Physics 2: Electricity & Magnetism
Faraday's Law of Induction

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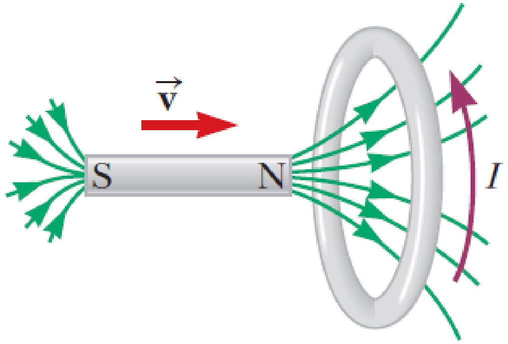

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Outline

-Faraday’s Law of Induction

-Motional EMF

-Lenz’s Law

-Induced EMF and Electric Fields

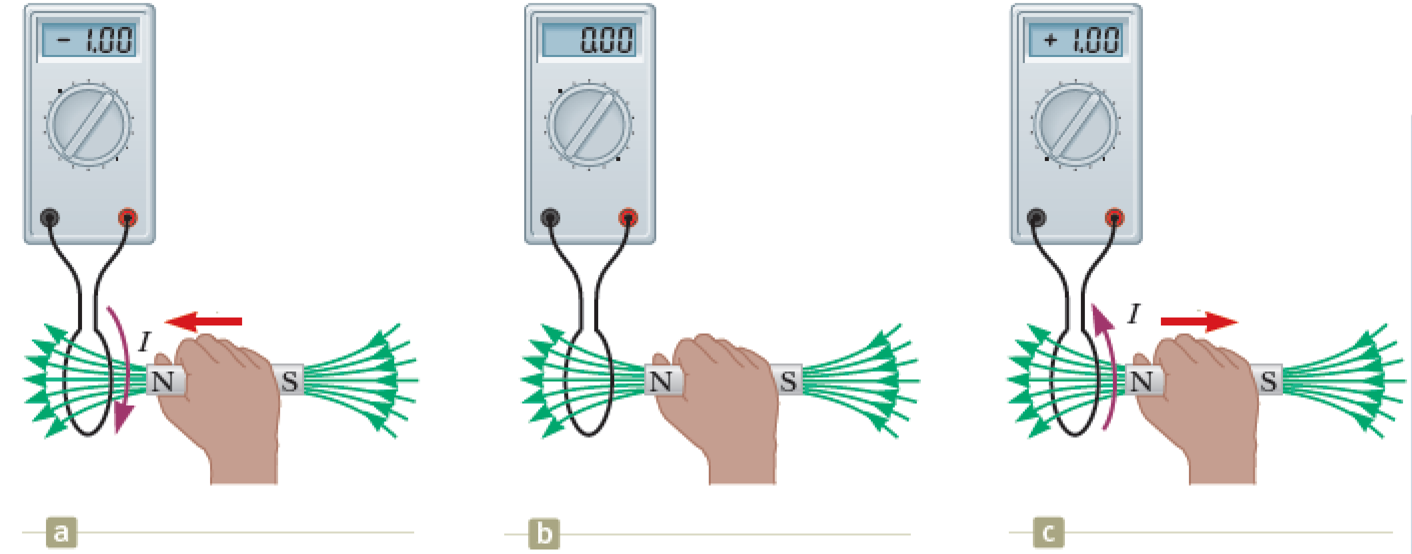
-Generators and Motors

-Maxwell’s Equations

This chapter explores the effects produced by magnetic fields that vary in time.

Faraday’s Law of Induction

No batteries are present in the circuit!



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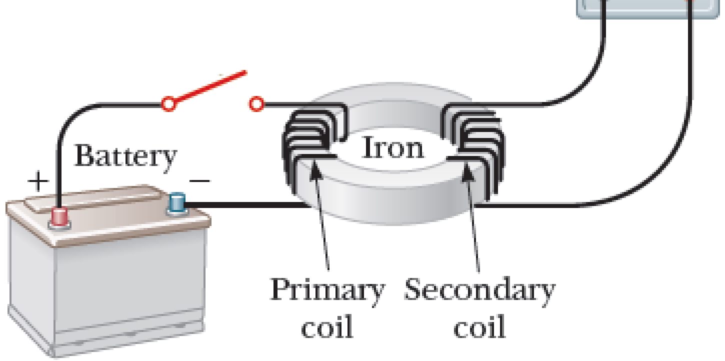
(a) Magnet is moved toward a loop. (Or loop is moved toward a magnet.) Ammeter shows that current is induced in the loop.

