Integration

$$ \int \frac{\mathrm{dx}}{\mathrm{x}} = \ln(\mathrm{x}) + \mathrm{C} \qquad \text{(standard integral)} $$

Nuclear Reactions

- Integrate both sides of equation

$$\int \frac{d(N)}{N} = \int -kdt = -k\int dt$$

$$\ln[N] = -kt + C$$

- At time zero ($t_o = 0$) $\quad N = N_0$

$$-kt = 0 \qquad C = \ln(N_o)$$

$$\ln(N) = -kt + \ln(N_0)$$

$$\ln\frac{N}{N_0} = -kt \qquad N = N_0 \times e^{-kt} \qquad \ln\frac{N_0}{N} = kt$$

- At $t_{1/2}$ = half-life, $N = ½ N_o$

$$t_{1/2} = \frac{\ln\frac{N_0}{1/2N_0}}{k} = \frac{\ln 2}{k} = \frac{0.693}{k}$$

- Half-life is not dependent on the number of nuclei and is inversely related to the decay constant

Practice Problem

If a sample of Sr-90 has an activity of 1.2×10^{12} d/s, what are the activity and the fraction of nuclei that have decayed after 59 yr. ($t_{½}$ Sr-90 = 29 yr)

Activity of sample is proportional to number of nuclei

$$\text{Fraction Decayed} = \frac{N_0 - N_t}{N_0} = \frac{A_0 - A_t}{A_0}$$

Calculate Decay Constant (k)

$$t_{1/2} = \frac{0.693}{k} \qquad k = \frac{0.693}{t_{1/2}} = \frac{0.693}{29\text{yr}} = 0.024\ \text{yr}^{-1}$$

$$\ln\frac{N_0}{N} = \ln\frac{A_0}{A_t} = kt \qquad \ln A_0 - \ln A_t = kt$$

$$\ln A_t = -kt + \ln A_0 = -(0.024\ \text{yr}^{-1} \times 59\ \text{yr}) + \ln(1.2 \times 10^{12}\ \text{d/s})$$

$$\ln A_t = -1.4 + \ln(27.81) = 26.4$$

$$A_t = 2.9 \times 10^{11}\ \text{d/s}$$

$$\text{Fraction Decayed} = \frac{A_0 - A_t}{A_0} = \frac{1.2 \times 10^{12}\ \text{d/s} - 2.9 \times 10^{11}\ \text{d/s}}{1.2 \times 10^{12}\ \text{d/s}} = 0.76$$

Nuclear Reactions

- Radioisotope Dating
  - Understanding of prehistory utilizes radioisotopes to determine the ages of trees, rocks, ice, oceanic deposits
  - Radiocarbon dating is based on measurements of the amounts of ^{14}C and ^{12}C in materials of biological origin
    - High energy cosmic rays, consisting mainly of protons, enter the atmosphere initiating a cascade of nuclear events
    - One of these reactions produces neutrons which bombard ordinary Nitrogen $^{14}_{7}N$ to form $^{14}_{6}C$

$$^{14}_{7}N + ^{1}_{0}n \rightarrow ^{14}_{6}C + ^{1}_{1}H$$

    - Carbon-14 is radioactive with a half-life of 5730 yrs
    - Useful for dating objects up to 6 half-lives of $^{14}_{6}C$ or about 36,000 yrs

Nuclear Reactions

- The C-14 atoms combine with CO_2 atoms, diffuse throughout the lower atmosphere, and enter the total carbon pool as gaseous $^{14}CO_2$ and aqueous $H^{14}CO_3^-$ (Bicarbonate)
- The ratio of C-12/C-14 in the environment – atmosphere, water, plants, and animals - is thought to have been a constant at about 10^{12}/l for thousands of years, thus useful for dating
- When an organism dies, it no longer absorbs or releases CO_2 and the ratio of C-12/C-14 increases as the amount of C-14 decays to N-14

$$^{14}_{6}\mathbf{C} \rightarrow \ ^{14}_{7}\mathbf{N} + \ ^{0}_{-1}\boldsymbol{B}$$

- The ratio of C-12/C-14 in a dead organism and the ratio in living organisms reflects the time elapsed since the organism dies

Nuclear Reactions

- The 1st order rate equation in the activity form can be rearranged to solve for the age of an object

$$\ln \frac{A_0}{A} = kt$$

$$t = \frac{1}{k} \ln \frac{A_0}{A}$$

Where: A_0 - is the activity in a living organism

A - is the activity in an object whose age is unknown

Practice Problem

A sample of a bone has a specific activity of 5.22 disintegrations per minute per gram of carbon (d/min•g)

If the C-12/C-14 ratio for living organisms results in a specific activity of 15.3 d/min•g, how old is this bone?

