Physics 2: Electricity & Magnetism – Magnetic Fields

Assoc. Prof. Dr. Fulya Bağcı

Ankara University, Department of Physics Engineering

Outline

-Particle in a Magnetic Field

-Motion of a Charged Particle in a Uniform Magnetic Field

-Applications Involving Charged Particles Moving in a Magnetic Field

-Magnetic Force Acting on a Current-Carrying Conductor

-Torque on a Current Loop in a Uniform Magnetic Field

Magnetism

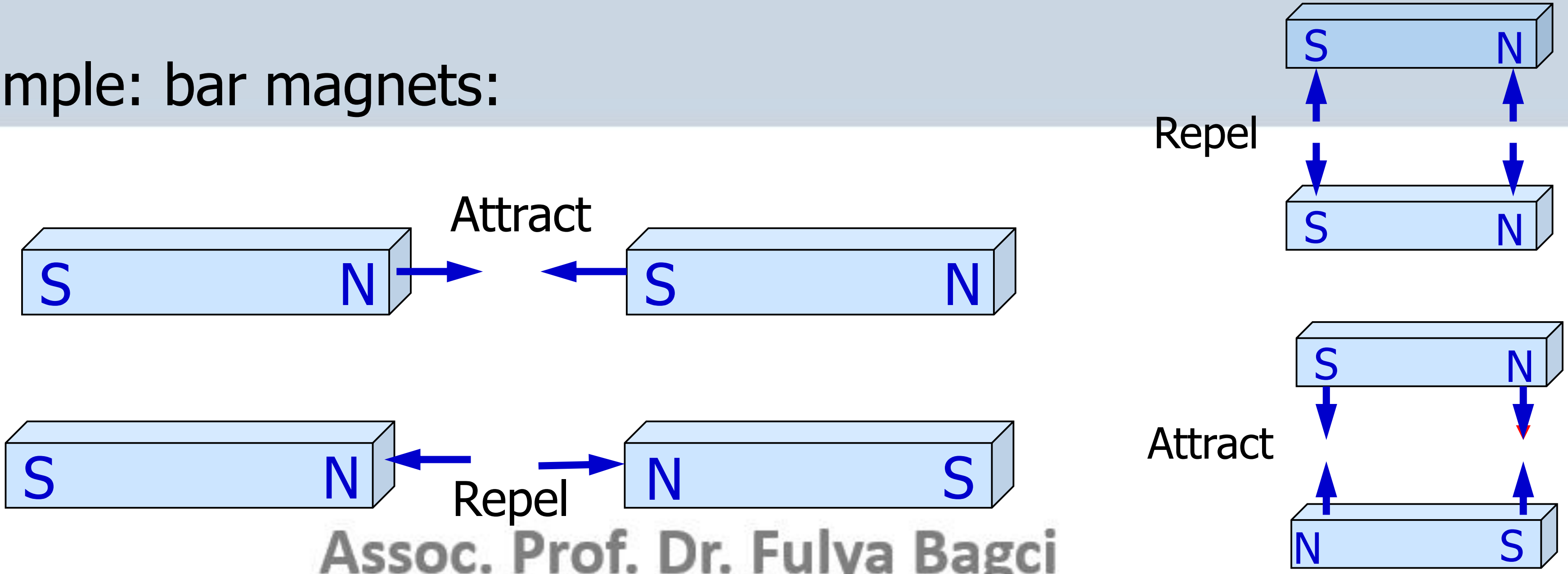
Recall: (lecture 1)

- electric (Coulomb) force between electric charges (+ and -), like charges repel, opposite charges attract

Analogously:

- two kinds of magnetic poles (north and south), like poles repel, opposites attract

Example: bar magnets:



