Ankara Ü. BLM bölümü

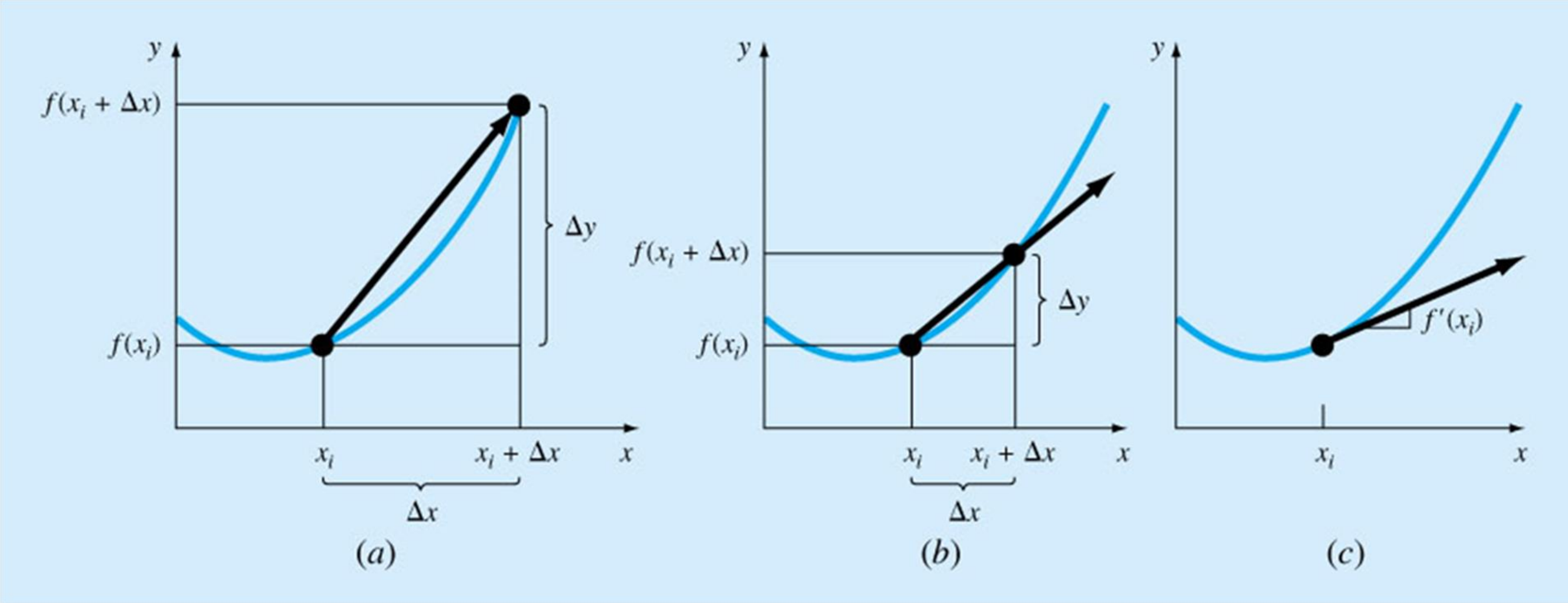
BLM433- Sayısal Analiz Teknikleri

Calculus is the mathematics of change. Because engineers must continuously deal with systems and processes that change, calculus is an essential tool of engineering. Standing in the heart of calculus are the mathematical concepts of differentiation and integration:

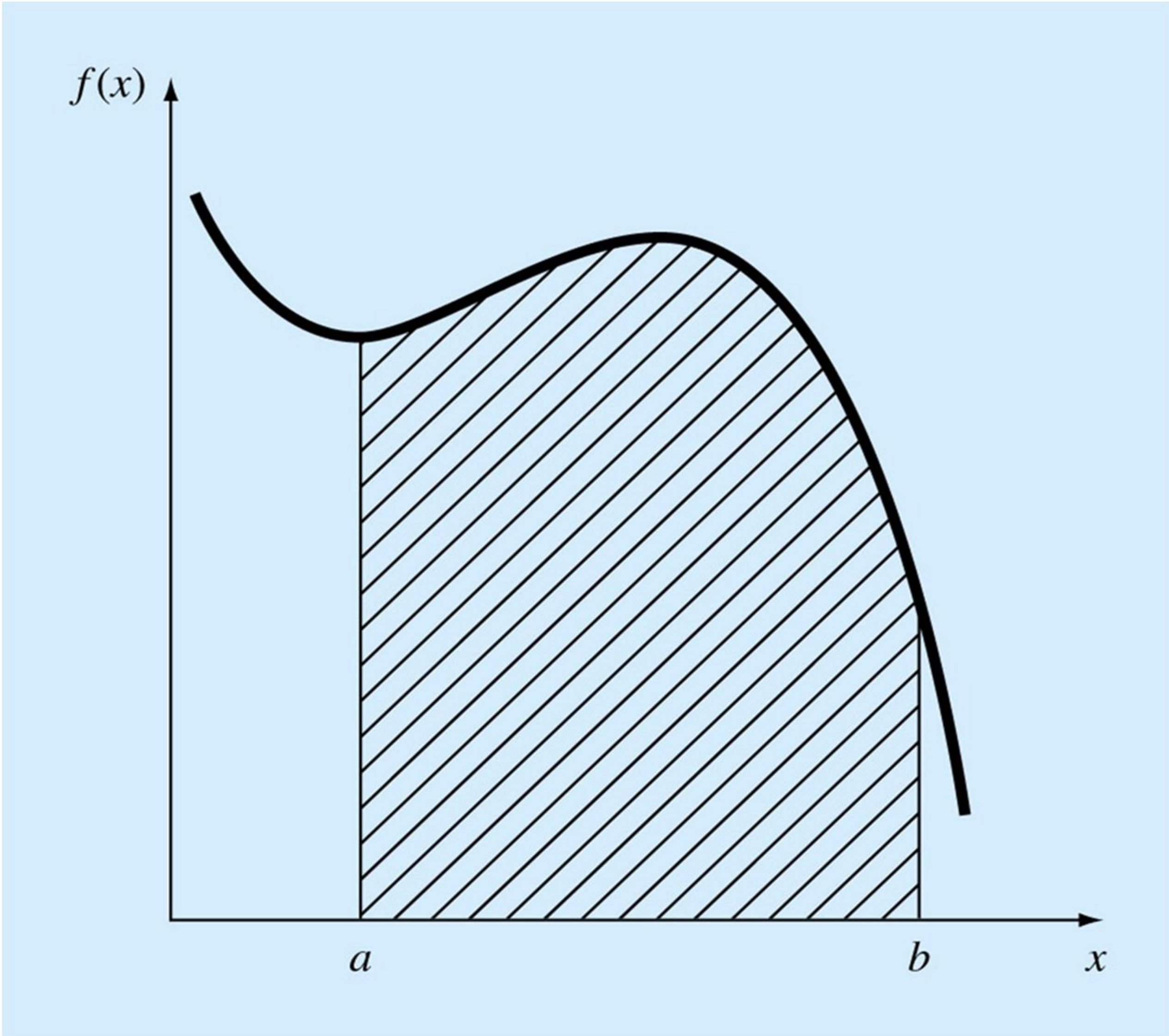
$$\frac{\Delta y}{\Delta x} = \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$$

