

Adi Diferansiyel Eşitlikler ve Problemler[1-6]

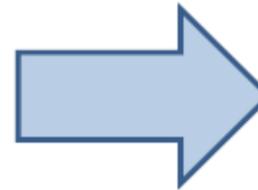
Kaynaklar:

1. Chapra S.C. and Canale R.P. “Numerical Methods for Engineers”, Sixth Edition, McGraw Hill, International Edition 2010.
2. Chapra S.C. and Canale R. P. “Yazılım ve programlama Uygulamalarıyla Mühendisler için Sayısal Yöntemler” 4.Basımdan Çevirenler: Hasan Heperkan ve Uğur Kesgin 2003.
3. Chapra S.C. “Applied Numerical Methods with MATLAB for engineers and Scientists” Third Edition, McGraw Hill, International Edition 2012.
4. Mathews J.H. and Fink K.D. “Numerical Methods using MATLAB”, Fourth Edition, Pearson P. Hall, International Edition 2004.
5. Fausett L.V. “Applied Numerical Analysis Using MATLAB, Second Edition, Pearson P. Hall, International Edition, 2008.
6. Gilat A. And Subramaniam V. “Numerical Methods, An introduction with Applications Using MATLAB”, Second Edition, John Wiley and Sons. Inc. 2011.

Dördüncü Mertebe Runge-Kutta Yöntemi

$$\frac{dy_1}{dx} = y_1 + 2y_2$$

$$\frac{dy_2}{dx} = 3y_1 + 2y_2$$



$$x = 0, y_1 = 6, y_2 = 4$$

Adım Boyu $h = 0.02$.

$$y_{1,i+1} = y_{1,i} + \frac{h}{6}(k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1})$$

$$y_{2,i+1} = y_{2,i} + \frac{h}{6}(k_{1,2} + 2k_{2,2} + 2k_{3,2} + k_{4,2})$$

$$k_{1,1} = f_1(x_i, y_{1,i}, y_{2,i})$$

$$k_{1,2} = f_2(x_i, y_{1,i}, y_{2,i})$$

$$k_{2,1} = f_1\left(x_i + \frac{h}{2}, \quad y_{1,i} + \frac{h}{2}k_{1,1}, \quad y_{2,i} + \frac{h}{2}k_{1,2}\right)$$

$$k_{2,2} = f_2\left(x_i + \frac{h}{2}, \quad y_{1,i} + \frac{h}{2}k_{1,1}, \quad y_{2,i} + \frac{h}{2}k_{1,2}\right)$$

$$k_{3,1} = f_1\left(x_i + \frac{h}{2}, \quad y_{1,i} + \frac{h}{2}k_{2,1}, \quad y_{2,i} + \frac{h}{2}k_{2,2}\right)$$

$$k_{3,2} = f_2\left(x_i + \frac{h}{2}, \quad y_{1,i} + \frac{h}{2}k_{2,1}, \quad y_{2,i} + \frac{h}{2}k_{2,2}\right)$$

$$k_{4,1} = f_1(x_i + h, \quad y_{1,i} + k_{3,1}h, \quad y_{2,i} + k_{3,2}h)$$

$$k_{4,2} = f_2(x_i + h, \quad y_{1,i} + k_{3,1}h, \quad y_{2,i} + k_{3,2}h)$$

$$k_{1,1} = f(0.0, 6.0, 4.0) = y_1 + 2 * y_2 = 6.0 + 2 * 4 = 14.0$$

$$k_{1,2} = g(0.0, 6.0, 4.0) = 3 * y_1 + 2 * y_2 = 3 * 6 + 2 * 4 = 26.0$$

$$y_{1,i} + \frac{h}{2} k_{1,1} = 6.0 + \frac{0.02}{2} * 14.0 = 6.14$$

$$y_{2,i} + \frac{h}{2} k_{1,2} = 4 + \frac{0.02}{2} * 26.0 = 4.26$$

$$k_{2,1} = f(0.01, 6.14, 4.26) = y_1 + 2 * y_2 = 6.14 + 2 * 4.26 = 14.66$$

$$k_{2,2} = g(0.01, 6.14, 4.26) = 3 * y_1 + 2 * y_2 = 3 * 6.14 + 2 * 4.26 = 26.94$$

$$y_{1,i} + \frac{h}{2} k_{2,1} = 6.0 + \frac{0.02}{2} * 14.66 = 6.1466$$

$$y_{2,i} + \frac{h}{2} k_{2,2} = 4 + \frac{0.02}{2} * 26.94 = 4.2694$$

$$k_{3,1} = f(0.01, 6.1466, 4.2694) = y_1 + 2 * y_2 = 6.1466 + 2 * 4.2694 = 14.6854$$

$$k_{3,2} = g(0.01, 6.1466, 4.2694) = 3 * y_1 + 2 * y_2 = 3 * 6.1466 + 2 * 4.2694 = 26.9786$$

$$y_{1,i} + k_{3,1}h = 6.0 + 0.02 * 14.6854 = 6.293708$$

$$y_{2,i} + k_{3,2}h = 4 + 0.02 * 26.9786 = 4.539572$$

$$k_{4,1} = f(0.02, 6.293708, 4.539575) = y_1 + 2 * y_2 = 6.293708 + 2 * 4.539575 = 15.372858$$

$$k_{4,2} = g(0.02, 6.293708, 4.539575) = 3 * y_1 + 2 * y_2 = 3 * 6.293708 + 2 * 4.539575 = 27.960274$$

$$y_{1,i} = 6 + \frac{0.02}{6} (14.0 + 2(14.66) + 2(14.6854) + 15.372858) = 6.29354553$$

$$y_{2,i} = 4 + \frac{0.02}{6} (26.0 + 2(26.94) + 2(26.9786) + 27.960274) = 4.53932491$$

Command window

```
>> xs=[0,0.02];  
>> y0=[6,4];  
>> [x,y]=ode23('biricitur', xs,y0);
```

y(1)	y(2)
6	4
6,02813233866667	4,05218851466667
6,05653071893120	4,10475613168349
6,08519719758201	4,15770597611329
6,11413384808782	4,21104119795967
6,14334276073131	4,26476497236765
6,17282604274384	4,31888049982589
6,20258581844128	4,37339100637066
6,23262422936093	4,42829974379126
6,26294343439949	4,48360998983724
6,29354560995219	4,53932504842723

$$y_{1,i} = 6 + \frac{0.02}{6} (14.0 + 2(14.66) + 2(14.6854) + 15.372858) \\ = 6.29354553$$

$$y_{2,i} = 4 + \frac{0.02}{6} (26.0 + 2(26.94) + 2(26.9786) + 27.960274) \\ = 4.53932491$$

```
>> plot(x,y)
```