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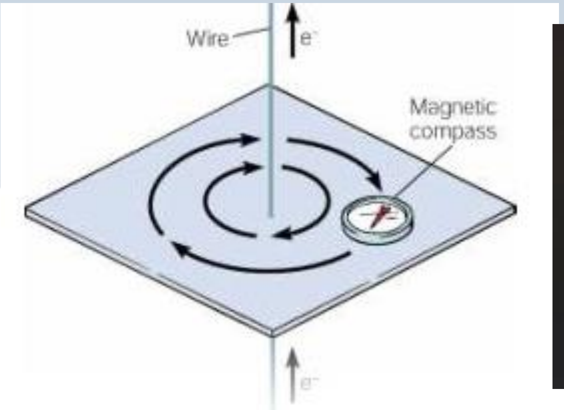
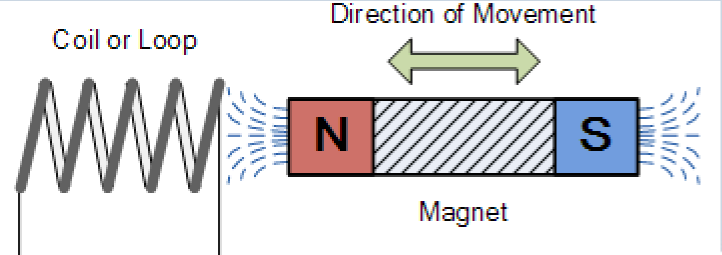
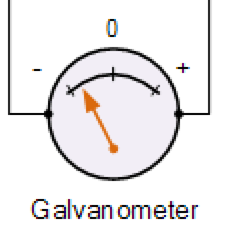
History of Magnetism

The relationship between magnetism and electricity was discovered in 1819 during a lecture demonstration. Hans Christian Oersted found that an electric current in a wire deflected a nearby compass needle. In the 1820s, further connections between electricity and magnetism were demonstrated independently by Faraday and Joseph Henry (1797–1878). They showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. These observations demonstrate that a changing magnetic field creates an electric field. Years later, theoretical work by Maxwell showed that the reverse is also true: a changing electric field creates a magnetic field.



Oersted
https://www.quora.com/



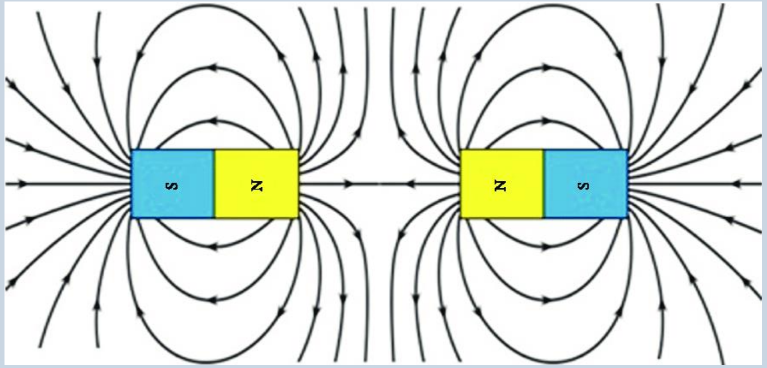


Faraday

https://www.electronics-tutorials.ws/electromagnetism/electromagnetic-induction.html

29.1 Magnetic Fields and Forces

- The region of space surrounding any *moving* electric charge also contains a magnetic field. A magnetic field also surrounds a magnetic substance, for example permanent magnets.



- Magnetic field lines outside the magnet point away from north poles and toward south poles.

Charged particles moving in a magnetic field

- The magnitude $\boldsymbol{F_B}$ of the magnetic force exerted on the particle is proportional to the charge q and to the speed $\boldsymbol{\vartheta}$ of the particle.
- The magnitude and direction of $\boldsymbol{F_B}$ depend on the magnitude and direction of the magnetic field $\boldsymbol{B}$.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- The magnitude of the magnetic force exerted on the moving particle is proportional to sinθ, where θ is the angle the particle's velocity vector makes with the direction of ***B***.

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

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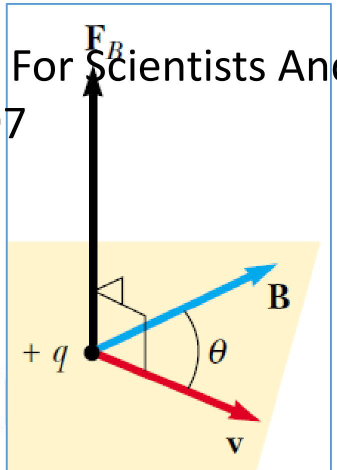