$$I = \int_a^b f(x)dx$$



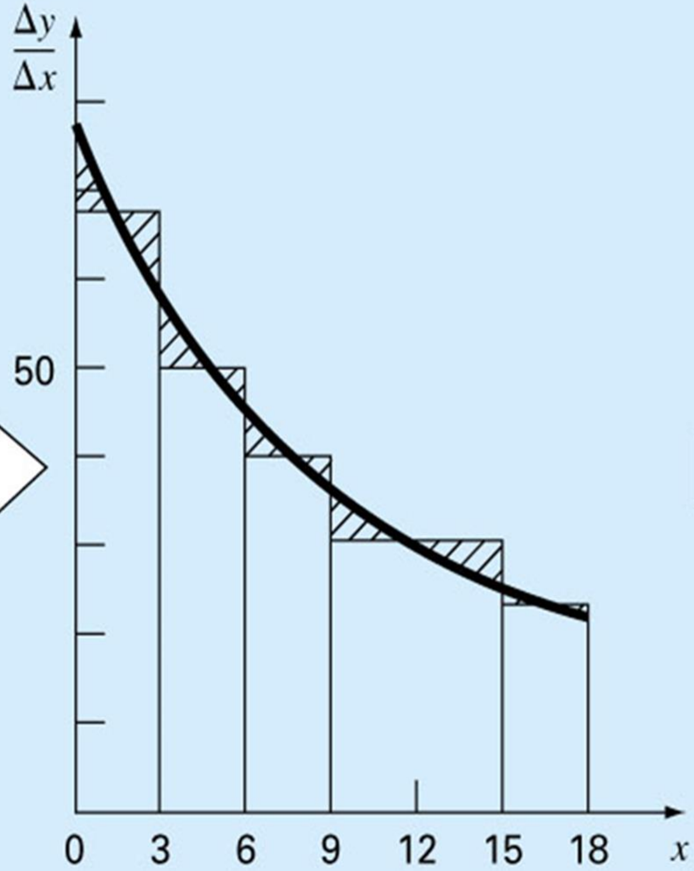
y
f(x_i + Δx)
Δy
f(x_i)
x_i
x_i + Δx
x
Δx
(a)
y
f(x_i + Δx)
Δy
f(x_i)
x_i
x_i + Δx
x
Δx
(b)
y
f′(x_i)
x_i
x
(c)



f(x)
a
b
x

x	y	$\Delta y/\Delta x$
0	0	
		66.7
3	200	
		50
6	350	
		40
9	470	
		30
15	650	
		23.3
18	720	

(*a*)



(*b*)

x	dy/dx
0	76.50
3	57.50
6	45.00
9	36.25
15	25.00
18	21.50

(*c*)

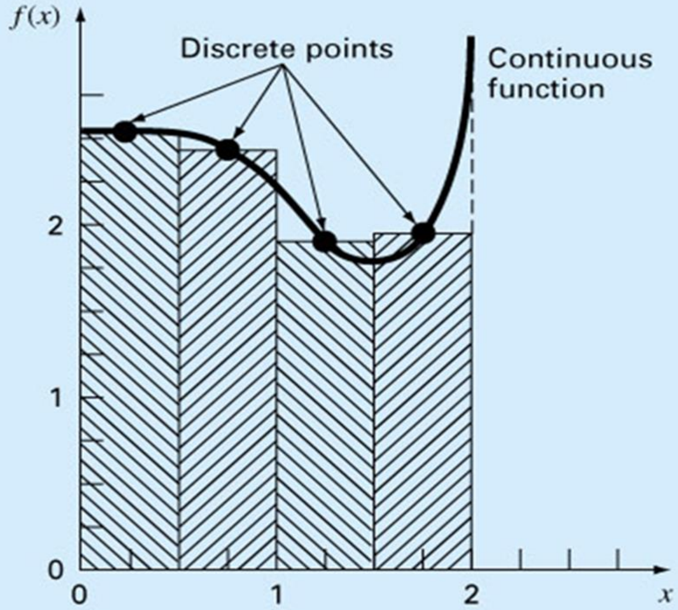
(a)