Ans: Compute k for C-14 decay

$$k = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{5730\ \text{yr}} = 1.21 \times 10^{-4}\ \text{yr}^{-1}$$

Calculate the age of the bone

$$t = \frac{1}{k} \ln \frac{A_0}{A} = \frac{1}{1.21 \times 10^{-4}\ \text{yr}^{-1}} \times \ln\left(\frac{15.3\ \text{d/min} \bullet \text{g}}{5.22\ \text{d/min} \bullet \text{g}}\right) = 8.89 \times 10^{3}\ \text{yr}$$

Nuclear Reactions

- Nuclear Transmutation - Alchemy???
  - Alchemists' dream of changing base metals into Gold has never materialized
  - Elements can be changed into another elements
  - Nuclear Transmutation is the induced conversion of one nucleus into another by high energy bombardment of a nucleus in a "particle accelerator"
  - First transmutation occurred in 1919 by Rutherford
    - Alpha particles from radium bombarded atmospheric nitrogen to form a proton and O-17

$$^{14}_{7}\mathrm{N} + {}^{4}_{2}\alpha \rightarrow {}^{1}_{1}\mathrm{p} + {}^{17}_{8}\mathrm{O}$$

    - Shorthand Notation

$$^{14}N(\alpha,\rho)^{17}O$$

Nuclear Reactions

- 1^{st} Artificial Radioisotope
  - Marie & Frederick Joliot-Curie created P-30 when they bombarded Aluminum foil with α-particles

$$^{27}_{13}\mathbf{Al} + {}^{4}_{2}\alpha \rightarrow {}^{1}_{0}\mathbf{n} + {}^{30}_{15}\mathbf{P} \quad \text{or} \quad {}^{27}\mathbf{Al}(\alpha, \mathbf{n})\,{}^{30}\mathbf{P}$$

- Particle Accelerators
  - Particle accelerators were invented to:
    - Impart high-energy to particles
    - Provide Electric fields in conjunction with magnetic fields to accelerate the particles
    - Provide detectors to record the results of the impact of these particles on selected target nuclei
    - Common particles – neutrons, alpha-particles, protons, deuterons (from stable deuterium)-

Nuclear Reactions

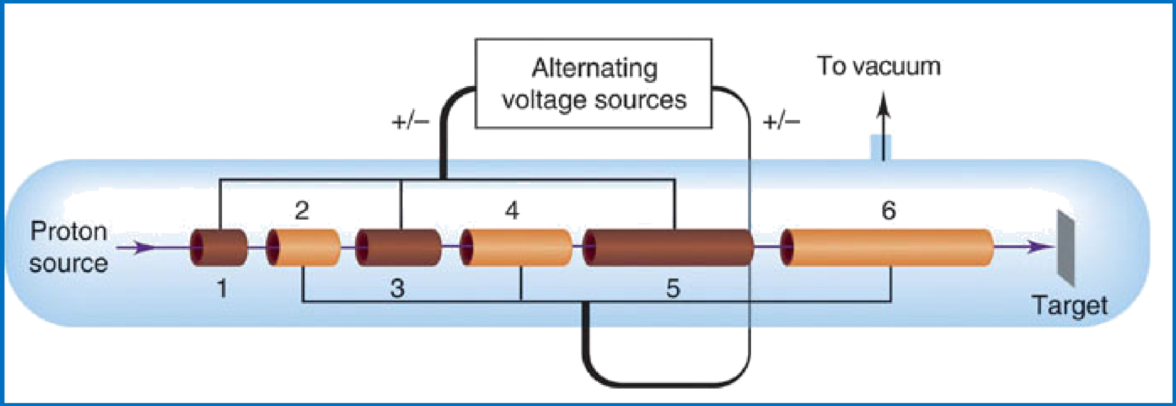
- Linear Accelerators
  - Series of separated tubes of increasing length that, through a source of "alternating voltage", change their charge from positive to negative in synchrony with the movement of the particle
  - A charged particle is first attracted to a tube, then repelled from the tube as the tube changes polarity
  - The particle is then attracted to the next larger tube
  - The particle is accelerated at higher energy across the gap between the tubes
  - As the tubes get larger the particle continues to gain energy