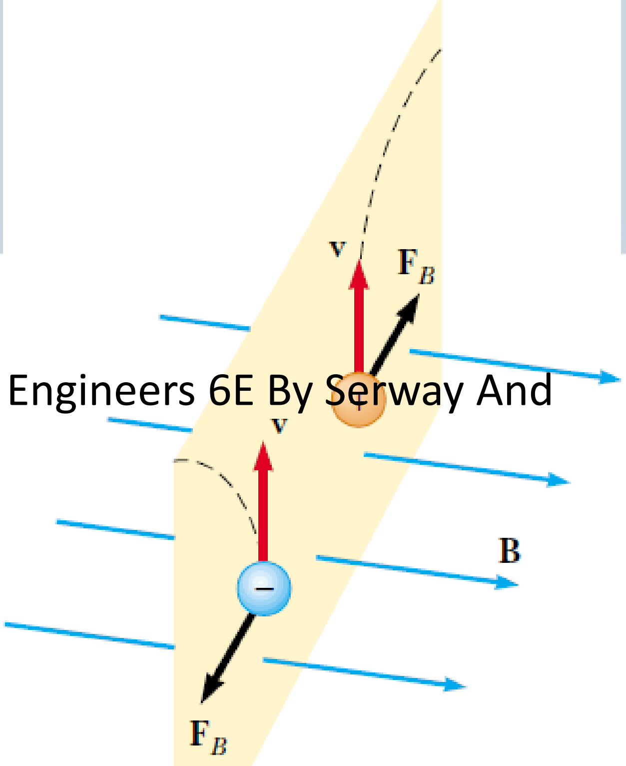

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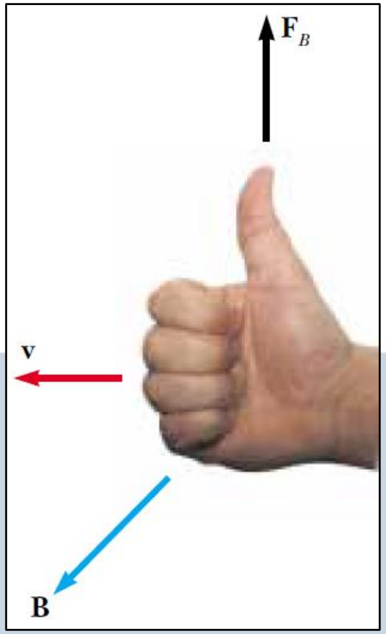
Two **right-hand rules** for determining the direction of the magnetic force acting on +q



- The fingers point in the direction of $\boldsymbol{v}$, with $\boldsymbol{B}$ coming out of your palm, so that you can curl your fingers in the direction of $\boldsymbol{B}$. The direction of $\boldsymbol{v} \times \boldsymbol{B}$, is the direction in which the thumb points

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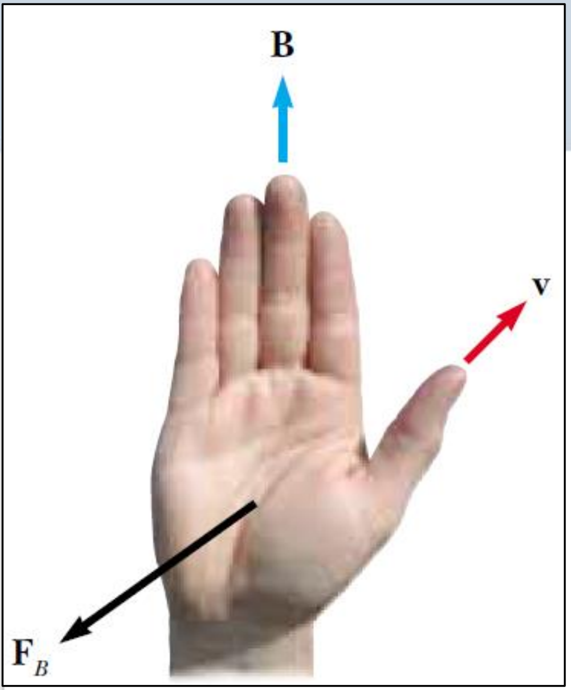

- The vector $\boldsymbol{v}$ is in the direction of your thumb and $\boldsymbol{B}$ in the direction of your fingers. The force $\boldsymbol{F}_B$ on a positive charge is in the direction of your palm, as if you are pushing the particle with your hand.

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- The **magnitude of the magnetic force** on a charged particle is

$$F_B = |q|\vartheta B \sin\theta$$

where θ is the small angle between ϑ and B.

- From this expression, we see that F_B is zero when ϑ is parallel or antiparallel to B (θ = 0 or 180°) and maximum when ϑ is perpendicular to B (θ = 90°).

Differences between electric and magnetic forces

- The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.

Unit of the Magnetic Field

- The SI unit of magnetic field is the newton per coulomb-meter per second, which is called the tesla (T)

$$F_B = |q| \vartheta B \sin\theta$$

$$N = \frac{Cm[B]}{s} \Rightarrow [B] = \frac{N}{C/s \times m} = T \qquad T = \frac{N}{A \cdot m}$$

- A non-SI magnetic-field unit in common use, called the *gauss* (G), is related to the tesla through the conversion 1 T = 10^4 G.

Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	0.5×10^{-4}
Inside human brain (due to nerve impulses)	10^{-13}

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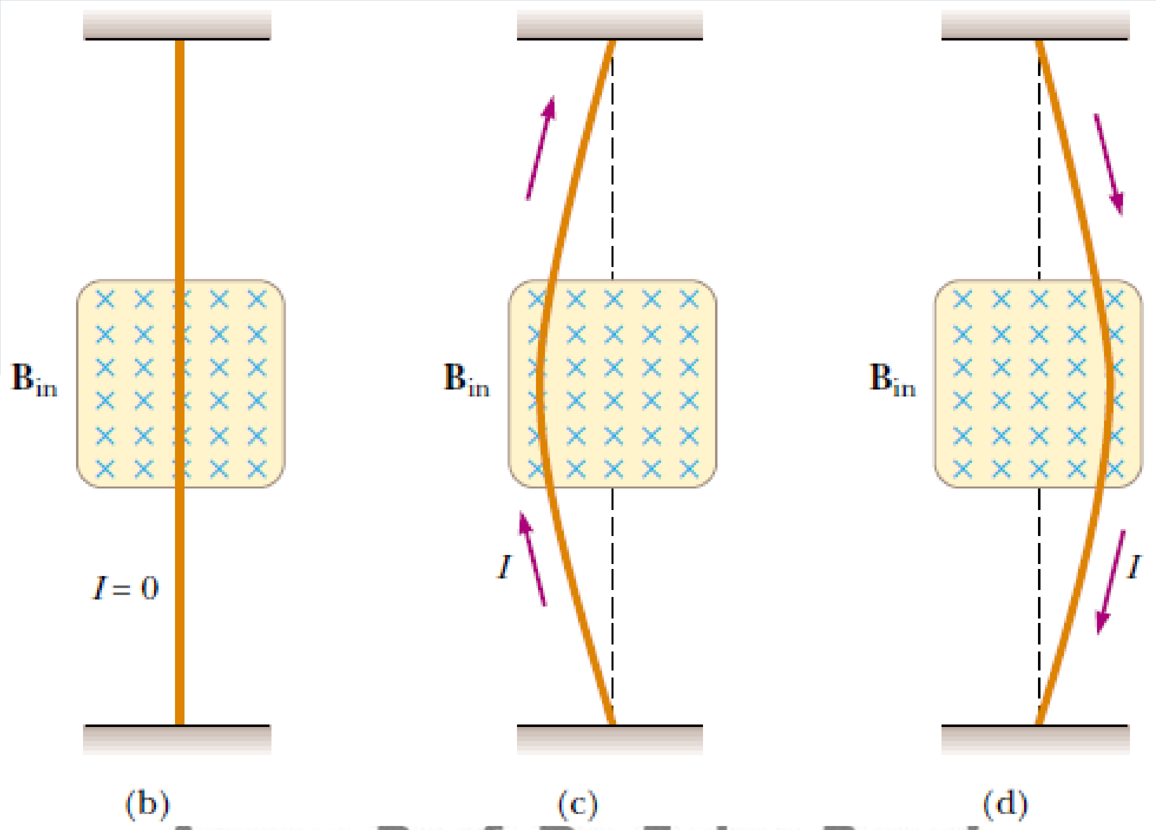
29.2 Magnetic Force Acting on a Current-Carrying Conductor

A current-carrying wire also experiences a force when placed in a magnetic field since current is a collection of many charged particles in motion. One can demonstrate the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet. When the wire carries a current directed upward, the wire deflects to the left. If we reverse the current, the wire deflects to the right.



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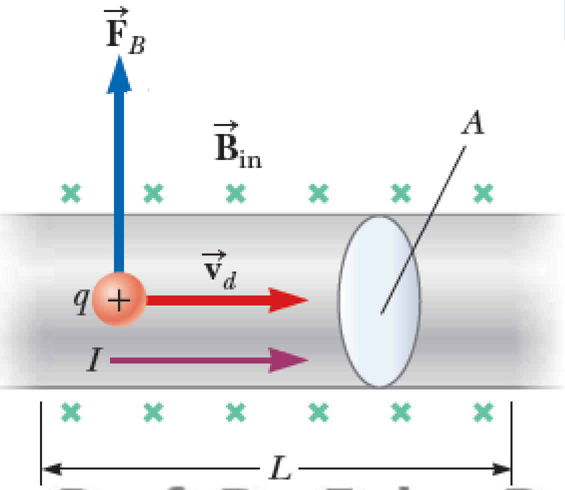
If there are *n* number of charges per unit volume *V*, there are *nAL* charges in a straight segment of wire carrying current *I* in a uniform magnetic field B. Hence, the total magnetic force is