(b) Magnet is stationary. (Loop is stationary.) There is no induced current in the loop.

(a) Magnet is moved away from the loop. (Or loop is moved away from the magnet.) The induced current is opposite to that in (a).

Faraday's Experiment



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- Secondary coil is neither connected to a battery nor primary coil.
- At the instant the switch is closed/opened, the ammeter reading momentarily changes from zero.
- The ammeter reads zero when there is either a steady current or no current in the primary circuit.
- **Change** in the current in the primary circuit induces a current in the secondary coil.
- Faraday concluded that an electric current can be induced in a loop by changing magnetic field.

Faraday's Law of Induction:

•The induced emf is directly proportional to the time rate of change of the magnetic flux through the loop.

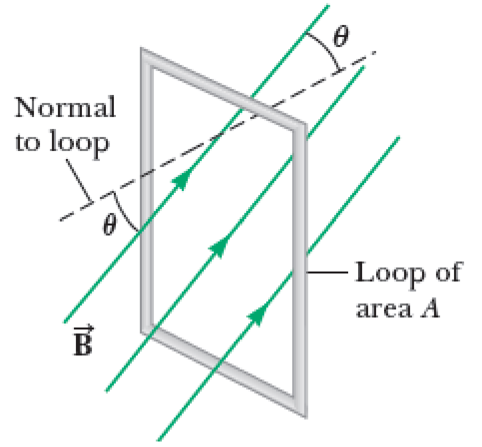
$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(\int \vec{B}.d\vec{A})}{dt}$$

- If a coil consists of N loops with the same area and is the magnetic flux through one loop, an emf is induced in every loop. The loops are in series so their emfs are add, the total induced emf in the coil is:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

- Suppose a loop enclosing an area A lies in a uniform magnetic field. θ is the angle between the magnetic field and the normal to the loop.The induced emf can be expressed as

$$\varepsilon = -\frac{d}{dt}(BAcos\theta)$$



From this expression, we see that an emf can be induced in the circuit in several ways:

- The magnitude of **B** can change with time.
- The area enclosed by the loop can change with time.
- The angle θ between **B** and the normal to the loop can change with time.
- Any combination of the above can occur.

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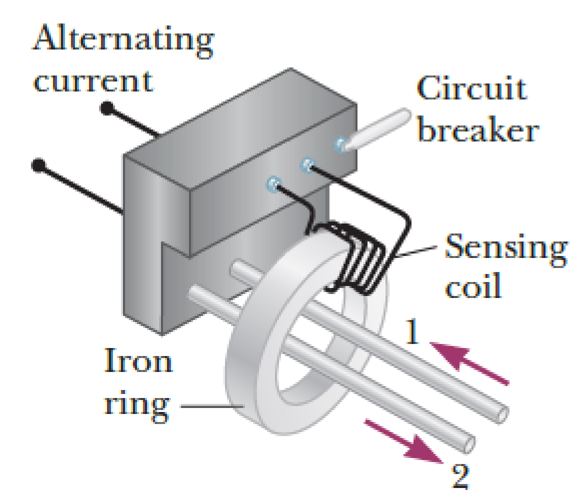
Some Applications of Faraday's Law

1. The ground fault circuit interrupter *(topraklama hatası devre kesicisi)*

-Wire 1 leads from the wall outlet to the appliance to be protected.
-wire 2 leads from the appliance back to the wall outlet.
-An iron ring surrounds the two wires
-A sensing coil is wrapped around part of the ring.