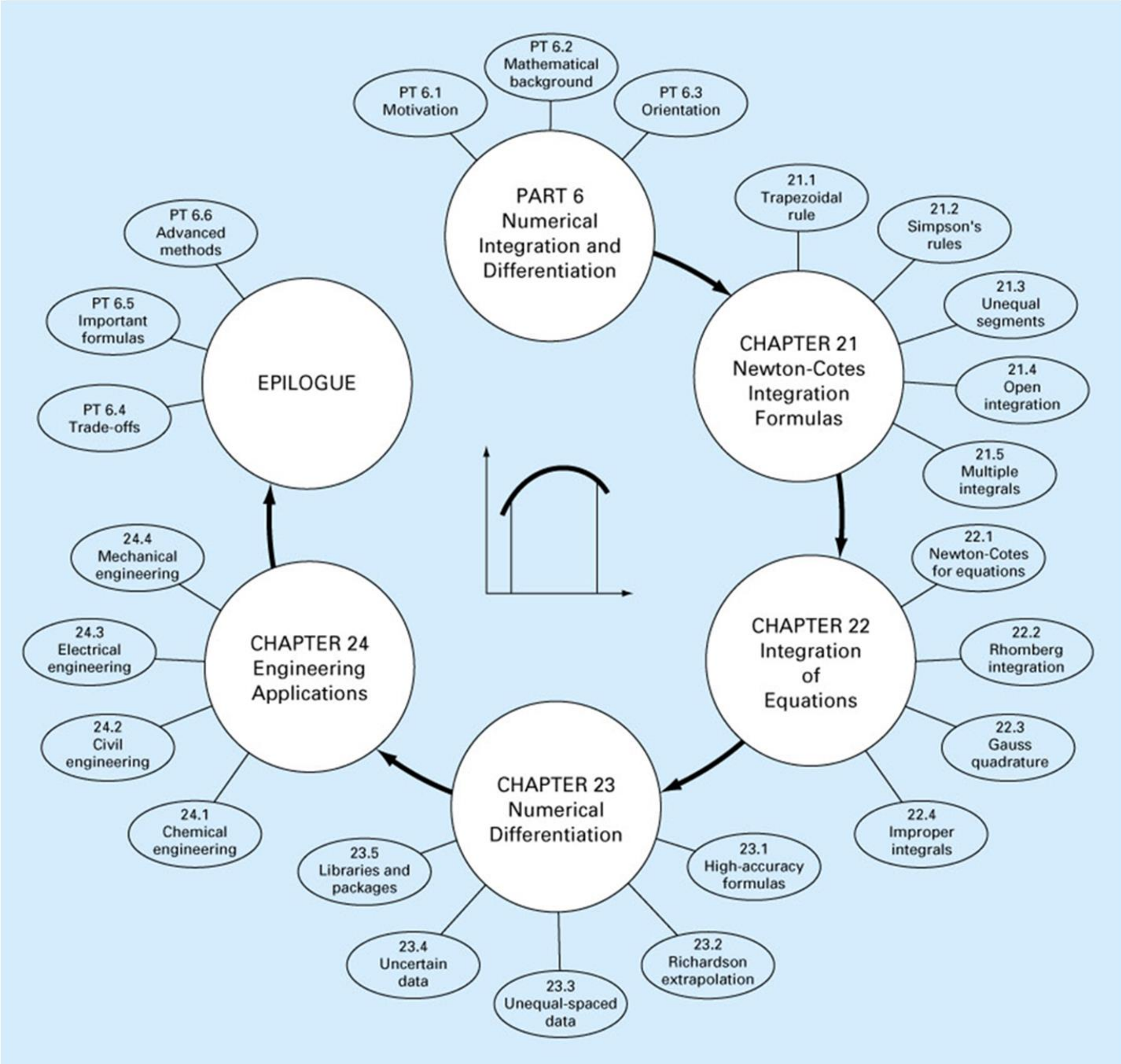
$$\int_0^2 \frac{2 + \cos(1 + x^{3/2})}{\sqrt{1 + 0.5 \sin x}} e^{0.5x} \, dx$$

(b)

x	$f(x)$
0.25	2.599
0.75	2.414
1.25	1.945
1.75	1.993

(c)