$$\vec{F}_B = \left(q\vec{\vartheta}_d \times \vec{B}\right) nAL$$

Remember the current in a wire was (Eq. 27.4) $I = nq\vartheta_d A$. Therefore,

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

This expression applies only to a straight segment of wire in a uniform magnetic field.



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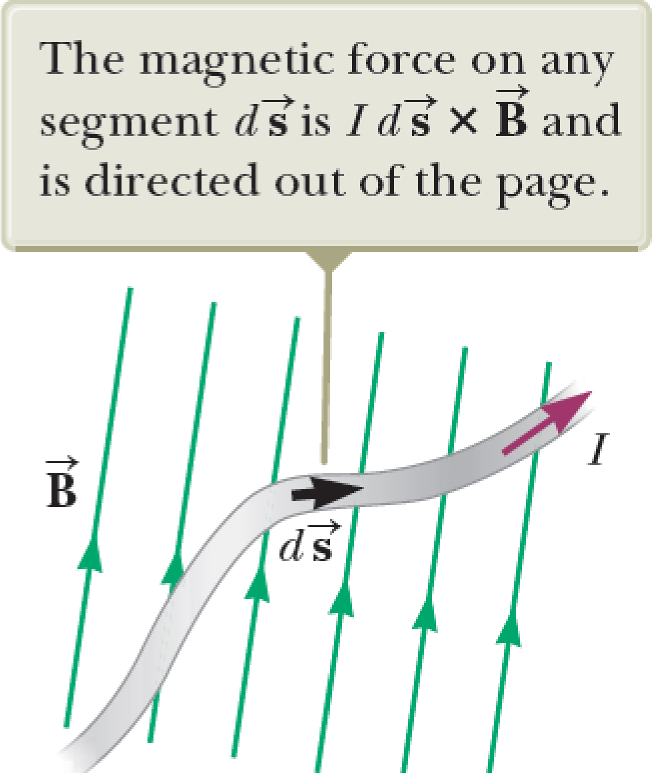
Now consider an arbitrarily shaped wire segment of uniform cross section in a magnetic field. The magnetic force exerted on a small segment is:

$$d\vec{\mathbf{F}}_B = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

The total force $\mathbf{F}_B$ is:

$$\vec{F}_B = I\int_a^b d\vec{s}\times\vec{B}$$

where *a* and *b* represent the end points of the wire.



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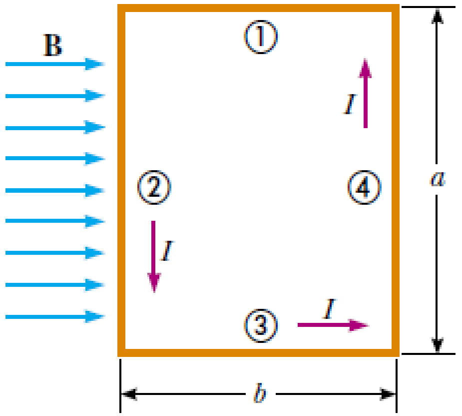
29.5 Torque on a Current Loop in a Uniform Magnetic Field



A torque is exerted on a current loop placed in a magnetic field.

Consider a rectangular loop carrying a current *I* in the presence of a uniform magnetic field directed parallel to the plane of the loop ($\mathbf{L} \times \mathbf{B} = 0$)

Magnetic forces do act on sides 2 and 4 because these sides are oriented perpendicular to the field.

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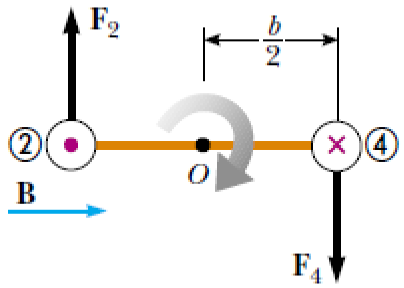
$$\vec{F}_2 = \vec{F}_4 = IaB$$

$\vec{F}_2$ is out of page and $\vec{F}_4$ is into the page

If the loop is pivoted so that it can rotate about point O, these two forces produce about O a torque that rotates the loop clockwise.



