İnterpolasyon, interpolasyon Polinomları, Şerit İnterpolasyonu [1-6]

Kaynaklar:

1. Chapra S.C. and Canale R.P. "Numerical Methods for Engineers", Sixth Edition,McGraw Hill, International Edition 2010.

2. Chapra S.C. and Canale R. P. "Yazılım ve programlama Uygulamalarıyla Mühendisler için Sayısal Yöntemler" 4.Basımdan Çevirenler: Hasan Heperkan ve Uğur Kesgin 2003.

3.Chapra S.C. "Applied Numerical Methods with MATLAB for engineers and Scientists" Third Edition, McGraw Hill, International Edition 2012.

4. Mathews J.H. and Fink K.D. "Numerical Methods using MATLAB", Fourth Edition, Pearson P. Hall, International Edition 2004.

5. Fausett L.V. "Applied Numerical Analysis Using MATLAB, Second Edition, PearsonP. Hall, International Edition, 2008.

6. Gilat A. And Subramaniam V. "Numerical Methods, An introduction with Applications Using MATLAB", Second Edition,John Wiley and Sons. Inc. 2011.

2.ci derece Lagrange İnterpolasyonu

v (m^3/kg)	0.10377	0.108	0.11144	0.1254
s(kJ/kgK)	6.4147	?	6.5453	6.7664

$$x_0 = 0.10377 \quad f(x_0) = 6.4147$$
$$x_1 = 0.11144 \quad f(x_1) = 6.5453$$
$$x_2 = 0.1254 \quad f(x_2) = 6.7664$$

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f_n(x)$$

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$f_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$f_2(x)=\frac{(0.108-0.11144)(0.108-0.1254)}{(0.10377-0.11144)(0.10377-0.1254)}6.4147+\frac{(0.108-0.10377)(0.108-0.1254)}{(0.11144-0.10377)(0.11144-0.1254)}6.5453$$

$$+\frac{(0.108-0.10377)(0.108-0.11144)}{(0.1254-0.10377)(0.1254-0.11144)}6.7664$$

$$f_2(0.108)=6.4874$$

Lagrange Kübik interpolasyon polinomu

$$P_3(x)=y_0\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}+y_1\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+y_2\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}+y_3\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$P_x(x) = 1.000000\frac{(x-0.4)(x-0.8)(x-1.2)}{(0.0-0.4)(0.0-0.8)(0.0-1.2)} + 0.921061\frac{(x-0.0)(x-0.8)(x-1.2)}{(0.4-0.0)(0.4-0.8)(0.4-1.2)}$$
$$+ 0.696707\frac{(x-0.0)(x-0.4)(x-1.2)}{(0.8-0.0)(0.8-0.4)(0.8-1.2)}$$
$$+ 0.362358\frac{(x-0.0)(x-0.4)(x-0.8)}{(1.2-0.0)(1.2-0.4)(1.2-0.8)}$$

$$P_3(x) = -2.604167(x-0.4)(x-0.8)(x-1.2) + 7.195789(x-0.0)(x-0.8)(x-1.2)$$
$$- 5.443021(x-0.0)(x-0.4)(x-1.2) + 0.943641(x-0.0)(x-0.4)(x-0.8)$$

$$P_3(0.6) = -0.062500008 + 0518096808 + 0.3918975 - 0.022647384$$
$$P_3(0.6) = 0.824847 \cong 0.825$$

KÜBİK ŞERİT

X=5.5 ' değerinde $f(x=5.5)=?$ tahmin ediniz.

x	3.0	4.5	5.0	7.0
f(x)	2.5	1.0	1.1	2.5

$$f_i(x)=\frac{f_i''(x_{i-1})}{6(x_i-x_{i-1})}(x_i-x)^3+\frac{f_i''(x_i)}{6(x_i-x_{i-1})}(x-x_{i-1})^3+\left[\frac{f(x_{i-1})}{(x_i-x_{i-1})}-\frac{f''(x_{i-1})(x_i-x_{i-1})}{6}\right](x_i-x)+$$

$$\left[\frac{f(x_i)}{(x_i-x_{i-1})}-\frac{f''(x_i)(x_i-x_{i-1})}{6}\right](x-x_{i-1})$$

$$(x_i-x_{i-1})f''(x_{i-1})+2(x_{i+1}-x_{i-1})f''(x_i)+(x_{i+1}-x_i)f''(x_{i+1})=\frac{6}{x_{i+1}-x_i}\left[f(x_{i+1})-f(x_i)\right]+\frac{6}{x_i-x_{i-1}}\left[f(x_{i-1})-f(x_i)\right]$$

$$(x_i - x_{i-1})f''(x_{i-1}) + 2(x_{i+1} - x_{i-1})f''(x_i) + (x_{i+1} - x_i)f''(x_{i+1}) = \frac{6}{x_{i+1} - x_i}\left[f(x_{i+1}) - f(x_i)\right] + \frac{6}{x_i - x_{i-1}}\left[f(x_{i-1}) - f(x_i)\right]$$

i=1

$$(x_1 - x_0)f''(x_0) + 2(x_2 - x_0)f''(x_1) + (x_2 - x_1)f''(x_2) = \frac{6}{x_2 - x_1}\left[f(x_2) - f(x_1)\right] + \frac{6}{x_1 - x_0}\left[f(x_0) - f(x_1)\right]$$

$$(4.5-3)f''(3) + 2(5-3)f''(4.5) + (5-4.5)f''(5) = \frac{6}{(5-4.5)}\left[1.1-1\right] + \frac{6}{(4.5-3)}\left[2.5-1\right]$$