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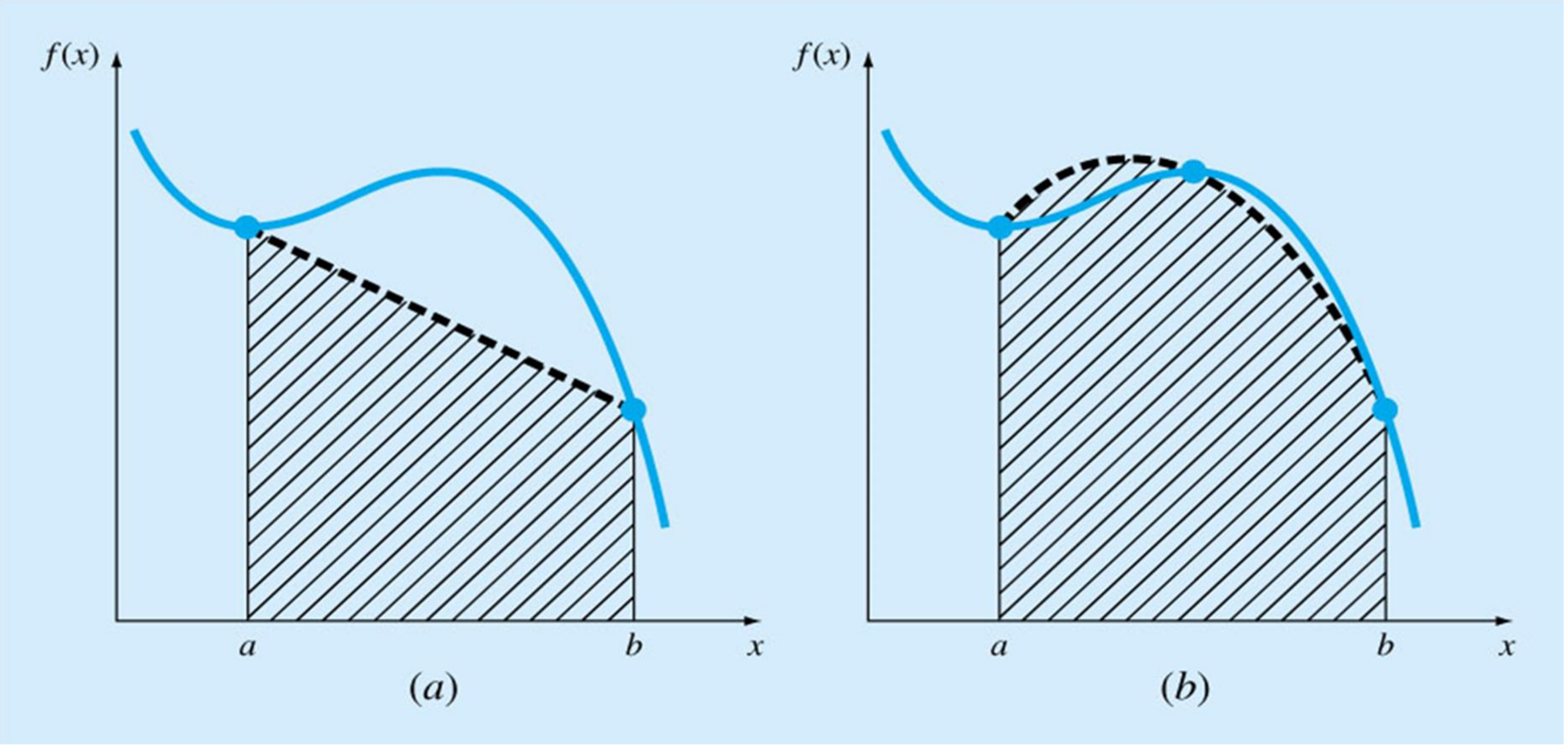
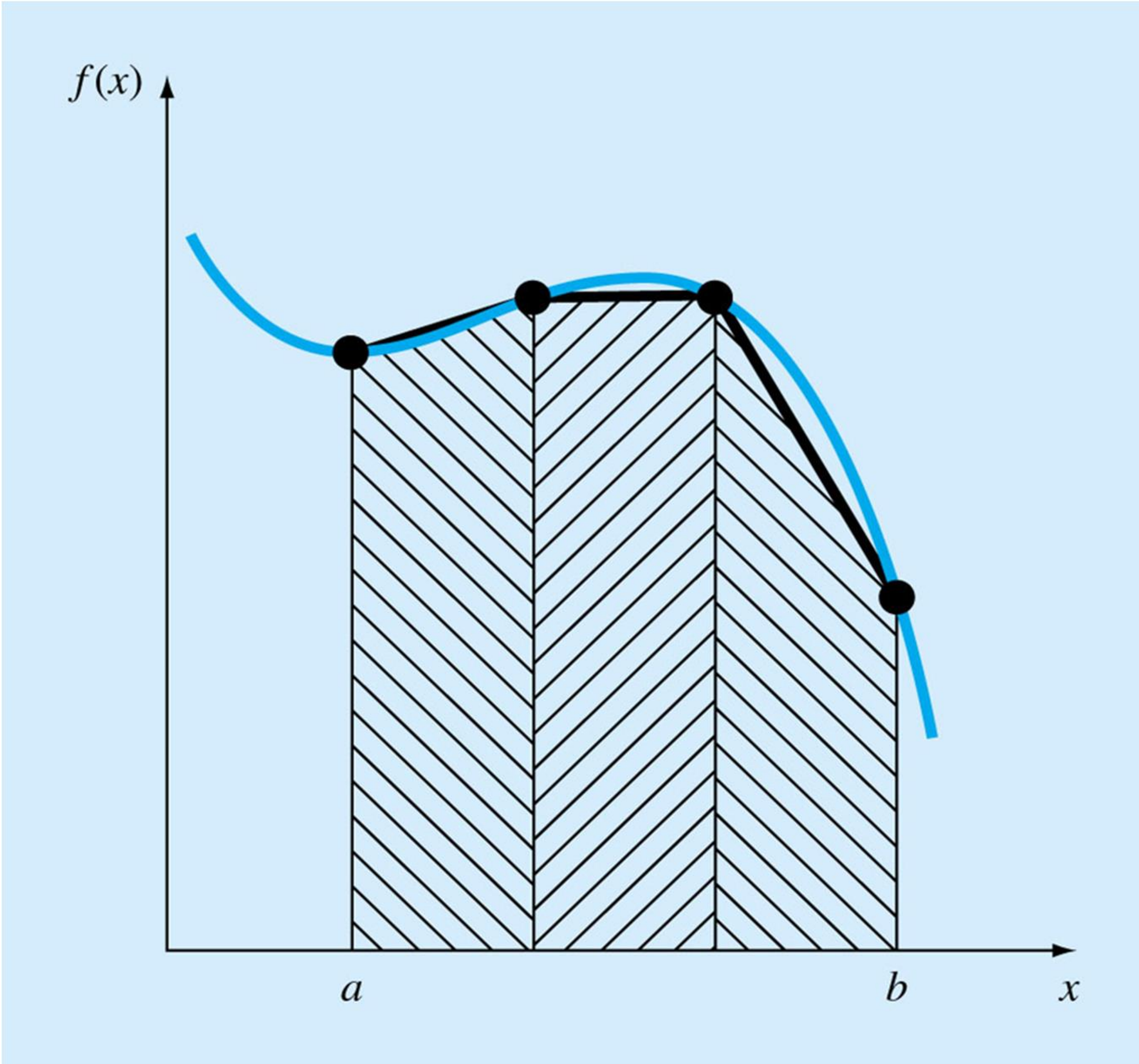
EPILOGUE