Nuclear Reactions

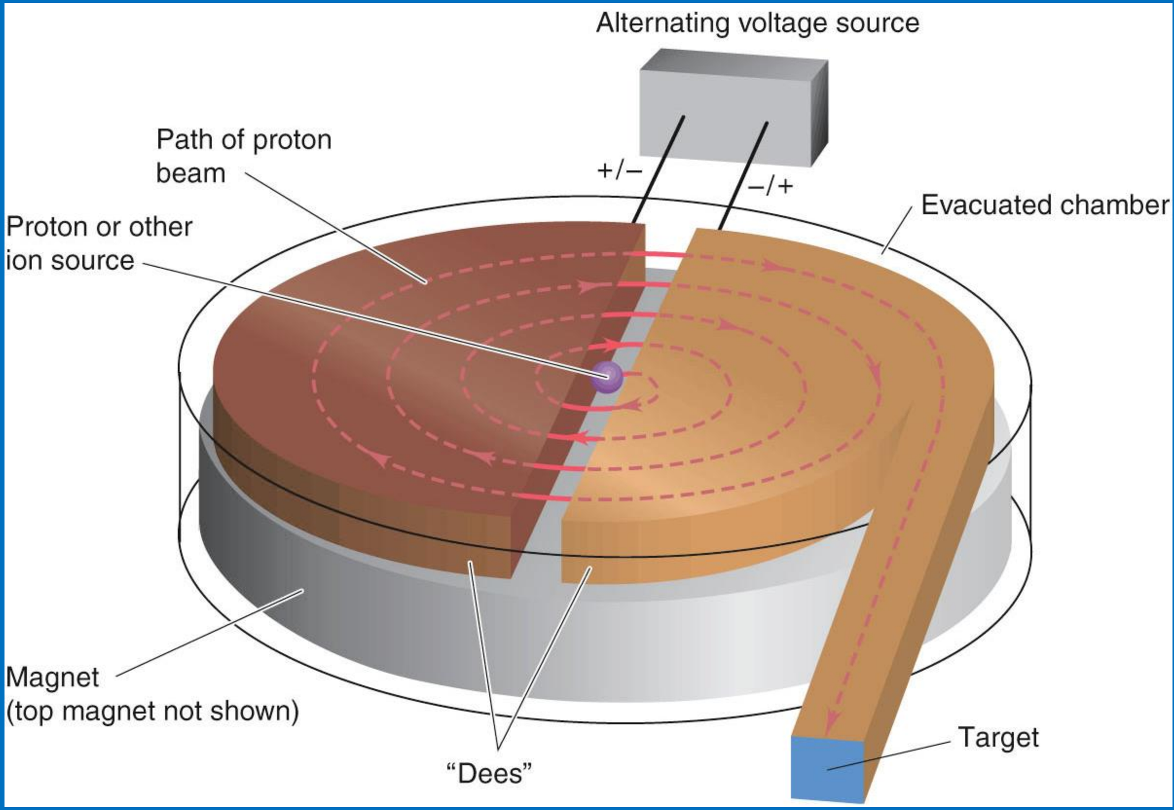
- Cyclotrons
  - Invented by E.O. Lawrence in 1930
  - Use principle of "linear" accelerator, but uses electromagnets to give the particle a "Spiral" path to save space
  - Magnets lie within an "evacuated chamber" above and below Dees
  - Dees are open, D-shaped electrodes that act like the tubes in the linear accelerator, continuously switching from positive to negative
  - The speed (energy) and radius of the particle trajectory is increased until it is deflected toward the target at the appropriate energy

Nuclear Reactions

Linear Accelerator



Cyclotron



Nuclear Reactions

- Synchrotron
  - Use a synchronously increasing magnetic field to make the particle's path "circular" rather than spiral
- Accelerator Applications
  - Production of radioisotopes for medical applications
  - Fundamental Research into the nature of matter
  - Synthesis of Transuranium Elements with atomic numbers greater than uranium
    - Uranium – Highest atomic number (92) and atomic mass of all naturally occurring elements
    - Transuranium elements include:
      - The remaining actinides (Z = 93 to 103 where the 5f sublevel is being filled)
      - Elements in the 4th transition series (Z=104 – 112; where the 6d sublevel is being filled)

Nuclear Reactions

Formation of some Transuranium Nuclides

Reaction							Half-life of Product
$^{239}_{94}Pu$	+	2^1_0n	$\longrightarrow$	$^{241}_{95}Am$	+	$^{0}_{-1}\beta$	432 yr
$^{239}_{94}Pu$	+	$^4_2\alpha$	$\longrightarrow$	$^{242}_{96}Cm$	+	^1_0n	163 days
$^{241}_{95}Am$	+	$^4_2\alpha$	$\longrightarrow$	$^{243}_{97}Bk$	+	2^1_0n	4.5 h
$^{242}_{96}Cm$	+	$^4_2\alpha$	$\longrightarrow$	$^{245}_{98}Cf$	+	^1_0n	45 min
$^{253}_{99}Es$	+	$^4_2\alpha$	$\longrightarrow$	$^{256}_{101}Md$	+	^1_0n	76 min
$^{243}_{95}Am$	+	$^{18}_{8}O$	$\longrightarrow$	$^{256}_{103}Lr$	+	5^1_0n	28 s

*Like chemical reactions, nuclear reactions may occur in several steps. For example, the first reaction here is actually an overall process that occurs in three steps:
(1) $^{239}_{94}Pu + ^1_0n \longrightarrow ^{240}_{94}Pu$ (2) $^{240}_{94}Pu + ^1_0n \longrightarrow ^{241}_{94}Pu$ (3) $^{241}_{94}Pu \longrightarrow ^{241}_{95}Am + ^{0}_{-1}\beta$

