Denklemlerin Kökleri Parantez (Kapalı) Yöntemler, , Açık Yöntemler [1-6]

Kaynaklar:

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2. Chapra S.C. and Canale R. P. "Yazılım ve programlama Uygulamalarıyla Mühendisler için Sayısal Yöntemler" 4.Basımdan Çevirenler: Hasan Heperkan ve Uğur Kesgin 2003.

3.Chapra S.C. "Applied Numerical Methods with MATLAB for engineers and Scientists" Third Edition, McGraw Hill, International Edition 2012.

4. Mathews J.H. and Fınk K.D. "Numerical Methods using MATLAB", Fourth Edition, Pearson P. Hall, International Edition 2004.

5. Fausett L.V. "Applied Numerical Analysis Using MATLAB, Second Edition, PearsonP. Hall, International Edition, 2008.

6. Gilat A. And Subramaniam V. "Numerical Methods, An introduction with Applications Using MATLAB", Second Edition,John Wiley and Sons. Inc. 2011.

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

$$f(x_l)f(x_r) < 0 \quad x_r = x_u$$

$$f(x_l)f(x_r) > 0 \quad x_r = x_l$$

$$f(x_l)f(x_r) = 0 \quad x_r = x_r$$

$$\varepsilon_a = \left| \frac{x_r^{new} - x_r^{old}}{x_r^{new}} \right| 100\%$$

$$\varepsilon_t = \left| \frac{x_{true} - x_r^{new}}{x_{true}} \right| 100\%$$

Yer Değiştirme Yöntemi:

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

Birinci iterasyon

$$f(c) = \frac{667.38}{c}(1 - e^{-0.146843\,c}) - 40$$

$$x_l = 12 \qquad f(x_l) = 6.0699$$

$$x_u = 16 \qquad f(x_u) = -2.2688$$

$$x_r = 16 - \frac{-2.2688(12 - 16)}{6.0699 - f(-2.2688)} = 14.9113$$

$$f(x_r) = f(14.9113) = \frac{667.38}{c}(1 - e^{-0.146843\,c}) - 40 = -0.2543$$

$$f(x_l)f(x_r) = f(12)f(14.9113) = 6.0699 * (-0.2543) < 0$$

$$\text{So: } x_r = x_u$$

ikinci iterasyon

$$x_l = 12 \qquad f(x_l) = 6.0699$$

$$x_u = 14.9113 \qquad f(x_u) = -0.2543$$

$$x_r = 14.9113 - \frac{-0.2543(12 - 14.9113)}{6.0699 - (-0.2543)} = 14.7942$$

$$\varepsilon_a = \left|\frac{x_r^{new} - x_r^{old}}{x_r^{new}}\right| 100\%$$

$$\varepsilon_a = \left|\frac{14.7942 - 14.9113}{14.7942}\right| 100\% = 0.7915$$

$$\varepsilon_t = \left|\frac{x_{true} - x_r^{new}}{x_{true}}\right| 100\%$$

$$\varepsilon_t = \left|\frac{14.7802 - 14.7942}{14.7802}\right| 100\% = 0.0947 < 0.5$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\varepsilon_a = \left|\frac{x_r^{new} - x_r^{old}}{x_r^{new}}\right| 100\%$$

Newton Raphson Yöntemi

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = e^{-x} - x$$

fonksiyonun birinci türevi:

$$f'(x) = -e^{-x} - 1$$

orijinal fonksiyonla birlikte Newton-Raphson denkleminde yerine konur:

$$x_{i+1} = x_i - \frac{e^{-x} - x}{-e^{-x} - 1}$$

$x_0 = 0.5$ komşuluğunda başlatarak

birinci iterasyon

$$x_{i+1} = 0.5 - \frac{e^{-0.5} - 0.5}{-e^{-0.5} - 1} = 0.5663$$

$$\varepsilon_a = \left|\frac{x_r^{new} - x_r^{old}}{x_r^{new}}\right| 100\%$$

$$\varepsilon_a = \left|\frac{0.5663 - 0.5}{0.5663}\right| 100\% = 11.7$$

ikinci iterasyon, :

$x_i = 0.5663$

$$x_{i+1} = 0.5663 - \frac{e^{-0.5663} - 0.5663}{-e^{-0.5663} - 1} = 0.5671$$

$$\varepsilon_a = \left|\frac{0.5671 - 0.5663}{0.5671}\right| 100\% = 0.14$$

So;

$x = 0.5671$

Ters Karesel İnterpolasyon yöntemi:

$$x_{i+1} = \frac{y_{i-1} y_i}{(y_{i-2} - y_{i-1})(y_{i-2} - y_i)} x_{i-2} + \frac{y_{i-2} y_i}{(y_{i-1} - y_{i-2})(y_{i-1} - y_i)} x_{i-1} + \frac{y_{i-2} y_{i-1}}{(y_i - y_{i-2})(y_i - y_{i-1})} x_i$$

$$\varepsilon_t = \left| \frac{x_{\text{gerçek}} - x_r^{\text{yeni}}}{x_{\text{gerçek}}} \right| \%100$$