Newton-Cotes integration

$$I = \int_a^b f(x)dx \cong \int_a^b f_n(x)dx$$

$$f_n(x) = a_0 + a_1 x + \cdots + a_{n-1}x^{n-1} + a_n x^n$$

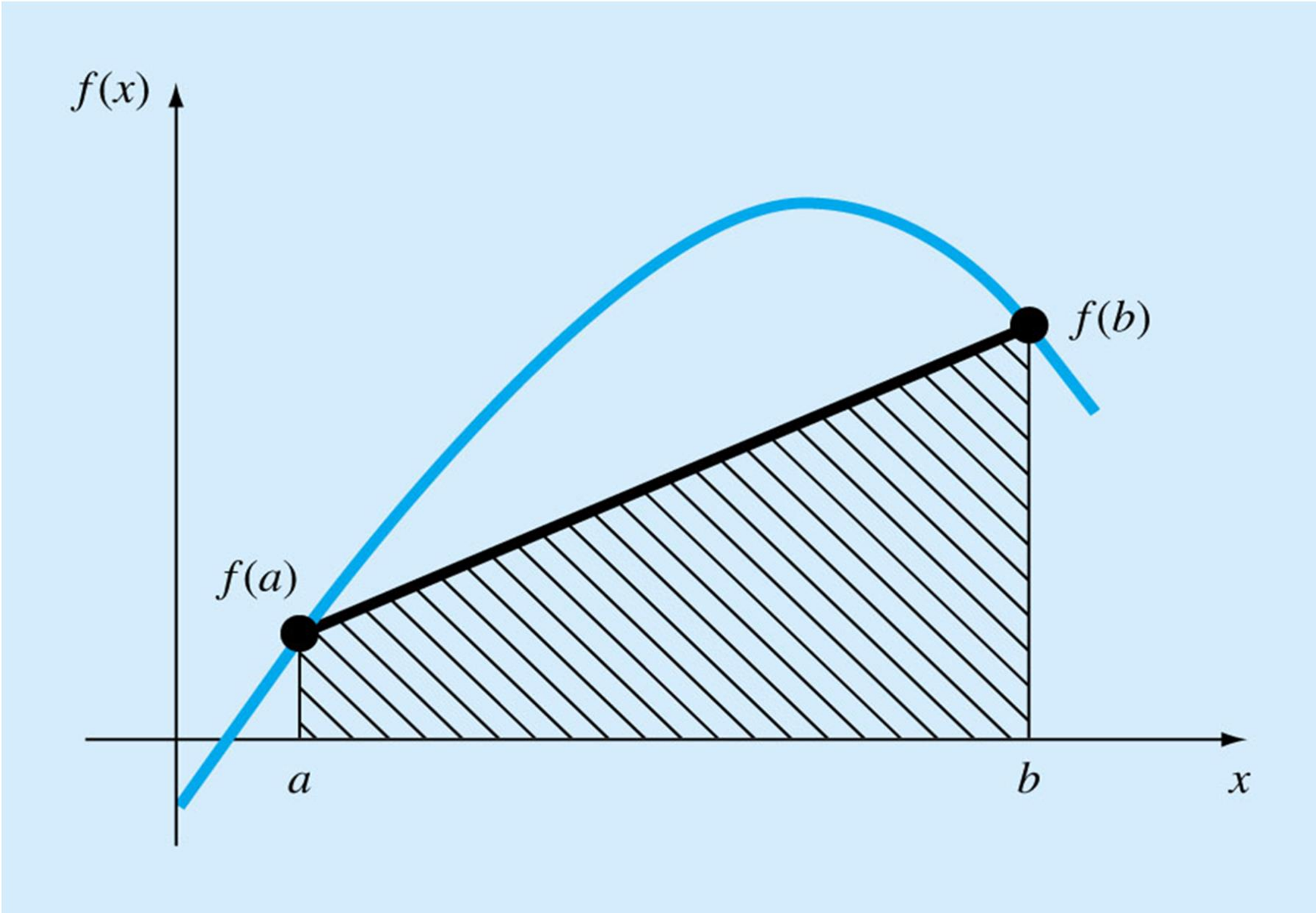


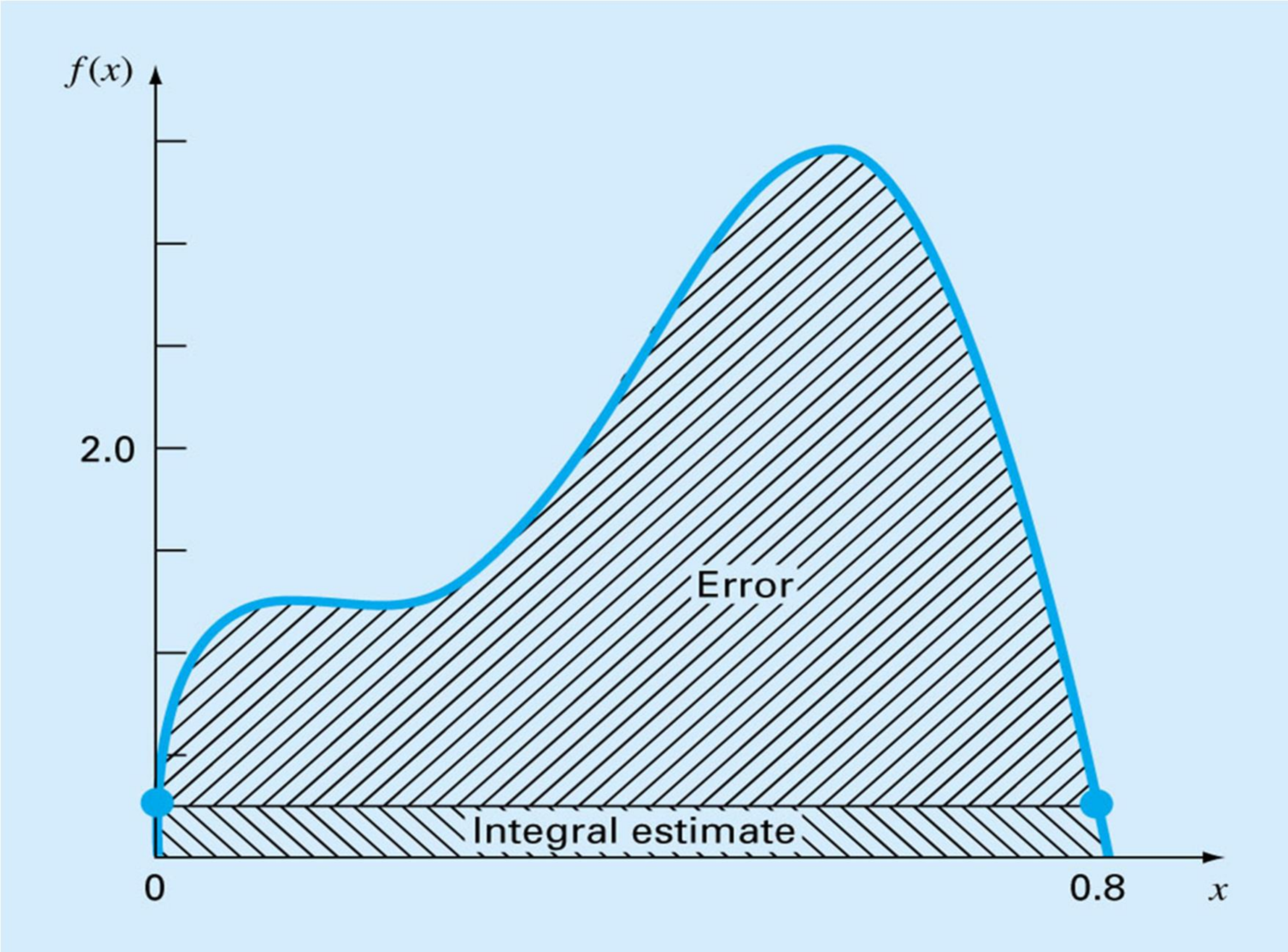


f(x)
a
b
x

$$I = (b-a)\frac{f(a)+f(b)}{2}$$

