Ankara Ü. BLM bölümü

BLM433 Sayısal analiz Teknikleri

Functions to be integrated numerically are in two forms:

A table of values. We are limited by the number of points that are given.

A function. We can generate as many values of $f(x)$ as needed to attain acceptable accuracy.

Will focus on two techniques that are designed to analyze functions:

Romberg integration
Gauss quadrature

ROMBERG integration

Is based on successive application of the trapezoidal rule to attain efficient numerical integrals of functions.

Richardson’s Extrapolation/

Uses two estimates of an integral to compute a third and more accurate approximation

$$I = I(h) + E(h)$$

$$h = (b-a)/n$$

$$I(h_1) + E(h_1) = I(h_2) + E(h_2)$$

$$n = (b-a)/h$$

$$E \cong \frac{b-a}{12} h^2 \bar{f}''$$

$$\frac{E(h_1)}{E(h_2)} \cong \frac{h_1^2}{h_2^2}$$

$$E(h_1) \cong E(h_2)\left(\frac{h_1}{h_2}\right)^2$$

I =exact value of integral

I(h) =the approximation from an n segment application of trapezoidal rule with step size h

E(h) =the truncation error

$$I(h_1) + E(h_2)\left(\frac{h_1}{h_2}\right)^2 \cong I(h_2) + E(h_2)$$

$$E(h_2) \cong \frac{I(h_1) - I(h_2)}{1 - \left(h_1/h_2\right)^2}$$

$$I = I(h_2) + E(h_2)$$

$$I \cong I(h_2) + \frac{1}{\left(h_1/h_2\right)^2 - 1}\left[I(h_2) - I(h_1)\right]$$

(a)

$O(h^2)$	$O(h^4)$
0.172800	1.367467
1.068800	

(b)

$O(h^2)$	$O(h^4)$	$O(h^6)$
0.172800	1.367467	1.640533
1.068800	1.623467	
1.484800		

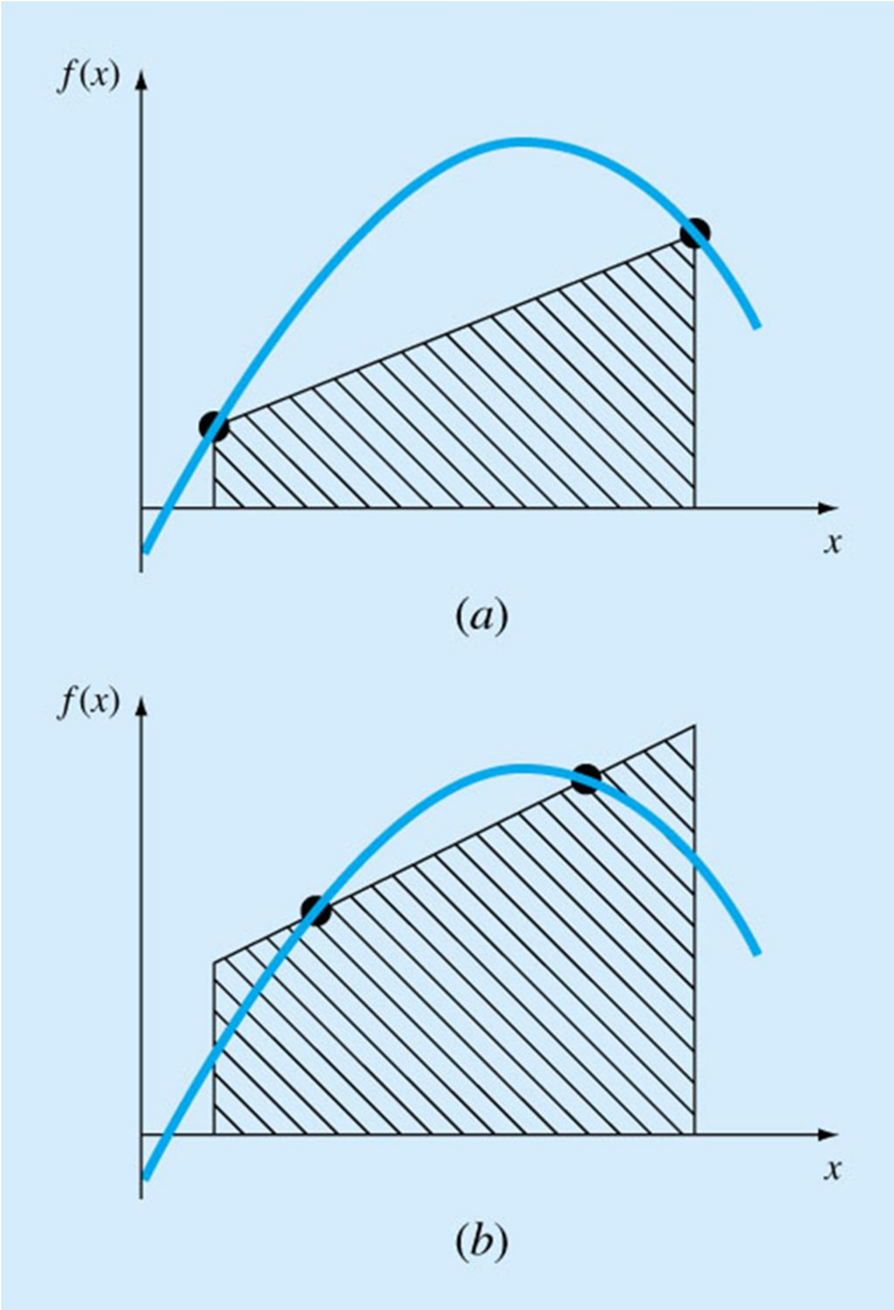
(c)