When the currents in the wires are in opposite directions and of equal magnitude, there is zero net current flowing through the ring and the net magnetic flux through the sensing coil is zero. If return current in wire 2 changes, there is magnetic flux through the sensing coil. Because household current is alternating (A.C), magnetic flux through the sensing coil changes with time, inducing an emf in the coil. This emf triggers a circuit breaker, which stops the current it reaches to harmful level.



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Some Applications of Faraday's Law

2. Production of sound in an electric guitar

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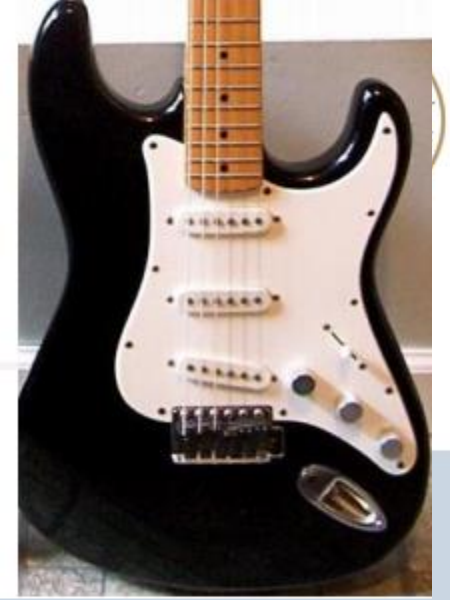
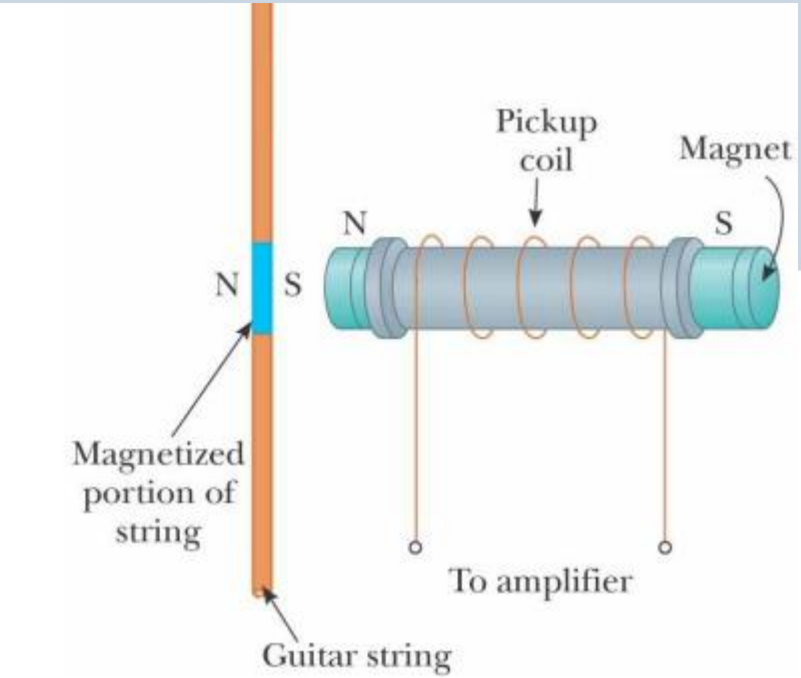


-Pickup coil is placed near the vibrating guitar string.
-Portion of the coil nearest to the string is magnetized.
-When the string vibrates at some frequency, its magnetized segment produces a changing magnetic flux through the coil.
-Induced emf is fed by the amplifier.
-The output of the amplifer is sent to loudspeaker, which produces the sound waves we hear.



Example 31.1 Inducing an emf in a Coil

A coil consists of 200 turns of wire. Each turn is a square of side d = 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

Solution:

$$|\varepsilon| = N\frac{\Delta\Phi_B}{\Delta t} = NA\frac{\Delta B}{\Delta t} = Nd^2\frac{B_f - B_i}{\Delta t}$$