Birinci iterasyon:

$$y = f(x) = e^{-x} - x = 0$$

$$x_{i-2} = 0.1 \qquad y_{i-2} = f(0.1) = e^{-0.1} - 0.1 = 0.8048$$

$$x_{i-1} = 0.5 \qquad y_{i-1} = f(0.5) = e^{-0.5} - 0.5 = 0.1065$$

$$x_i = 1.0 \qquad y_i = f(1.0) = e^{-1.0} - 1.0 = -0.6321$$

$$x_{i+1} = \frac{y_{i-1} y_i}{(y_{i-2} - y_{i-1})(y_{i-2} - y_i)} x_{i-2} + \frac{y_{i-2} y_i}{(y_{i-1} - y_{i-2})(y_{i-1} - y_i)} x_{i-1} + \frac{y_{i-2} y_{i-1}}{(y_i - y_{i-2})(y_i - y_{i-1})} x_i$$

$$x_{i+1} = \frac{0.1065(-0.6321)}{(0.8048-0.1065)(0.8048--0.6321)}0.1 + \frac{0.8048(-0.6321)}{(0.1065-0.8048)(0.1065--0.6321)}0.5$$

$$+\frac{0.8048(0.1065)}{(-0.6321-0.8048)(-0.6321-0.1065)}1.0$$

$$x_{i+1} = -0.0067 + 0.4931 + 0.0807 = 0.5671$$

$$y_{i+1} = f(0.5671) = e^{-0.5671} - 0.5671 = 0.000068 \approx 0$$

$$\boldsymbol{\varepsilon_t = \left|\frac{x_{true} - x_r{}^{new}}{x_{true}}\right| 100\% = \varepsilon_t = \left|\frac{0.56714 - 0.5671}{0.56714}\right| 100\% = 0.007\ \%}$$

İkiye bölme yöntemiyle fonksiyonun kökünü belirleme:

$$f(x) = e^{-3x} - 2x$$

Başlangıç komşulukları Xalt =0.23 ve Xüst= 0.26

$$x_r = \frac{0.23 + 0.26}{2} = 0.2450$$

Fonksiyon değerinin ürününün alt sınırda ve orta noktada hesaplanması: :

$$f(0.23) \times f(0.2450) = (0.0416) * (-0.0105) = -0.0004$$

Sıfırdan daha düşük olan ve bu nedenle alt sınır ve orta nokta arasında bir işaret değişikliği meydana gelir. Bu nedenle kök 0.23 ve 0.2450 arasındadır. Üst sınır 0.2450 olarak yeniden tanımlanır ve ikinci yineleme için kök tahmini şu şekilde hesaplanır:

Xalt $= 0.23, \quad x_r = x_U = 0.2450$

$$x_r = \frac{0.2300 + 0.2450}{2} = 0.2375$$

$$\varepsilon_a = \left|\frac{0.2375 - 0.2450}{0.2375}\right| 100\% = 3.16$$

$$f(0.23) * f(0.2375) = (0.0416) * (0.0154) = 0.0006 > 0$$

$$x_r = x_L = 0.2375, \quad x_U = 0.2450$$

$$x_r = \frac{0.2375 + 0.2450}{2} = 0.2412$$

$$\varepsilon_a = \left|\frac{0.2412 - 0.2375}{0.2412}\right| 100\% = 1.53$$

iterasyon	Xalt	Xüst	x_r	ε_a (%)
1	0.2300	0.2600	0.2450	
2	0.2300	0.2450	0.2375	3.16
3	0.2375	0.2450	0.2412	1.53

$x = 0.2412$

Denklemlerin köklerini hesaplamak için çoklu denklem Newton-Raphson yöntemi

Doğru kök çifti x= -0.1868 **ve** y=0.5283.

$x_0 = -0.1600$ ve $y_0 = 0.5000$ **komşuluğunda**

$$u(x,y) = -x^2 + x - y + 0.75$$
$$v(x,y) = x^2 - y - 5xy$$

$$x_{i+1} = x_i - \frac{u_i \frac{\partial v_i}{dy} - vi \frac{\partial u_i}{dy}}{\frac{\partial u_i}{dx}\frac{\partial v_i}{dy} - \frac{\partial u_i}{dy}\frac{\partial v_i}{dx}} \qquad y_{i+1} = y_i - \frac{v_i \frac{\partial u_i}{dx} - ui \frac{\partial v_i}{dx}}{\frac{\partial u_i}{dx}\frac{\partial v_i}{dy} - \frac{\partial u_i}{dy}\frac{\partial v_i}{dx}}$$

$$u(x,y) = x^2 - x + y - 0.75 = 0 \qquad v(x,y) = y + 5xy - x^2 = 0$$

$$\frac{\partial u_i}{dy} = 1 \qquad \frac{\partial u_i}{dx} = 2x_i - 1$$

$$\frac{\partial v_i}{dy} = 1 + 5x_i \qquad \frac{\partial v_i}{dx} = 5y_i - 2x_i$$