Düğüm noktalarının başında ve sonunda ikinci türevler sıfırdır.

$$f''(3) = 0$$

$$4f''(4.5) + 0.5f''(5) = 7.2 \qquad (1)$$

Aynı eşitlik ikinci iç noktaya da uygulanabilir

i=2

$$(x_2 - x_1)f''(x_1) + 2(x_3 - x_1)f''(x_2) + (x_3 - x_2)f''(x_3) = \frac{6}{x_3 - x_2}[f(x_3) - f(x_2)] + \frac{6}{x_2 - x_1}[f(x_1) - f(x_2)]$$

$$(5 - 4.5)f''(4.5) + 2(7 - 4.5)f''(5) + (7 - 5)f''(7) = \frac{6}{7 - 5}[f(7) - f(5)] + \frac{6}{5 - 4.5}[f(4.5) - f(5)]$$

$$(5 - 4.5)f''(4.5) + 2(7 - 4.5)f''(5) + (7 - 5)f''(7) = \frac{6}{7 - 5}[2.5 - 1.1] + \frac{6}{5 - 4.5}[1 - 1.1]$$

$$0.5f''(4.5) + 5f''(5) = 3 \qquad (2)$$

$$4f''(4.5) + 0.5f''(5) = 7.2 \qquad (1)$$

$$(-8)\{0.5f''(4.5) + 5f''(5) = 3\} \qquad (2)$$

$$4f''(4.5) + 0.5f''(5) = 7.2 \qquad (1)$$
$$-4f''(4.5) - 40f''(5) = -24 \qquad (2)$$

(1) ile (2) toplanırsa $-39.5f''(5) = -16.8$ $\qquad f''(5) = 0.42531$

Eşitlik (1)'de $f''(5) = 0.42531$ **koyarsak**

$$4f''(4.5) + 0.5(0.4253\ 1) = 7.2 \qquad f''(4.5) = 1.74683$$

$$f_i(x) = \frac{f_i''(x_{i-1})}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{f_i''(x_i)}{6(x_i - x_{i-1})}(x - x_{i-1})^3 + \left[\frac{f(x_{i-1})}{(x_i - x_{i-1})} - \frac{f''(x_{i-1})(x_i - x_{i-1})}{6}\right](x_i - x) +$$
$$\left[\frac{f(x_i)}{(x_i - x_{i-1})} - \frac{f''(x_i)(x_i - x_{i-1})}{6}\right](x - x_{i-1})$$

i=1

$$f_1(x) = \frac{f_1''(x_0)}{6(x_1 - x_0)}(x_1 - x)^3 + \frac{f_1''(x_1)}{6(x_1 - x_0)}(x - x_0)^3 + \left[\frac{f(x_0)}{(x_1 - x_0)} - \frac{f''(x_0)(x_1 - x_0)}{6}\right](x_1 - x) + \left[\frac{f(x_1)}{(x_1 - x_0)} - \frac{f''(x_1)(x_1 - x_0)}{6}\right](x - x_0)$$

$$f_1(x) = \frac{f_1''(3)}{6(4.5-3)}(4.5 - x)^3 + \frac{f_1''(4.5)}{6(4.5-3)}(x - 3)^3 + \left[\frac{f(3)}{(4.5-3)} - \frac{f''(3)(4.5-3)}{6}\right](4.5 - x) + \left[\frac{f(4.5)}{(4.5-3)} - \frac{f''(4.5)(4.5-3)}{6}\right](x - 3)$$

Düğümlerin sonunda ikinci türevler sıfırdır.

$$f''(3) = f_1''(3) = 0$$

$$f_1(x) = \frac{1.74683}{9}(x - 3)^3 + \left[\frac{2.5}{1.5}(4.5 - x)\right] + \left[\frac{1}{1.5} - 1.74683 * 0.25\right](x - 3)$$