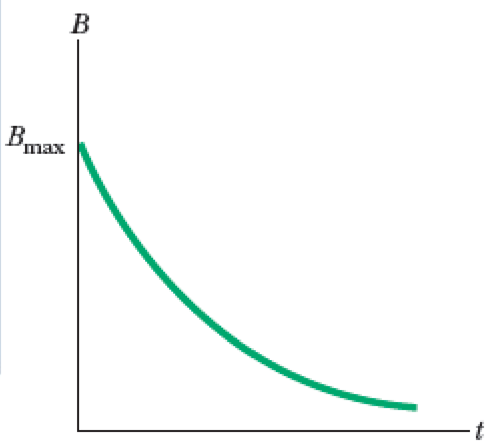
$$|\varepsilon| = 200\times 0.18^2\times\frac{0.5-0}{0.8} = 4.0V$$

Example 31.2 **An Exponentially Decaying Magnetic Field**

A loop of wire enclosing an area *A* is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of **B** varies in time according to the expression $B = B_{max}e^{-at}$, where a is some constant. That is, at t = 0, the field is B_{max}, and for t > 0, the field decreases exponentially (Fig. 31.6). Find the induced emf in the loop as a function of time.

Solution:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(AB_{max}e^{-at}) = -AB_{max}\frac{d}{dt}e^{-at} = aAB_{max}e^{-at}$$



This expression indicates that the induced emf decays exponentially in time. The maximum emf occurs at *t* = 0, where $\varepsilon_{max} = aAB_{max}$The plot of ε versus *t* is similar to the *B*-versus-*t* curve.

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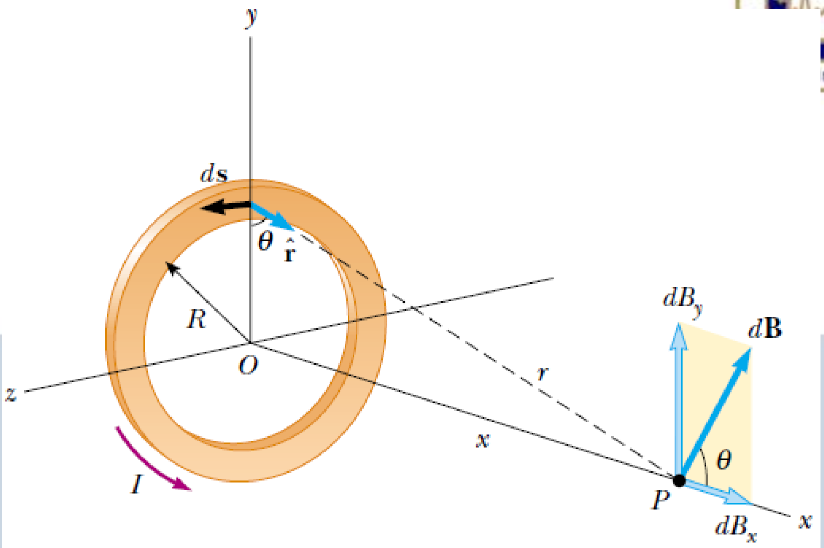
$$B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi}\oint \frac{ds\cos\theta}{x^2+R^2}$$

$$\cos\theta = R/(x^2 + R^2)^{1/2}$$



$$B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi\left(x^2+R^2\right)^{3/2}}\oint ds$$

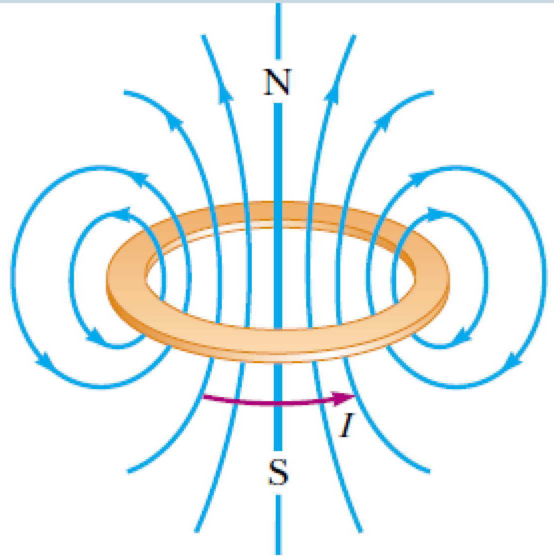
$$\oint ds = 2\pi R$$

$$B_x = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$$

Finalize:

To find the magnetic field at the center of the loop, we set $x = 0$.

$$B = \frac{\mu_0 I}{2R} \qquad \text{at } x = 0$$



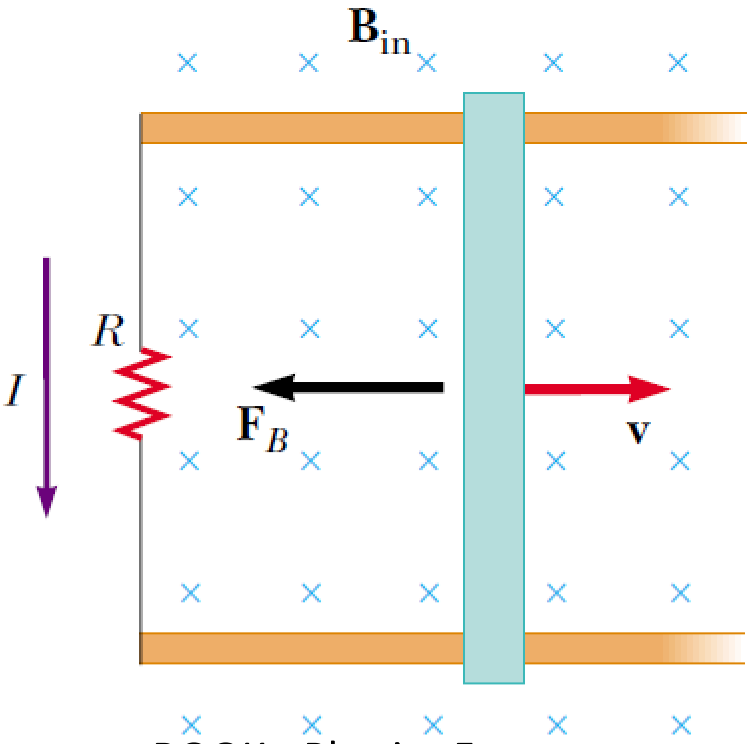
31.3 Lenz's Law

- Faraday's law of induction

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- The induced emf and the change in flux have opposite signs.
- Physical Meaning: The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop. This law is a consequence of the law of conservation of energy.

Example 1: Bar moving to the right on two parallel rails in the presence of a uniform magnetic field



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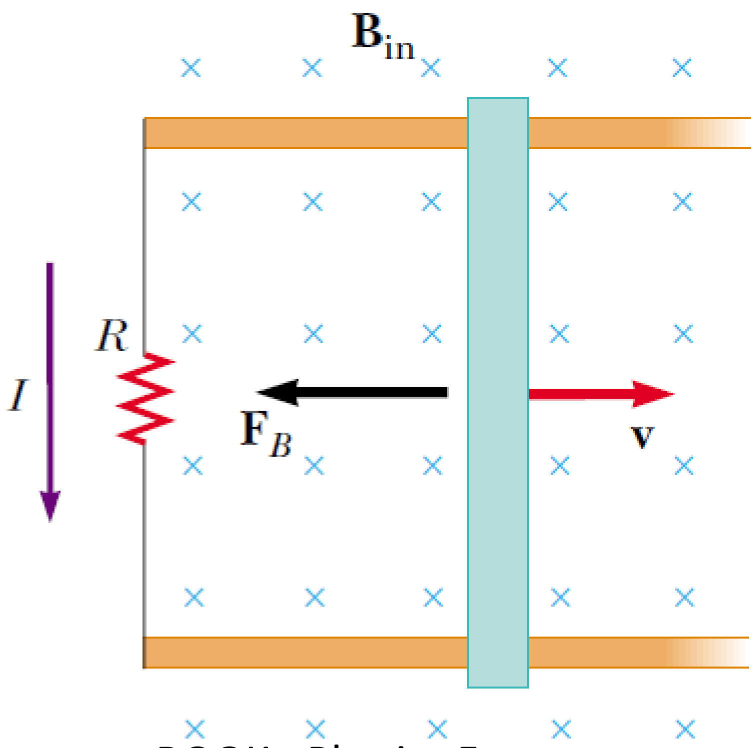
- As the bar moves to the right, the magnetic flux through the area enclosed by the circuit increases.
- The induced current opposes this change by producing a field directed out of the page.
- Hence, the induced current must be directed counterclockwise when the bar moves to the right. If it was clockwise, this would accelerate the rod and increase the induced current with no input of energy.