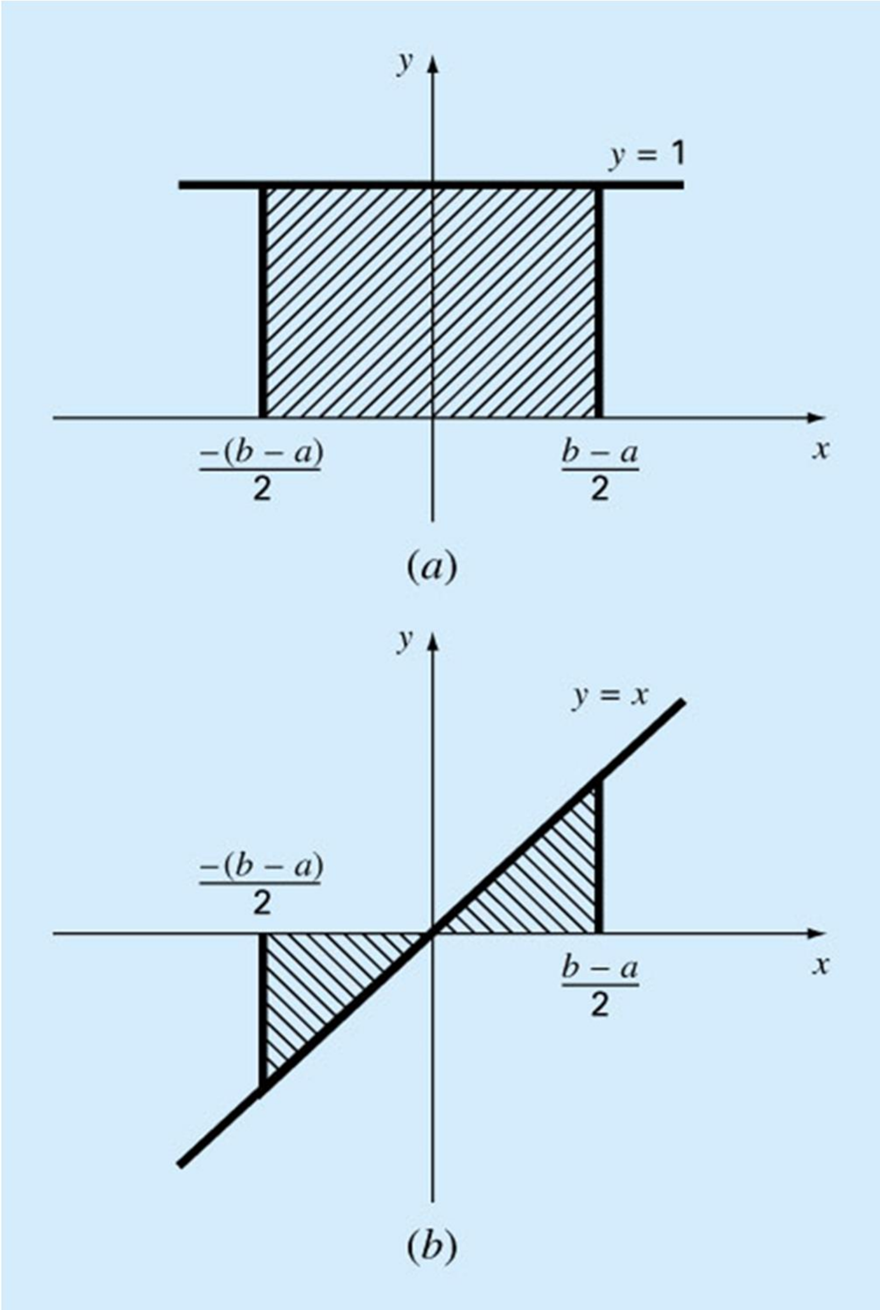
$O(h^2)$	$O(h^4)$	$O(h^6)$	$O(h^8)$
0.172800	1.367467	1.640533	1.640533
1.068800	1.623467	1.640533	
1.484800	1.639467		
1.600800			