Nuclear Reactions

- Effects of Nuclear Radiation on Matter
  - Nuclear Changes cause chemical changes in surrounding matter
    - The change in the nucleus does not affect the electron configuration
    - The radiation from a nuclear change, however, does affect the electrons in "nearby" atoms
    - Virtually all radioactivity causes "ionization" in surrounding matter, as the emission particles collide with the atoms and dislodged electrons ionizing the atom or molecule

$$\text{Atom} \xrightarrow{\text{ionizing radiation}} \text{ion}^{+} + e^{-}$$

    - From each ionization event, a cation and a free electron result
    - The number of cation-electron pairs formed is directly related to the energy of ionizing radiation

Nuclear Reactions

- Effects of Ionizing Radiation of Living Matter
  - Ionizing radiation has a destructive effect on living tissue
  - Ionization effects on key biological macromolecules or cell membranes can devastate the cell, even the organism
  - Danger from a radionuclide depends on three (3) factors
    - Type of radiation
    - Half-life
    - Biochemical Behavior

Nuclear Reactions

- Type of Radiation
  - Uranium-235 (^{235}U)
    - Long half-life
    - Excreted rapidly from body
    - Little effect on tissue
  - Plutonium-239 (^{239}Pu)
    - Long half-life
    - Behave like "calcium" and is incorporated into bones and teeth
  - Strontium-90 ($^{90}Sr^{2+}$) from nuclear explosions
    - Also behaves similar to calcium

Nuclear Reactions

- Units of Radiation Dosage
  - The number of cation-electron pairs produced in a given amount of living tissue is a measure of the energy absorbed by the tissue
  - The standard metric system unit for such energy absorption is the "Gray" – 1 Joule of energy absorbed per "kilogram" of body tissue

$$1\,\text{Gy} = 1\,\text{J/kg}$$

  - A more widely used unit is the "Rad" (radiation-absorbed dose) – 0.01 Gy

$$1\ \text{rad} = 0.01\ \text{J/kg} = 0.01\,\text{Gy}$$

Nuclear Reactions

- Measure of Actual Tissue Damage must account for:

  Strength of Radiation

  Exposure Time

  Type of Tissue

- Multiply the number of "rads" by a "Relative Biological Effectiveness (RBE) factor", which depends on the effect of given type of radiation on a given tissue or body part
- The Product of "Rads" x RBE is defined as the "Rem"

The "Rem" is the unit of radiation dosage equivalent of a given amount of damage in a human

$$\textbf{no. of rems} = \textbf{no. of rads} \times \textbf{RBE}$$

$$\textbf{1 rem} = \textbf{0.01 Sv} \quad \textbf{(Sievert (Sv) - SI unit for dosage equivalent)}$$

$$\textbf{'rems' often expressed in "millirems" } (\mathbf{10^{-3}} \textbf{ rem})$$

Nuclear Reactions

- Penetrating Power of Emissions
  - Effect of radiation on tissue depends on the penetrating power and ionizing ability of the radiation
  - Penetrating Power is "inversely" related to the "mass, charge, and energy" of the emission
  - If a particle interacts "strongly" with matter it does not penetrate very far
    - Alpha (α) particles
      - Massive
      - Highly charged
      - Interact most strongly with matter
      - Penetrate very lightly, even paper can stop it
      - Internally, α particles cause localized damage

Nuclear Reactions

- Beta Particles (β^-) Positrons (β^+)
  - Have less charge and much less mass
  - Interact less strongly than α particles
  - Lower chance of producing ionization
  - More potential destructive external source because penetrating power is greater
  - Heavy clothing or thick metal (0.5 cm) required to stop penetration
- Gamma Rays (γ)
  - Massless
  - Interact least with matter (penetrate most)
  - Most dangerous, ionizing many layers of tissue
  - Several inches lead required to stop penetration

Nuclear Reactions

- Molecular Interactions
  - Ionizing radiation causes the loss of an electron from a bond or a long pair of electrons
  - Resulting species is "charged" and proceeds to form a "Free Radical"

<u>Free Radical</u>

Molecular or atomic Species with one or more unpaired electrons

- Free Radicals are very reactive
- Free Radicals form electron pairs by attacking bonds in other molecules, sometimes forming more free radicals

Nuclear Reactions

- Gamma Radiation most likely to react with water in living tissue to form free radicals

$$H_2O + \gamma \rightarrow H_2O\bullet^{+} + e^{-}$$

- $H_2O\bullet^{+}$ and e^{-} collide with more water to form more free radicals

$$H_2O\bullet^{+} + H_2O \rightarrow H_3O^{+} + \bullet OH$$

$$e^{-} + H_2O \rightarrow H\bullet + OH^{-}$$

- Double bonds are particularly susceptible to free radical attack

$$H\bullet + RCH=CHR' \rightarrow RCH_2-\dot{C}HR'$$

  - Note: One electron of the π bond forms a C-H bond between one of the double-bonded carbons and the $H^{\bullet}$, and the other electron resides on the other carbon to form a free radical