Example 2: Bar moving to the left on two parallel rails in the presence of a uniform magnetic field



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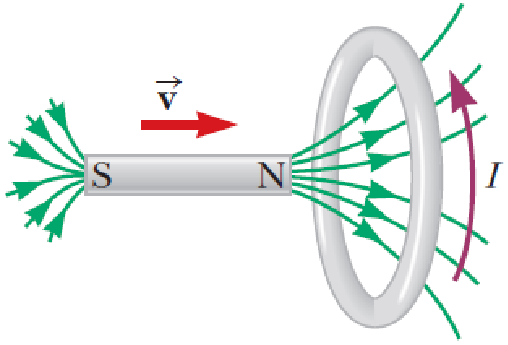
- As the bar moves to the left, the magnetic flux through the area enclosed by the circuit decreases.
- The induced current prevents this change by producing a field directed into the page.
- Hence, the induced current produces a field that also is directed into the page.

Example 31.6 **Application of Lenz's Law**

A magnet is placed near a metal loop as shown in Figure 31.13a.

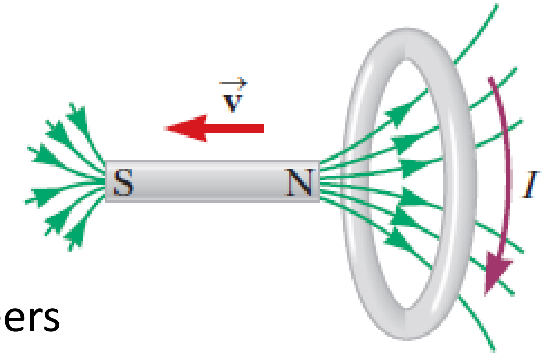
(A) Find the direction of the induced current in the loop when the magnet is pushed toward the loop.

Answer: The induced current produces its own magnetic field directed to the left that counteracts the increasing external flux.



(B) Find the direction of the induced current in the loop when the magnet is moved away from the loop.

Answer: The induced current produces a magnetic field directed to the right and so counteracts the decreasing external flux.



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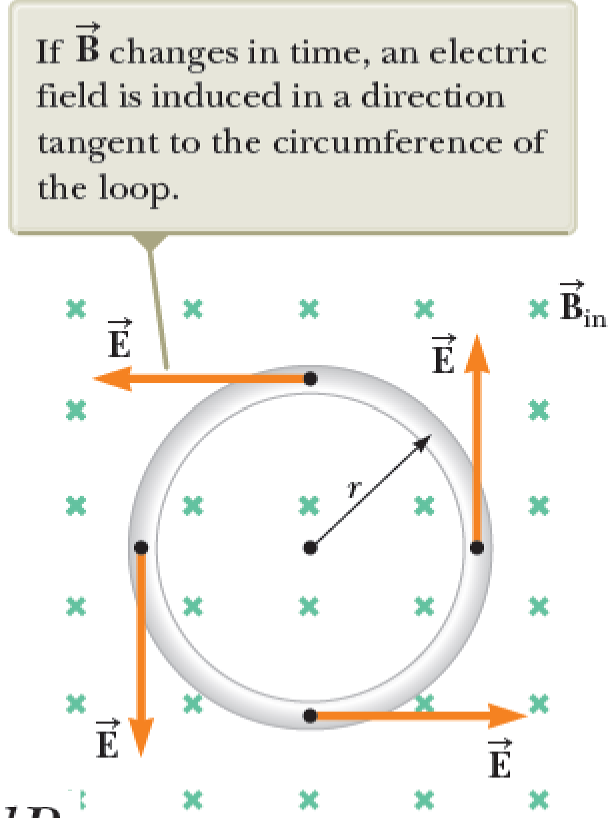
31.4 Induced emf and electric fields

- Changing magnetic field generates an electric field in the conductor and also in empty space.
- This induced electric field is *nonconservative,* unlike the electrostatic field produced by stationary charges. To illustrate this point, consider a conducting loop of radius *r* situated in a uniform magnetic field. If the magnetic field changes with time, an emf is induced in the loop.