Gauss quadrature

implements a strategy of positioning any two points on a curve to define a straight line that would balance the positive and negative errors.

Hence the area evaluated under this straight line provides an improved estimate of the integral.


f(x)
x
(a)
f(x)
x
(b)



$$c0 + c1 = \int_{-(b-a)/2}^{(b-a)/2} 1\, dx$$

$$-c_0 \frac{b-a}{2} + c_1 \frac{b-a}{2} = \int_{-(b-a)/2}^{(b-a)/2} x\, dx$$

$$c_0 + c_1 = b - a$$

$$-c_0 \frac{b-a}{2} + c_1 \frac{b-a}{2} = 0$$

$$c_0 = c_1 = \frac{b-a}{2}$$

$$I = \frac{b-a}{2} f(a) + \frac{b-a}{2} f(b)$$

$$c_0 f(x_0) + c_1 f(x_1) = \int_{-1}^{1} 1\, dx = 2$$

$$c_0 f(x_0) + c_1 f(x_1) = \int_{-1}^{1} x\, dx = 0$$

$$c_0 f(x_0) + c_1 f(x_1) = \int_{-1}^{1} x^2\, dx = \frac{2}{3}$$

$$c_0 f(x_0) + c_1 f(x_1) = \int_{-1}^{1} x^3\, dx = 0$$