Nuclear Reactions

- Ionization damage to critical biological structures
  - Changes to "Lipid" structure causes changes in membrane fluidity resulting in leakage through cell membrane and destruction of protective fatty tissue around organs
  - Changes to critical bonds in "enzyme" lead to their malfunction as catalysts of metabolic reactions
  - Changes in "Nucleic" acids and proteins that govern the rate of cell division can cause cancer
  - Genetic damage and mutations may occur when bonds in the DNA of sperm and egg cells are altered by free radicals

Nuclear Reactions

- Sources of Ionizing Radiation
  - Humans are continuously exposed to ionizing radiation from:

    Natural sources

    Artificial sources
  - Life evolved in the presence of “background” radiation
  - Some “radiation damage” in the evolution of life, mankind included, actually caused beneficial mutations allowing to species to adapt to change in order to survive
  - Exposure to some “artificial” sources of radiation – nuclear testing, nuclear waste dumps, medical diagnostic tests - is minimal and controllable
  - Excessive exposure to the sun’s rays is potentially dangerous

Nuclear Reactions

Typical Radiation Doses from Natural and Artificial Sources

Source of Radiation	Average Adult Exposure
Natural	
Cosmic radiation	30−50 mrem/yr
Radiation from the ground	
From clay soil and rocks	~25−170 mrem/yr
In wooden houses	10−20 mrem/yr
In brick houses	60−70 mrem/yr
In concrete (cinder block) houses	60−160 mrem/yr
Radiation from the air (mainly radon)	
Outdoors, average value	20 mrem/yr
In wooden houses	70 mrem/yr
In brick houses	130 mrem/yr
In concrete (cinder block) houses	260 mrem/yr
Internal radiation from minerals in tap water and daily intake of food (^{40}K, ^{14}C, Ra)	~40 mrem/yr
Artificial	
Diagnostic x-ray methods	
Lung (local)	0.04−0.2 rad/film
Kidney (local)	1.5−3 rad/film
Dental (dose to the skin)	≤1 rad/film
Therapeutic radiation treatment	Locally ≤ 10,000 rad
Other sources	
Jet flight (4 h)	~1 mrem
Nuclear testing	<4 mrem/yr
Nuclear power industry	<1 mrem/yr
Total average value	100−200 mrem/yr

Nuclear Reactions

- Applications of Radioisotopes
  - Radioactive Tracers can used to trace a labeled substance:
    - Through a complex process
    - From one region of a system to another
  - Tracers are prepared by combining a small amount of a radioactive species – the "beacon" - combined with a larger amount of the stable isotope

Practice Problem

Reaction Pathways

- Reaction Example – Periodate and Iodide ions

$$IO_4^-(aq) + 2I^-(aq) + H_2O \rightarrow I_2(s) + IO_3^-(aq) + 2OH^-(aq)$$

- Note the 2 species of Iodine as reactants
- Which species are reduced or oxidized to form the products, i.e., does I_2 come from IO_4^- or I^-?
  - Add non-radioactive IO_4^- to solution
  - Add "hot" radioactive I_2 ($^{131}I^-$) to solution

$$IO_4^-(aq) + 2\,^{131}I^-(aq) + H_2O \rightarrow {}^{131}I_2(s) + IO_3^-(aq) + 2OH^-(aq)$$

  - The iodide (I^-) is indeed oxidized (loses e^-) to form Radioactive I_2
  - The Periodate (IO_4^-) is reduced (gains e^-) to form non-radioactive Iodate (IO_3^-)

$$2H^+ + IO_4^- + 2e^- \rightarrow IO_3^- + H_2O$$

Nuclear Reactions

- Tracers in Material Flow
  - Metal exchange between deep layers of solid and surrounding solution
  - Material movement in semiconductors, paint, metal plating, detergent action, corrosion
  - Hydrologists and Hydrologic engineers trace the volume and flow of large bodies of water
    - Tritium (^{3}H) in H_2O, $^{90}Sr^{2+}$, and $^{137}Cs^{+}$ have been used to map the flow of water from the land to lakes to streams to the oceans
    - Naturally occurring Uranium isotopes (^{234}U, ^{235}U & ^{238}U have been used to trace and identify sources of ground water flow
    - (Oxygen isotopes (^{18}O & ^{16}O) used to date ice cores

Nuclear Reactions

- Neutron Activation
  - Neutrons bombard a nonradioactive sample, converting a small fraction of its atoms to radioisotopes
  - The characteristic decay patterns, such as gamma spectra (γ) reveal minute amounts of elements present
  - Leaves sample intact
  - Useful in both geochemical investigations and industrial processes