1.ci iterasyon

$$\frac{\partial u_0}{dy}=1 \qquad \frac{\partial u_0}{dx}=2x_0-1=2\times(-0.16)-1=-1.32$$

$$\frac{\partial v_0}{dy}=1+5x_0=1+5\times(-0.16)=0.2 \qquad \frac{\partial v_0}{dx}=5y_0-2x_0=5\times 0(.5)-2\times(-0.16)=2.82$$

$$J=\begin{bmatrix}\frac{\partial u_0}{\partial x} & \frac{\partial u_0}{\partial y}\\ \frac{\partial v_0}{\partial x} & \frac{\partial v_0}{\partial y}\end{bmatrix}=\begin{bmatrix}-1.32 & 1\\ 2.82 & 0.2\end{bmatrix}$$

$$|J|=\frac{\partial u_0}{\partial x}\frac{\partial v_0}{\partial y}-\frac{\partial u_0}{\partial y}\frac{\partial v_0}{\partial x}=-1.32\times 0.2-1\times 2.82=-3.084$$

$$u(x_0,y_0)=x_0{}^2-x_0+y_0-0.75=(-0.16)^2-(-0.16)+0.50-0.75=-0.0644$$

$$v(x_0,y_0)=y_0+5x_0y_0-x_0{}^2=0.50+5*(-0.16)*(0.50)-(-0.16)^2=0.0744$$

Bu değerler aşağıdaki denklemlerde yerine konursa x_1 ve y_1 çözülür.

$$x_{i+1}=x_i-\frac{u_i\frac{\partial v_i}{dy}-vi\frac{\partial u_i}{dy}}{\frac{\partial u_i}{dx}\frac{\partial v_i}{dy}-\frac{\partial u_i}{dy}\frac{\partial v_i}{dx}} \qquad y_{i+1}=y_i-\frac{v_i\frac{\partial u_i}{dx}-ui\frac{\partial v_i}{dx}}{\frac{\partial u_i}{dx}\frac{\partial v_i}{dy}-\frac{\partial u_i}{dy}\frac{\partial v_i}{dx}}$$

$$x_1=x_0-\frac{u_0\frac{\partial v_0}{dy}-v_0\frac{\partial u_0}{dy}}{\frac{\partial u_0}{dx}\frac{\partial v_0}{dy}-\frac{\partial u_0}{dy}\frac{\partial v_0}{dx}}=(-0.16)-\frac{(-0.0644)*(0.2)-(0.0744)*(1)}{-3.084}=-0.1883$$

$$y_1=y_0-\frac{v_0\frac{\partial u_0}{dx}-u_0\frac{\partial v_0}{dx}}{\frac{\partial u_0}{dx}\frac{\partial v_0}{dy}-\frac{\partial u_0}{dy}\frac{\partial v_0}{dx}}=0.50-\frac{(0.0744)*(-1.32)-(-0.0644)*(2.82)}{-3.084}=0.5270$$

2.ci iterasyon

$$\frac{\partial u_1}{dy}=1 \qquad \frac{\partial u_1}{dx}=2x_1-1=2\times(-0.1883)-1=-1.3766$$

$$\frac{\partial v_1}{dy}=1+5x_1=1+5\times(-0.1883)=0.0585 \qquad \frac{\partial v_1}{dx}=5y_1-2x_1=5\times 0.5270-2\times(-0.1883)=3.0116$$

$$J=\begin{bmatrix}\frac{\partial u_0}{\partial x} & \frac{\partial u_0}{\partial y}\\ \frac{\partial v_0}{\partial x} & \frac{\partial v_0}{\partial y}\end{bmatrix}=\begin{bmatrix}-1.3766 & 1\\ 3.0116 & 0.0585\end{bmatrix}$$

$$u(x_1,y_1)=x_1^2-x_1+y_1-0.75=(-0.1883)^2-(-0.1883)+0.5270-0.75=7.5689*10^{-4}$$

$$v(x_1,y_1)=y_1+5x_1y_1-x_1^2=0.5270+5*(-0.1883)*(0.5270)-(-0.1883)^2=-4.6273*10^{-3}$$

$$x_2=x_1-\frac{u_1\frac{\partial v_1}{dy}-v_1\frac{\partial u_1}{dy}}{\frac{\partial u_1}{dx}\frac{\partial v_1}{dy}-\frac{\partial u_1}{dy}\frac{\partial v_1}{dx}}=(-0.1883)-\frac{(7.5689*10^{-4})*(0.0585)-(-4.627*10^{-3})*(1)}{(-1.3766)*(0.0585)-(1)*(3.0116)}\cong 0.1868$$

$$y_2=y_1-\frac{v_1\frac{\partial u_1}{dx}-u_1\frac{\partial v_1}{dx}}{\frac{\partial u_1}{dx}\frac{\partial v_0}{dy}-\frac{\partial u_1}{dy}\frac{\partial v_0}{dx}}=0.5270-\frac{(-4.627*10^{-3})*(-1.3766)-(7.5689*10^{-4})*(3.0116)}{(-1.3766)*(0.0585)-1*(3.0116)}\cong 0.5283$$