$$E_t = -\frac{1}{12} f''(\xi)(b-a)^3$$

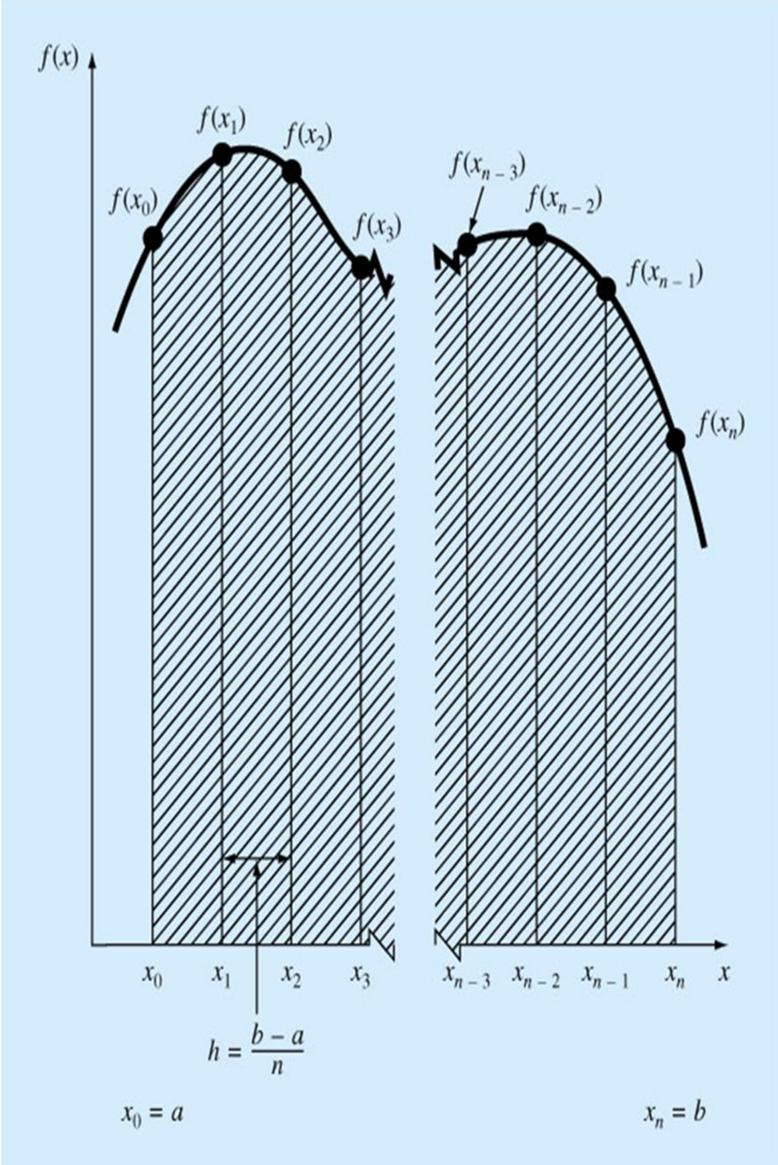



f(x)
2.0
Error
Integral estimate
0
0.8
x

$$h = \frac{b-a}{n} \qquad a = x_0 \quad b = x_n$$

$$I = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \cdots + \int_{x_{n-1}}^{x_n} f(x)dx$$