View from side 3

$$\tau_{\max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2} = IabB$$

Area=*ab* so the maximum torque is $\tau_{\max} = IAB$

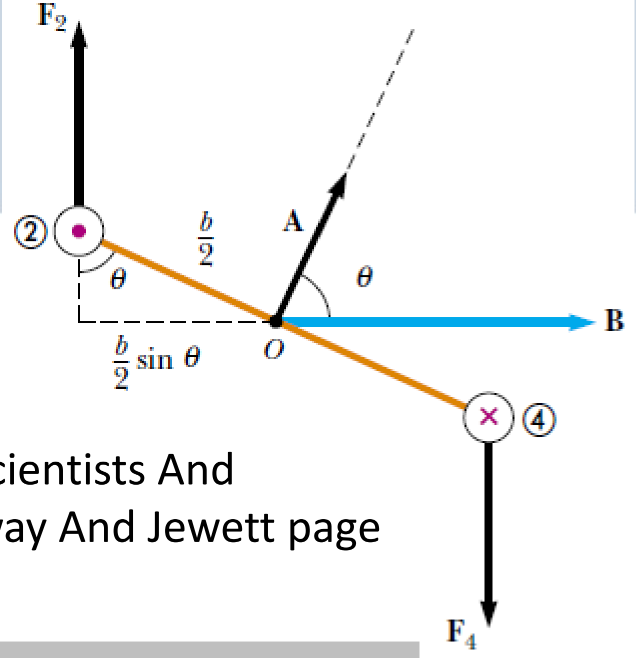
This maximum-torque result is valid only when the magnetic field is parallel to the plane of the loop.

When magnetic field makes an angle θ < 90° with a line perpendicular to the plane of the loop

B is perpendicular to sides 2 and 4. In this case, the magnetic forces F1 and F3 exerted on sides 1 and 3 cancel each other and produce no torque because they pass through a common origin. However, the magnetic forces F2 and F4 acting on sides 2 and 4 produce a torque about *any point.* The net torque about *O* is

$$\begin{aligned}\tau &= F_2 \frac{b}{2} \sin\theta + F_4 \frac{b}{2}\sin\theta \\ &= IaB\left(\frac{b}{2}\sin\theta\right) + IaB\left(\frac{b}{2}\sin\theta\right) = IabB\sin\theta \\ &= IAB\sin\theta\end{aligned}$$

(Area=A=*ab*)



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The torque is zero when when the field is parallel to the normal to the plane of the loop (θ=0)

- A convenient expression for the torque exerted on a loop placed in a uniform magnetic field B is

$$\boldsymbol{\tau} = I\boldsymbol{A} \times \boldsymbol{B}$$

where A is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop.

- The product *I*A is defined to be the magnetic dipole moment (often simply called the “magnetic moment”) of the loop.

$$\vec{\mu} = I\vec{\boldsymbol{A}}$$

The SI unit of magnetic dipole moment is ampere-meter^2

- The torque exerted on a current-carrying loop in a magnetic field B is

$$\vec{\boldsymbol{\tau}} = \vec{\mu} \times \vec{\boldsymbol{B}}$$

- Note that this result is analogous to $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$, for the torque exerted on an electric dipole in the presence of an electric field **E**, where **p** is the electric dipole moment.

- If a coil consists of *N* turns of wire, each carrying the same current and enclosing the same area, the total magnetic dipole moment of the coil is *N* times the magnetic dipole moment for one turn.

$$\vec{\boldsymbol{\tau}} = N\vec{\mu} \times \overrightarrow{\boldsymbol{B}} \qquad \vec{\boldsymbol{\tau}} = \vec{\mu}_{coil} \times \overrightarrow{\boldsymbol{B}}$$

- Remember that the potential energy of a system of an electric dipole in an electric field is given by

$$U = -\vec{\boldsymbol{p}}.\overrightarrow{\boldsymbol{E}}$$

- Likewise, the potential energy of a system of a magnetic dipole in a magnetic field is given by

$$U = -\vec{\mu}.\overrightarrow{\boldsymbol{B}}$$

- From this expression, we see that the system has its lowest energy when μ points in the same direction as B. The system has its highest energy when μ points in the direction opposite B.

29.4 Motion of a Charged Particle in a Uniform Magnetic Field

- $\vec{F}_B \perp \vec{\vartheta}$ Therefore, the work done by the magnetic field is zero.
- We know that if the force is always perpendicular to the velocity, the path of the particle is a circle. Figure shows a positively charged particle moving in a circle in a plane perpendicular to the magnetic field.

