Physics 122: Electricity & Magnetism
Capacitance and Dielectrics

Radio receiver, Tune the frequency!

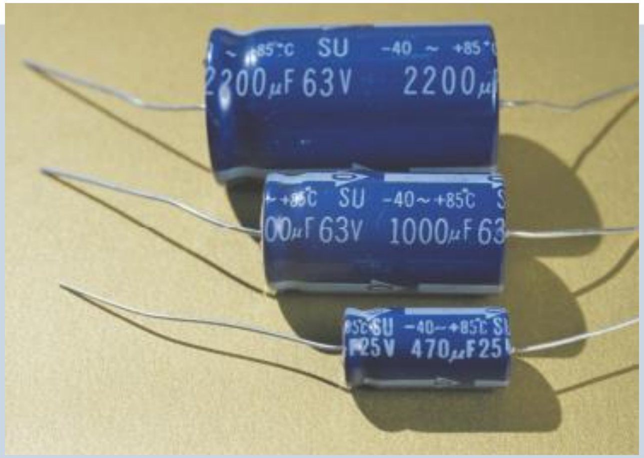

Figure from
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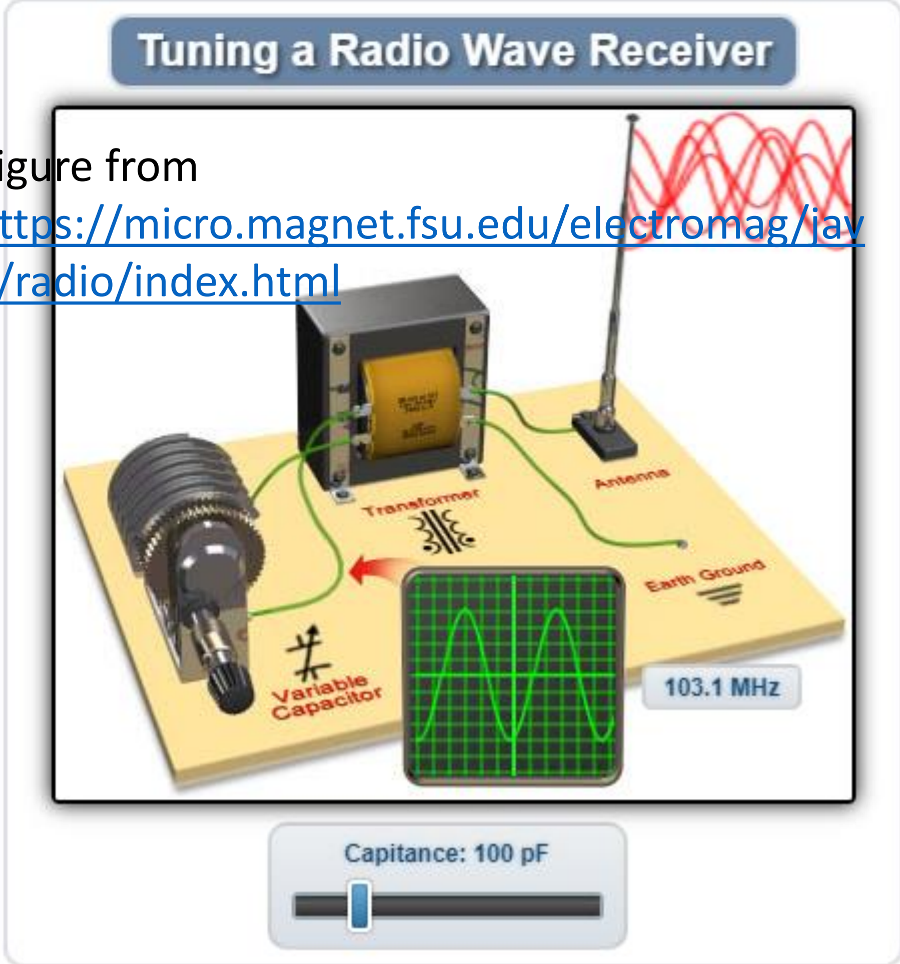

Figure from
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In this chapter, we introduce the first of three (Capacitors, resistors and inductors) simple circuit elements that can be connected with wires to form an electric circuit.

Capacitors are used to tune the frequency of radio receivers, as filters in power supplies,as energy-storing devices in electronic flash units, etc.

Consider two conductors as shown in Figure. Such a combination of two conductors is called a **capacitor.** The conductors are called *plates.* If the conductors carry charges of equal magnitude and opposite sign, a potential difference ΔV exists between them.

$$Q \propto \Delta V$$

The proportionality constant depends on the shape and separation of the conductors.