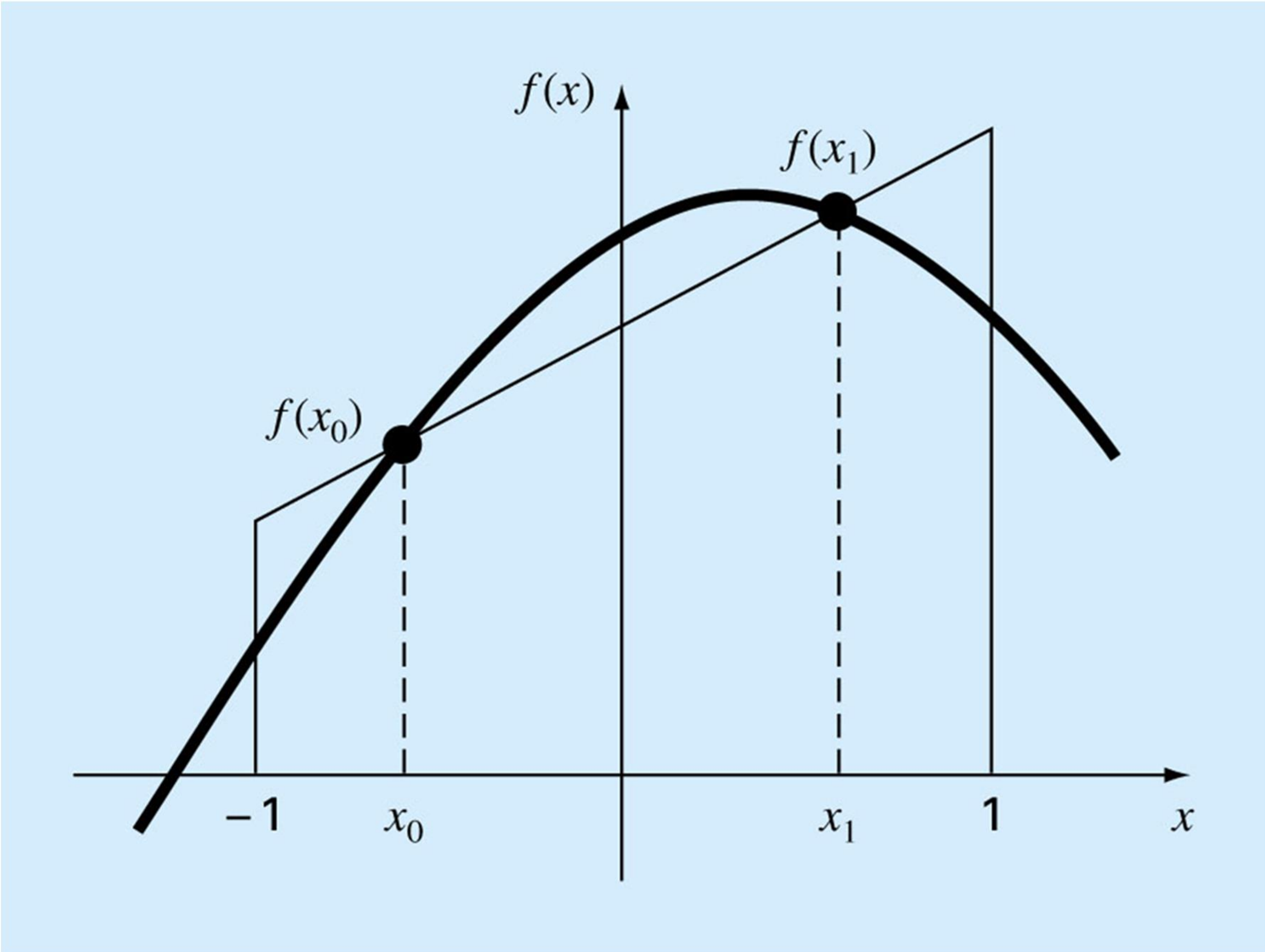
$$c_0 = c_1 = 1$$

$$x_0 = -\frac{1}{\sqrt{3}} = -0.5773503\ldots$$

$$x_1 = \frac{1}{\sqrt{3}} = -0.5773503\ldots$$

$$I \cong f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$E_t = \frac{2^{2n+3}\left[(n=1)!\right]^4}{(2n+3)\left[(2n+2)!\right]^3} f^{(2n+2)}(\xi)$$



f(x)
f(x_1)
f(x_0)
−1
x_0
x_1
1
x

Improper integrals can be evaluated by making a change of variable that transforms the infinite range to one that is finite,

$$\int_a^b f(x)dx = \int_{1/b}^{1/a} \frac{1}{t^2} f\left(\frac{1}{t}\right) dt \qquad ab \succ 0$$

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{-A} f(x)dx + \int_{-A}^{b} f(x)dx$$

where –A is chosen as a sufficiently large negative value so that the function has begun to approach zero asymptotically at least as fast as 1/x2