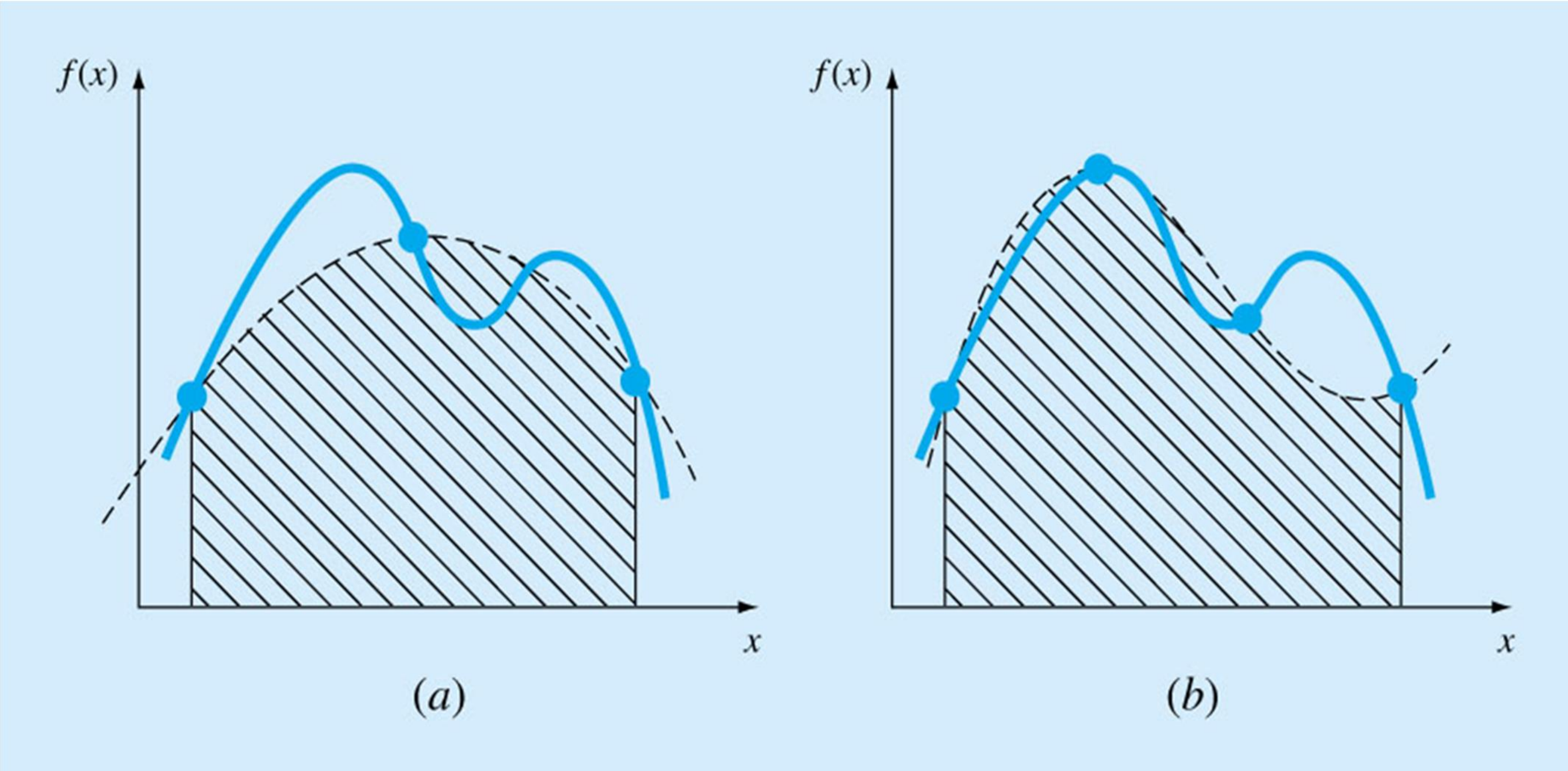
$$I = h\frac{f(x_0)+f(x_1)}{2} + h\frac{f(x_1)+f(x_2)}{2} + \cdots + h\frac{f(x_{n-1})+f(x_n)}{2}$$



$$\sum f''(\xi i) \cong n\bar{f}''$$

$$E_a = -\frac{(b-a)^3}{12n^2}\bar{f}''$$

Simpson rules



Simpson 1/3 rules

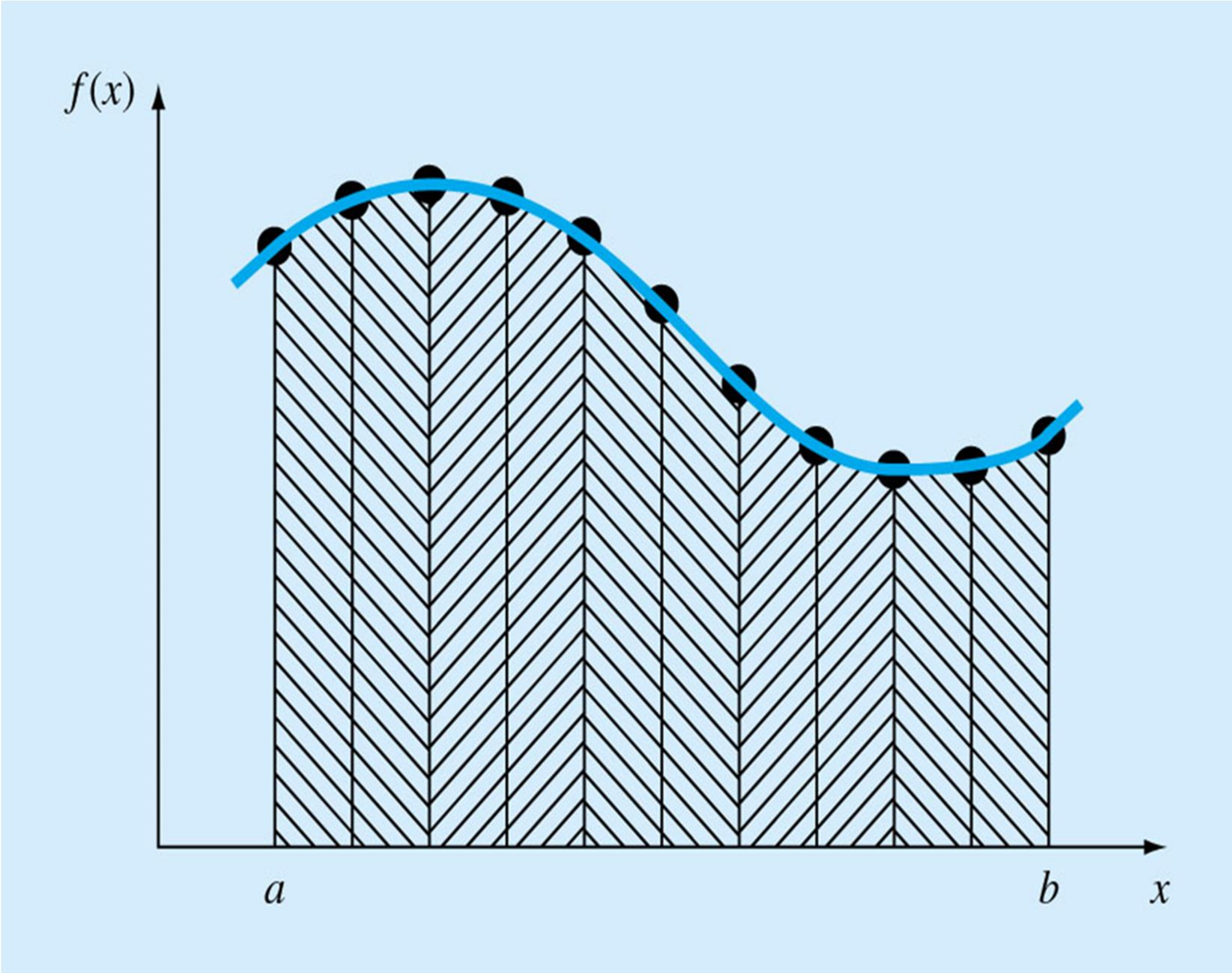
$$I = \int_a^b f(x)dx \cong \int_a^b f_2(x)dx$$