If the force is always perpendicular to the velocity, the path is circle!

$$\vec{F}_B = \vec{F}_{centripetal}$$

$$q\vec{\vartheta}\vec{B} = \frac{m\vec{\vartheta}^2}{\vec{r}}$$

$$r = \frac{m\vec{\vartheta}}{q\vec{B}}$$

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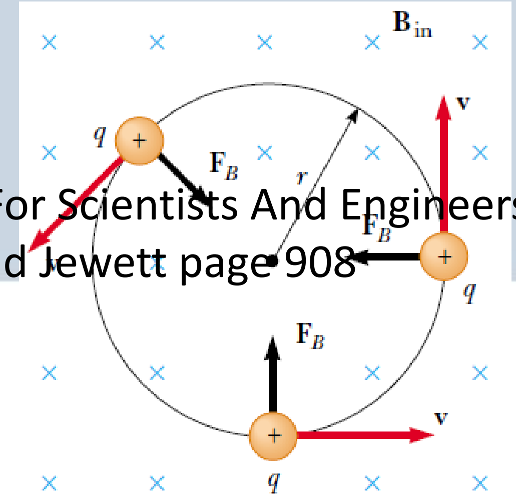


- The radius of the path is proportional to the linear momentum $m\vartheta$ of the particle and inversely proportional to the magnitude of the charge on the particle and to the magnitude of the magnetic field.

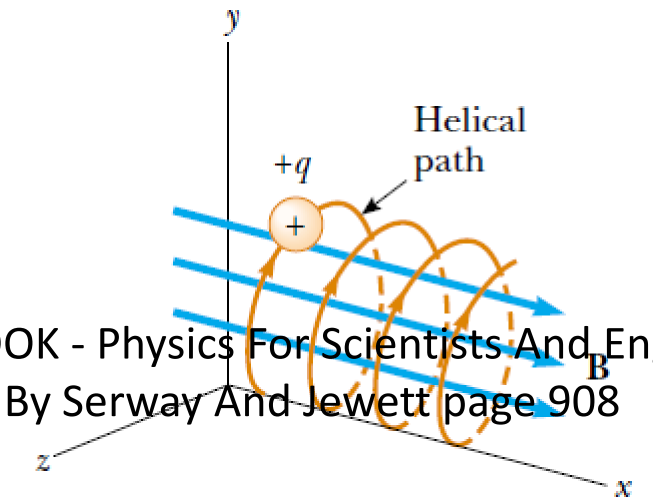
- The angular speed of the particle is $$\omega = \frac{\vec{\vartheta}}{r} = \frac{q\vec{B}}{m}$$

- The period of the motion is $$T = \frac{2\pi \vec{r}}{\vec{\vartheta}} = \frac{2\pi}{\vec{\omega}} = \frac{2\pi m}{q\vec{B}}$$

- These results show that the angular speed of the particle and the period of the circular motion do not depend on the linear speed of the particle or on the radius of the orbit.

- The angular speed ω is often referred to as the cyclotron frequency because charged particles circulate at this angular frequency in the type of accelerator called a *cyclotron.*

- If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to B, its path is a helix.



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Example 29.7 **Bending an Electron Beam**

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm. (Fig. 29.20 shows such a curved beam of electrons.) If the magnetic field is perpendicular to the beam, **(a)** what is the magnitude of the field?

Solution:

For the isolated electron–electric field system, the loss of potential energy as the electron moves through the 350-V potential difference appears as an increase in the kinetic energy of the electron.

$$K_i = 0,\ K_f = \frac{1}{2} m\vartheta^2$$

$$\Delta K + \Delta U = 0$$

Henry Leap and Jim Lehman

$$\frac{1}{2} m_e \vartheta^2 + (-e)\Delta V = 0$$

$$\vartheta = \sqrt{\frac{2e\Delta V}{m_e}} = \sqrt{\frac{2(1.6\times10^{-19} C)350V}{9.11\times10^{-31} kg}}$$

$$\vartheta = 1.11\times10^{7} m/s \quad \Rightarrow \quad B = \frac{m_e\vartheta}{er} = \frac{9.11\times10^{-31}\times1.11\times10^{7}}{1.6\times10^{-19}\times0.075} = 8.4\times10^{-4} T$$

(b) What is the angular speed of the electrons?

$$\omega = \frac{\vartheta}{r} = \frac{1.11\times10^{7}}{0.075} = 1.5\times10^{8}\, rad/s$$

To finalize this problem, note that the angular speed can be represented as

$$\omega = \pi\left(1.5\times10^{8}\, rad/s\right)\left(1\, rev/2\pi\ rad\right)$$
$$= 2.4\times10^{7}\, rev/s$$

The electrons travel around the circle 24 million times per second!