Nuclear Reactions

- Medical Diagnosis
  - 25% of hospital admissions are for diagnosis based on radioisotopes
  - Thyroid abnormalities are treated based on radioactive Iodide tracers used to monitor how well the thyroid gland incorporates dietary iodine into iodine –containing hormones
  - ^{59}Fe is used to measure physiological process dealing with blood flow, such as the rate at which the heart pumps blood
  - Positron-Emission Tomography (PET) is a powerful imaging tool for observing brain structures and function

Nuclear Reactions

- Medical Treatment
  - High energy radiation is used in radiological procedures to treat cancer
  - Cancer cells divide more rapidly than normal cells
  - Radioisotopes that interfere with cell division kill more cancer cells than normal cells
  - ^{198}Au (gold) and/or ^{90}Sr (strontium) have been used to destroy pituitary and breast tumor cells
  - γ (gamma) rays from ^{60}Co (Cobalt) have been used to destroy tumors in the brain and other body parts

Nuclear Reactions

- Interconversion of Mass and Energy
  - In addition to nuclear decay, there are two other nuclear processes
    - Nuclear Fission – The splitting of a "Heavy" nucleus into two much lighter nuclei, emitting several small particles at the same time
    - Nuclear Fusion – the combining of two light nuclei to form a heavier nuclide
    - Both Fission and Fusion release enormous amounts of energy
    - Both Fission and Fusion involve the transformation of Mass into Energy

Nuclear Reactions

- Mass Difference Between Nucleus & its Nucleons
  - The relation between mass and energy is not important for chemical reactions
  - The energy changes in a chemical reaction involve the breaking and forming of chemical bonds
  - Small mass changes are negligible

$$H_2O(g) \rightarrow 2H(g) + O(g) \qquad \Delta H^o_{rxn} = \Delta E = 2 \times \text{Bond Energy (BE) O - H} = 934\ kJ/mol$$

  - The mass equivalent to this energy is given by Einstein equation

$$E = mc^2 \qquad \text{or} \qquad \Delta E = \Delta mc^2 \qquad \Delta m = \frac{\Delta E}{c^2}$$

$$\Delta m = \frac{(9.34 \times 10^5\ J/mol) \times \dfrac{1(kg \bullet m^2)/s^2}{1\ J}}{\left(2.9979 \times 10^8\ m/s\right)^2} = 1.04 \times 10^{-11}\ kg/mol$$

  - Such small mass changes in chemical reactions allows us to assume conservation of mass

Nuclear Reactions

- Mass difference that accompanies a nuclear process
  - Carbon-12 breaks apart into its neutron and proton nucleons

$$^{12}C \rightarrow 6\,^1_1p + 6\,^1_0n$$

  - Determine mass of the nucleus and then the mass of the nucleons that make up the nucleus
  - By combining the mass of the 6 protons and 6 neutrons above and then subtracting the mass of one ^{12}C atom, the electrons cancel

    (6 e^- in 6 protons cancel 6 e^- in one ^{12}C atom

    - Mass of 1 proton – 1.007825 amu
    - Mass of one neutron – 1.008665 amu

Nuclear Reactions

$$\text{Mass of six } {}^{1}\text{H atoms} = 6 \times 1.007825 \text{ amu} = 6.046950 \text{ amu}$$

$$\text{Mass of six neutrons} = 6 \times 1.008665 \text{ amu} = 6.051990 \text{ amu}$$

$$\text{Total Mass} = 12.09840 \text{ amu}$$

- The mass of the reactant, one ^{12}C atom, is 12 amu exactly
- The mass difference (Δm) is obtained from the difference of the total mass of the nucleons and the mass of the nucleus

$$\Delta m = 12.098940 \text{ amu} - 12.000000 \text{ amu}$$

$$= 0.098940 \text{ amu} / {}^{12}\text{C} = 0.098940 \text{ g/mol } {}^{12}\text{C}$$

- The mass of the nucleus is "less" than the combined mass of its nucleons
- There is always a mass decrease when nucleons form a nucleus

Nuclear Reactions

- Nuclear Binding Energy
  - Compare Mass Change in the chemical reaction decomposing water into Hydrogen and Oxygen, with the breaking up of C-12 into protons and neutrons,

$$\Delta m_{(H_2O)} = 10.4 \times 10^{-12}\ \mathrm{kg/mol} \quad \text{(from slide 66)}$$

$$\Delta m_{(^{12}C)} = 9.89 \times 10^{-5}\ \mathrm{kg/mol} \quad \text{(from slide 68)}$$

  - Compute Energy equivalent of C-12 change using Einstein's equation

$$\Delta E = \Delta mc^2 = (9.8940 \times 10^{-5}\ \mathrm{kg/mol})(2.9979 \times 10^{8}\ \mathrm{m/s})^2$$

$$\Delta E = 8.8921 \times 10^{12} \frac{\mathrm{kg \bullet m^2}}{\mathrm{mol} \bullet s^2} = 8.8921 \times 10^{12}\ \mathrm{J/mol} = 8.8921 \times 10^{9}\ \mathrm{kJ/mol}$$