ε: induced emf

$$q\mathcal{E} = qE(2\pi r)$$

$$\Phi_B = BA = B\pi r^2$$

$$E = \frac{\mathcal{E}}{2\pi r}$$

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

- If dB/dt is known, the induced electric field can be calculated.

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$$\varepsilon = -\frac{d\Phi}{dt}$$

$$\oint \vec{E}.d\vec{s} = -\frac{d\Phi}{dt}$$

The induced electric field E is a nonconservative field. If the field was electrostatic and conservative, the line integral over the closed loop would be zero.

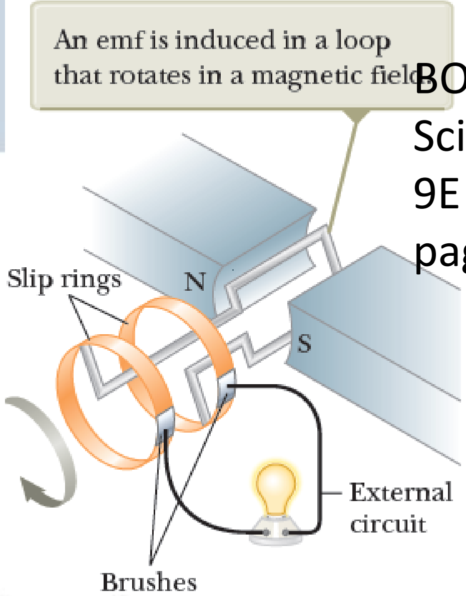
31.5 Generators and Motors

Electric generators are devices that take in energy by work and transfer it out by electrical transmission. To understand how they operate, let us consider the alternating current (AC) generator. In its simplest form, it consists of a loop of wire rotated by some external means in a magnetic field.

The energy required to rotate the loop can be derived from a variety of sources. Ex: falling water, burning coal.

- As a loop rotates in a magnetic field, the magnetic flux through the area enclosed by the loop changes with time; this induces an emf and a current in the loop according to Faraday's law. The ends of the loop are connected to slip rings that rotate with the loop.
- Connections from these slip rings, which act as output terminals of the generator, to the external circuit are made by stationary brushes in contact with the slip rings.

Hydroelectric power plant



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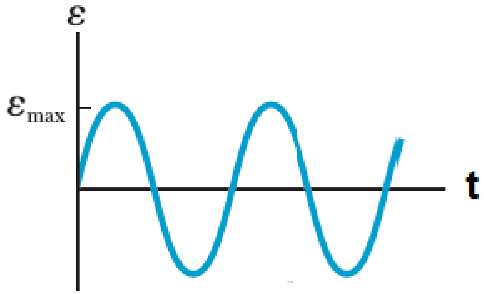


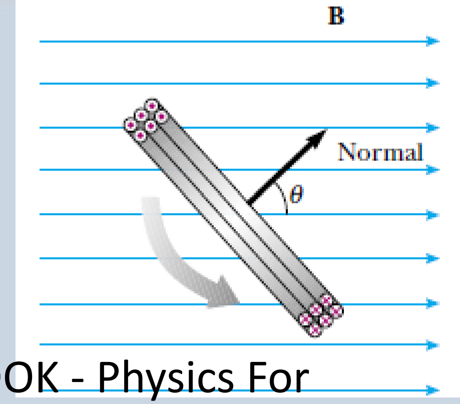
Suppose that, instead of a single turn, the loop has *N* turn

- If θ is the angle between the magnetic field and the normal to the plane of the loop, then the magnetic flux through the loop at any time *t* is,

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

- Hence, the induced emf in the coil is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}(\cos\omega t) = NAB\omega\sin\omega t$$