29.5 Applications Involving Charged Particles Moving in a Magnetic Field

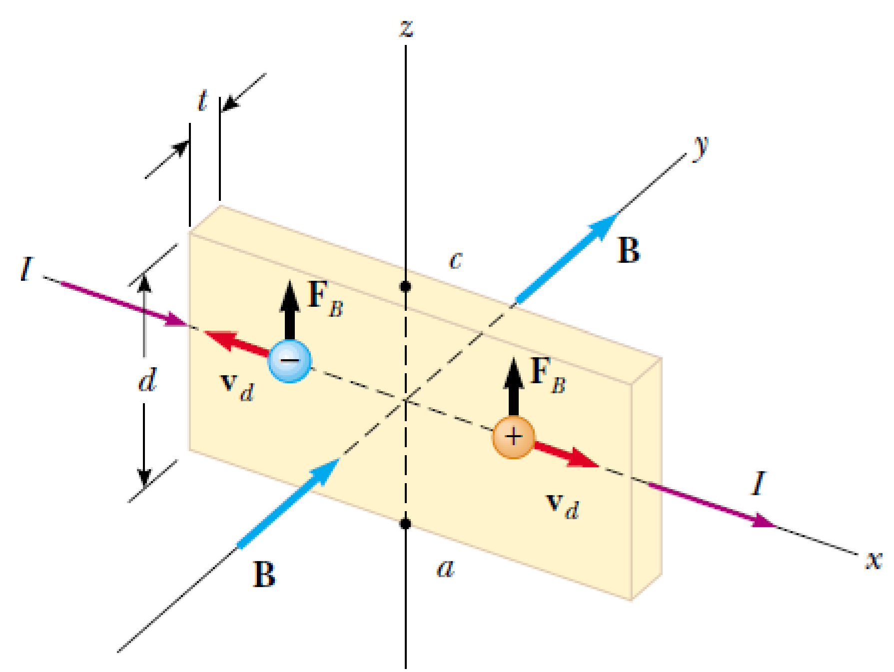
- A charge moving with a velocity v in the presence of both an electric field E and a magnetic field B experiences Lorenz force

$$\vec{F} = q\vec{E} + q\vec{\vartheta} \times \vec{B}$$

- A *mass spectrometer* separates ions according to their mass-to-charge ratio. A variation of this technique was used by J. J. Thomson (1856–1940) in 1897 to measure the ratio e/m_e for electrons.
- A *cyclotron* is a device that can accelerate charged particles to very high speeds. The energetic particles produced are used to bombard atomic nuclei and thereby produce nuclear reactions of interest to researchers. A number of hospitals use cyclotron facilities to produce radioactive substances for diagnosis and treatment.

29.6 The Hall Effect

- When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field. This phenomenon, first observed by Edwin Hall in 1879, is known as the *Hall effect*. The Hall effect gives information regarding the sign and density of the charge carriers .



Electrons experience an upward magnetic force, leaving an excess of positive charge at the lower edge.
This establishes an electric field and potential difference (Hall voltage) in the conductor. In equilubrum:

$$qv_d B = qE_H$$

$$E_H = v_d B$$

$$\Delta V_H = E_H d = \vartheta_d B d$$

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Summary

- The magnetic force that acts on a charge *q* moving with a velocity ϑ in a magnetic field B is $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
- If a straight conductor of length *L* carries a current *I* and is placed in a uniform magnetic field B, the force exerted on that conductor is $\mathbf{F}_B = I\mathbf{L} \times \mathbf{B}$
- The magnetic dipole moment μ of a loop carrying a current *I* is $\boldsymbol{\mu} = I\mathbf{A}$
- The torque τ on a current loop placed in a uniform B is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$
- The potential energy of the system of a magnetic dipole in a magnetic field is $U = -\boldsymbol{\mu} \cdot \mathbf{B}$
- If a charged particle moves in a uniform magnetic field so that its initial velocity is perpendicular to the field, the particle moves in a circle, the plane of which is perpendicular to the magnetic field. The radius of the circular path and the angular speed of the charged particle is $r = \frac{mv}{qB}$ $\omega = \frac{qB}{m}$

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