    - This quantity of energy is called the "Binding Energy"

Nuclear Reactions

- The positive value of the Binding Energy means "energy is absorbed.
- Binding energy is the energy required to break 1 mol of nuclei into individual nucleons

Nucleus + nuclear binding energy → nucleons

- Nuclear Binding Energy is qualitatively analogous to the sum of bond energies of a covalent compound or the lattice energy of an ionic compound
- Nuclear binding energies are several million times greater than bond energies

Nuclear Reactions

- The "Joule" is too large a unit to use for binding energies of individual nucleons
  - Nuclear scientists use the "Electron Volt" as the standard unit

$$1\,\text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

  - Binding Energies are commonly expressed in "millions" of electron volts

mega-electron volts (MeV)

$$1\,\text{MeV} = 10^{6}\ \text{eV} = 1.602 \times 10^{-13}\ \text{J}$$

  - Atomic Mass Units (amu) equivalence to electron volts

$$1\,\text{amu} = 931.5 \times 10^{6}\ \text{eV} = 931.5\,\text{MeV}$$

Practice Problem

Express the binding energy of the C-12 nucleus and the 12 nucleons in the nucleus in electron volts

$$\frac{\textbf{Binding energy}}{{}^{12}\textbf{C nucleus}} = \textbf{0.098940 amu} \times \frac{\textbf{931.5 MeV}}{\textbf{1 amu}} = \textbf{92.16 MeV}$$

$$\textbf{Binding energy per nucleon} = \frac{\textbf{binding erergy}}{\textbf{no. nucleons}} = \frac{\textbf{92.16 MeV}}{\textbf{12 nucleons}} = \textbf{7.680 MeV / nucleon}$$

Practice Problem

Compute the Binding Energy of ^{56}Fe; compare it with that for ^{12}C

mass ^{56}Fe atom	– 55.9834939 amu
mass ^{1}H atom (proton)	– 1.007825 amu
mass neutron	– 1.008665 amu

Iron-56 has 26 protons and 30 neutrons

Compute mass difference Δm of one ^{56}Fe atom and the sum of the masses of the 26 protons & 30 neutrons

$$\Delta m = [(26 \times \text{mass } ^{1}H) + (30 \times \text{neutron mass})] - \text{mass } ^{56}Fe \text{ atom}$$

$$\Delta m = [(26)(1.007825 \text{ amu}) + (30)(1.008665 \text{ amu})] - 55.93439 \text{ amu}$$

$$\Delta m = 0.52846 \text{ amu}$$

Compute binding energy per nucleon

$$\text{BE / nucleon} = \frac{0.52846 \text{ amu} \times 931.5 \text{ MeV / amu}}{56 \text{ nucleons}} = 8.790 \text{ MeV / nucleon}$$

$$\text{BE / nucleon} \quad ^{12}C(7.680 \text{ MeV}) \quad \text{vs} \quad ^{56}Fe(8.790 \text{ MeV})$$

More energy required to breakup ^{56}Fe $\therefore$ ^{56}Fe more stable

Practice Problem

Use 2 methods to compute the Binding Energy of $^{18}O_8$ in kJ / mole

Mass 1 atom $^{18}O_8$ = 15.994915 amu

Mass 8 protons = 8 x 1.007825 = 8.062600 amu

Mass 8 Neutrons = 8 x 1.008665 = 8.069320 amu

total mass = 16.131.920 amu

Mass Defect (Δm) = 16.131920 – 15.994915 = 0.137005 amu (g/mol)

$$\text{BE / nucleon} = \frac{0.137005\ \text{amu} \times 931.5\ \text{MeV / amu}}{16\ \text{nucleons}} = 7.976\ \text{MeV / nucleon}$$

$$\text{BE / atom} = \frac{0.137005\ \text{amu} \times 931.5\ \text{MeV / amu}}{1\ \text{atom}} = 127.6\ \text{MeV / atom}$$

$$\text{BE (kJ / mol)} = (127.6\ \text{MeV / atom})(6.022 \times 10^{23}\ \text{atoms / mol})(1.602 \times 10^{-16}\ \text{kJ / MeV})$$