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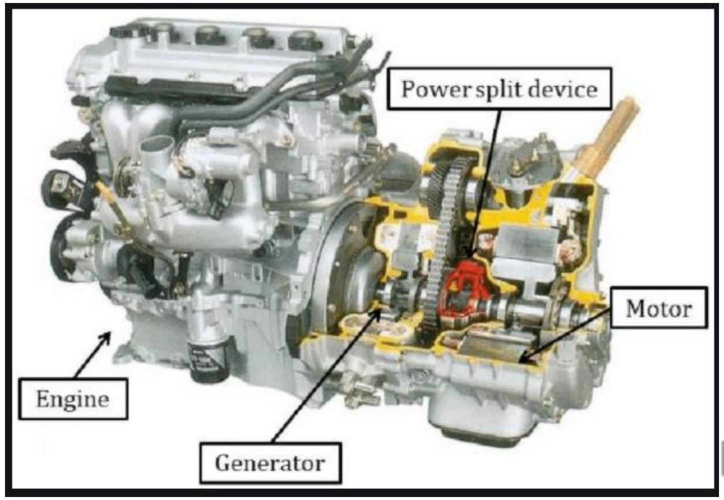
- Emf varies sinusoidally with time.
- $\varepsilon = \varepsilon_{max}$ is when the magnetic field is in the plane of the coil
- $\varepsilon = 0$ when $\omega t = 0$ or 180°, that is, when B is perpendicular to the
- plane of the coil.

- A motor is a generator operating in reverse. Instead of generating a current by rotating a coil, a current is supplied to the coil by a battery, and the torque acting on the current-carrying coil causes it to rotate.
- Useful mechanical work can be done by attaching the rotating coil to some external device. However, as the coil rotates in a magnetic field, the changing magnetic flux induces an emf in the coil; this induced emf always acts to reduce the current in the coil.
- In automobiles with *hybrid drive systems*, a gasoline engine and an electric motor are combined to increase the fuel economy of the vehicle and reduce its emissions.



Akın Oktav, New Trends and Recent Developments in Automotive Engineering, 2017

31.7 Maxwell's Equations

- 4 equations that are regarded as the basis of all electrical and magnetic phenomena
- Developed by James Clerk Maxwell
- Fundamental to electromagnetic phenomena as Newton's laws are to mechanical phenomena
- In agreement with the special theory of relativity, as Einstein showed in 1905
- These equations predict the existence of electromagnetic waves travel with $c = 1/\sqrt{\mu_0 \varepsilon_0} = 3\times10^8 m/s$ - the speed of light:
- The theory shows that such waves are radiated by accelerating charges.

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Maxwell's equations in free space and Lorentz Force Law

Gauss's Law: $\oint \vec{E} d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$

The total electric flux through any closed surface equals the net charge inside that surface divided by ε_0.

Gauss's Law in Magnetism: $\oint \vec{B} d\vec{a} = 0$

The net magnetic flux through a closed surface is zero. That is, the number of magnetic field lines that enter a closed volume must equal the number that leave that volume. Magnetic field lines cannot begin or end at any point. Because isolated magnetic monopoles do not exist (have not been observed.)

Faraday's Law of Induction: $\oint \vec{E} d\vec{s} = -\frac{\partial \Phi_B}{\partial t}$

The electromotor force equals to rate of change of magnetic flux through any surface area bounded by that path. Consequence: current is induced in a conducting loop placed in a time-varying magnetic field.

Ampere-Maxwell Law: $\oint \vec{B} d\vec{s} = \mu_0 I + \varepsilon_0 \mu_0 \frac{\partial \Phi_E}{\partial t}$

The line integral of electric field around any closed path is the sum of μ_0 times the net current through that path and $\varepsilon_0\mu_0$ times the rate of change of electric flux through any surface bounded by that path. Consequence: Magnetic field is created by a time-varying electric field and electric currents.

Lorentz Force Law: $\vec{F} = q\vec{E} + q\vec{\vartheta} \times \vec{B}$

The force acting on a particle of charge *q* in a specific electric field and magnetic field

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Summary

- Faraday’s law of induction
- Motional emf
- Lenz’s law
- General form of Faraday’s law of induction
- Maxwell’s equations