$$a = x_0 \quad b = x_2$$

$$I = \int_{x_0}^{x_2} \left[\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2) \right] dx$$

$$I \cong \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] \qquad h = \frac{b-a}{2}$$

$$E_t = -\frac{(b-a)^5}{2880} f^{(4)}(\xi) \qquad a \prec \xi \prec b$$


f(x)
a
b
x

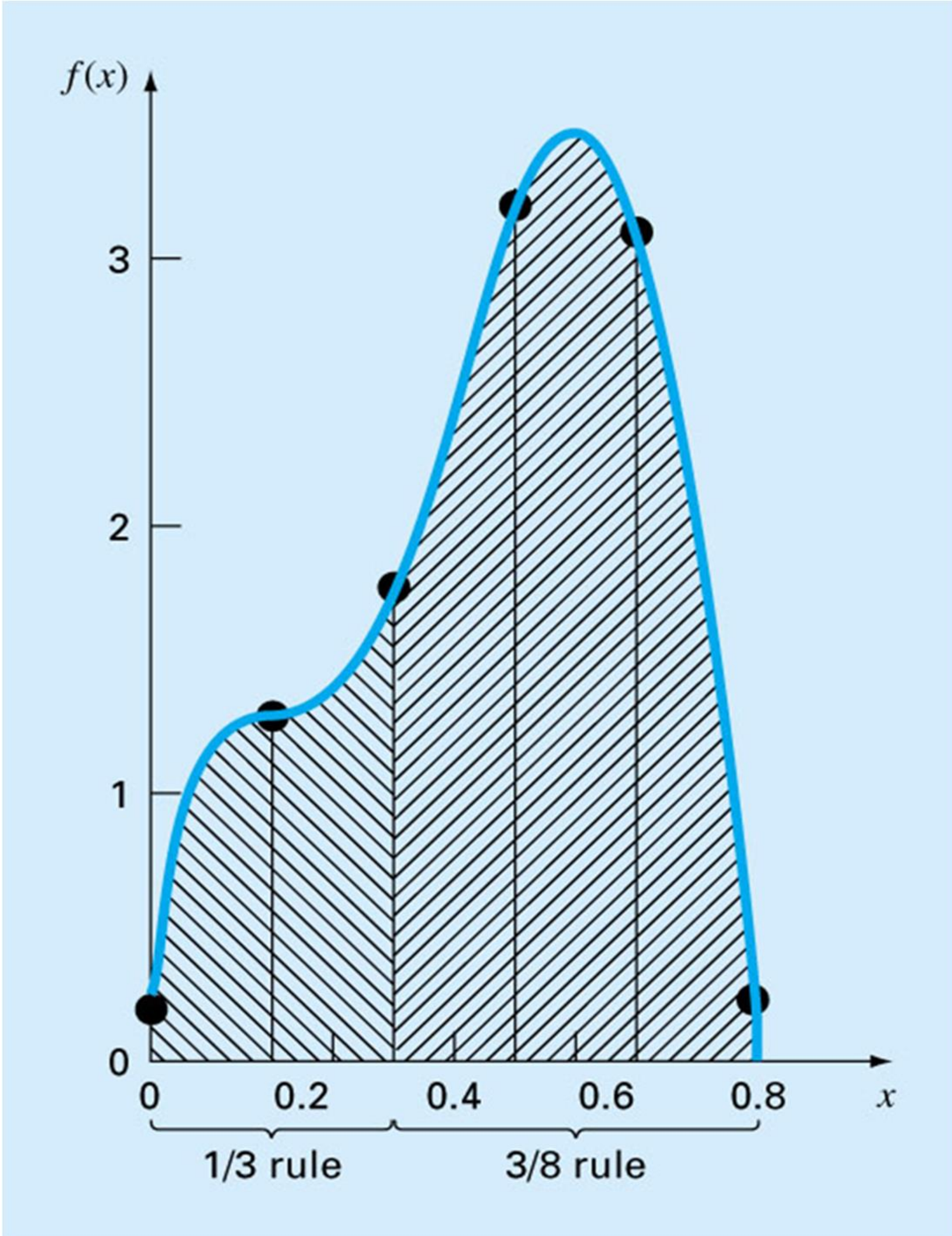
Simpson 3/8 rules

$$I = \int_a^b f(x)dx \cong \int_a^b f_3(x)dx$$

$$I \cong \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

$$h = \frac{(b-a)}{3}$$

$$E_t = -\frac{(b-a)5}{6480} f^{(4)}(\xi)$$



f(x)
3
2
1
0
0
0.2
0.4
0.6
0.8
x
1/3 rule
3/8 rule