$$\text{BE (kJ / mol)} = 1.231 \times 10^{10}\ \text{kJ / mol}$$

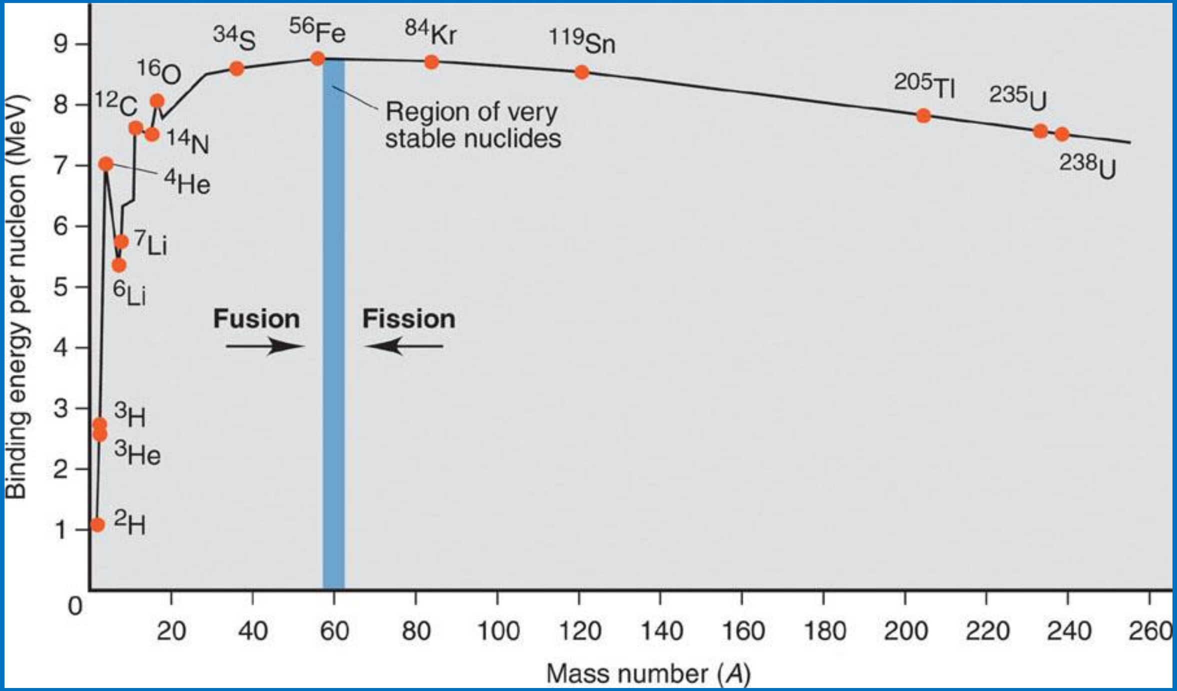
Or using $\Delta E = \Delta mc^2$

$$\text{BE} = 0.137005\ \text{amu (g / mol)}\ {}^{16}_{8}\text{O} \left(\frac{1\ \text{kg}}{10^3\ \text{g}}\right)(2.99792 \times 10^8\ \text{m / s})^2 \left(\frac{1\ \text{J}}{\text{kg} \bullet \text{m}^2 / \text{s}^2}\right)\left(\frac{1\ \text{kJ}}{10^3\ \text{J}}\right)$$

$$\text{BE (kJ / mol)} = 1.231 \times 10^{10}\ \text{kJ / mol}$$

Nuclear Reactions

- Fission or Fusion
  - Binding energy per nucleon varies considerable
  - The greater the binding energy the more stable the nucleon
  - Binding energy per nucleon "peaks out" at about mass no. 60



Nuclear Reactions

- The existence of a peak of stability suggests that there are two ways nuclides can increase their binding energy per nucleon
  - Fission – Heavier nuclei can split into lighter nuclei that are closer to A ≈ 60
    - The lighter nuclei products would have higher binding energies per nucleon than the reactant nuclei, and would release energy
    - Current nuclear power plants employ fission, where uranium nuclei are split
  - Fusion – lighter nuclei can combine to form heavier nuclei that are closer to A ≈ 60
    - The heavier product nuclei would be more stable than the lighter reactant, releasing energy
    - The sun, hydrogen bombs, & future power plants use fusion

Summary Equations

Isotope Notation $\,^{A}_{Z}X$

X - Symbol for Particle (H - hydrogen; Cl - Chlorine; Mg - Magnesium, etc.)

A - AtomicMass Number (Total number of Nucleons)

Z - Number of Protons

$$\text{Decay Rate (A)} = -\frac{\Delta N}{\Delta t}$$

$$A = kN$$

$$A = -\frac{\Delta N}{\Delta t} = kN$$

Activity of sample is proportional to number of nuclei

$$\text{Fraction Decayed} = \frac{N_0 - N_t}{N_0} = \frac{A_0 - A_t}{A_0}$$

Summary Equations

$$\int \frac{d(N)}{N} = \int -k\,dt = -k\int dt$$

$$\ln[N] = -kt + C$$

$$-kt = 0 \qquad C = \ln(N_o)$$

$$\ln(N) = -kt + \ln(N_0)$$

$$N = N_0 \times e^{-kt}$$

$$t_{1/2} = \frac{\ln \frac{N_0}{1/2N_0}}{k} = \frac{\ln 2}{k} = \frac{0.693}{k}$$

$$t = \frac{1}{k} \ln \frac{A_0}{A}$$