Birinci aralık için Kübik Şerit

$$f_1(x) = 0.194092\,(x-3)^3 + [1.66667\,(4.5-x)] + 0.229959\,(x-3)$$

i=2

$$f_2(x) = \frac{f_2''(x_1)}{6(x_2-x_1)}(x_2-x)^3 + \frac{f_2''(x_2)}{6(x_2-x_1)}(x-x_1)^3 + \left[\frac{f(x_1)}{(x_2-x_1)} - \frac{f''(x_1)(x_2-x_1)}{6}\right](x_2-x) + \left[\frac{f(x_2)}{(x_2-x_1)} - \frac{f''(x_2)(x_2-x_1)}{6}\right](x-x_1)$$

$$f_2(x) = \frac{f_2''(4.5)}{6(5-4.5)}(5-x)^3 + \frac{f_2''(5)}{6(5-4.5)}(x-4.5)^3 + \left[\frac{f(4.5)}{(5-4.5)} - \frac{f''(4.5)(5-4.5)}{6}\right](5-x) + \left[\frac{f(5)}{(5-4.5)} - \frac{f''(5)(5-4.5)}{6}\right](x-4.5)$$

$$f_2(x) = \frac{1.74683}{3}(5-x)^3 + \frac{0.42531}{3}(x-4.5)^3 + \left[\frac{1}{0.5} - 1.74683\,x0.08333\right](5-x) + \left[\frac{1.1}{0.5} - 0.42531\,x0.08333\right](x-4.5)$$

ikinci aralık için kübik şerit

$$f_2(x) = 0.582276\,(5-x)^3 + 0.14177\,(x-4.5)^3 + 1.854436\,(5-x) + 2.164558\,(x-4.5)$$

i=3

$$f_3(x) = \frac{f_3''(x_2)}{6(x_3-x_2)}(x_3-x)^3 + \frac{f_3''(x_3)}{6(x_3-x_2)}(x-x_2)^3 + \left[\frac{f(x_2)}{(x_3-x_2)} - \frac{f''(x_2)(x_3-x_2)}{6}\right](x_3-x) + \left[\frac{f(x_3)}{(x_3-x_2)} - \frac{f''(x_3)(x_3-x_2)}{6}\right](x-x_2)$$

$$f_3(x) = \frac{f_3''(5)}{6(7-5)}(7-x)^3 + \frac{f_3''(7)}{6(7-5)}(x-5)^3 + \left[\frac{f(5)}{(7-5)} - \frac{f''(5)(7-5)}{6}\right](7-x) + \left[\frac{f(7)}{(7-5)} - \frac{f''(7)(7-5)}{6}\right](x-5)$$

düğüm noktalarının sonunda ikinci türevler sıfırdır.

$$f''(7) = f_3''(7) = 0$$

$$f_3(x) = \frac{0.42531}{12}(7-x)^3 + \left[\frac{1.1}{2} - 0.42531\,x0.33333\right](7-x) + \left[\frac{2.5}{2}\right](x-5)$$

üçüncü aralık için Kübik şerit

$$f_3(x) = 0.035442\,(7-x)^3 + 0.408231\,(7-x) + 1.25(x-5)$$

x=5.5 **üçüncü aralığın içine düşer.**

$$f_3(5.5) = 0.035442\,(7-5.5)^3 + 0.408231\,(7-5.5) + 1.25(5.5-5) = 1.356963$$

Ters Quadratik İnterpolasyon yöntemi:

Birinci iterasyon:

$$y = f(x) = e^{-x} - x = 0$$

$$x_{i-2} = 0.1 \qquad y_{i-2} = f(0.1) = e^{-0.1} - 0.1 = 0.8048$$

$$x_{i-1} = 0.5 \qquad y_{i-1} = f(0.5) = e^{-0.5} - 0.5 = 0.1065$$

$$x_i = 1.0 \qquad y_i = f(1.0) = e^{-1.0} - 1.0 = -0.6321$$

$$x_{i+1} = \frac{y_{i-1}y_i}{(y_{i-2} - y_{i-1})(y_{i-2} - y_i)} x_{i-2} + \frac{y_{i-2}y_i}{(y_{i-1} - y_{i-2})(y_{i-1} - y_i)} x_{i-1} + \frac{y_{i-2}y_{i-1}}{(y_i - y_{i-2})(y_i - y_{i-1})} x_i$$

$$x_{i+1} = \frac{0.1065\,(-0.6321)}{(0.8048 - 0.1065)(0.8048 - -0.6321)} 0.1 + \frac{0.8048\,(-0.6321)}{(0.1065 - 0.8048)(0.1065 - -0.6321)} 0.5$$

$$+ \frac{0.8048\,(0.1065)}{(-0.6321 - 0.8048)(-0.6321 - 0.1065)} 1.0$$

$$x_{i+1} = -0.0067 + 0.4931 + 0.0807 = 0.5671$$

$$y_{i+1} = f(0.5671) = e^{-0.5671} - 0.5671 = 0.000068 \approx 0$$

$$\varepsilon_t = \left| \frac{x_{true} - x_r^{new}}{x_{true}} \right| 100\% = \varepsilon_t = \left| \frac{0.56714 - 0.5671}{0.56714} \right| 100\% = 0.007\ \%$$