The capacitance C of a capacitor is defined as,

$$C \equiv \frac{Q}{\Delta V}$$

C is always a positive quantity. The SI unit of C is Farad.

$$1\ \mathrm{F} = 1\ \mathrm{C/V}$$

Typical devices have capacitances ranging from microfarads to picofarads.

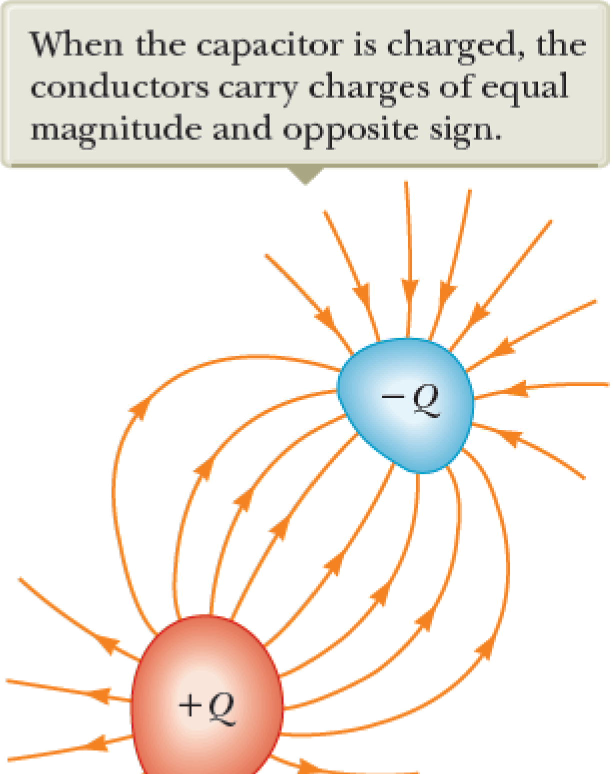
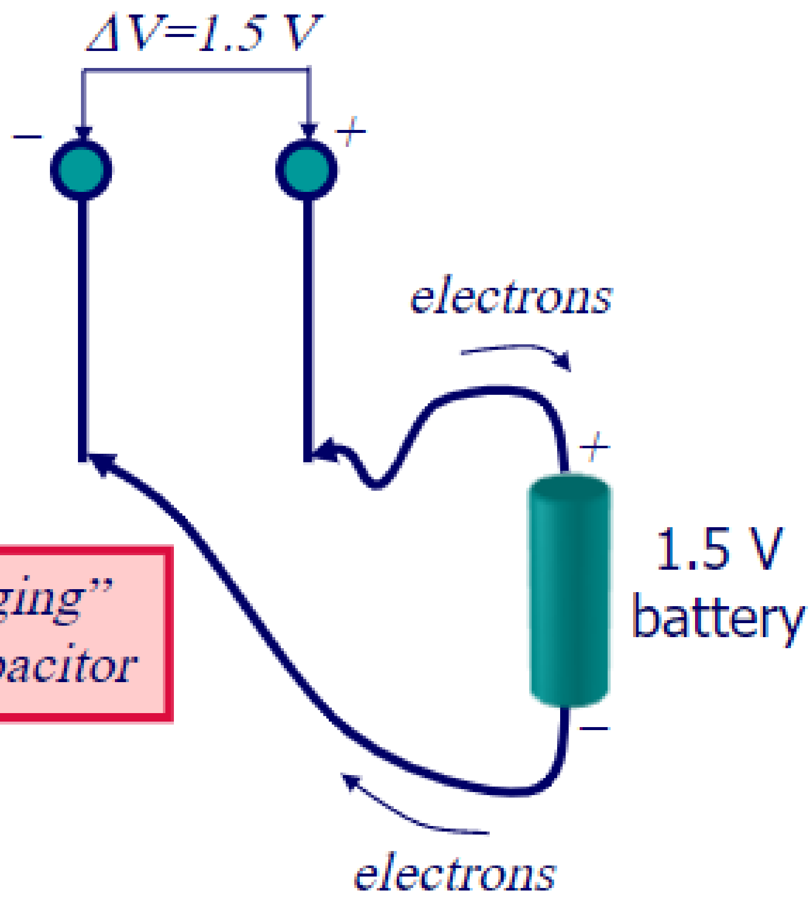


Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett page 796

What is Capacitance?

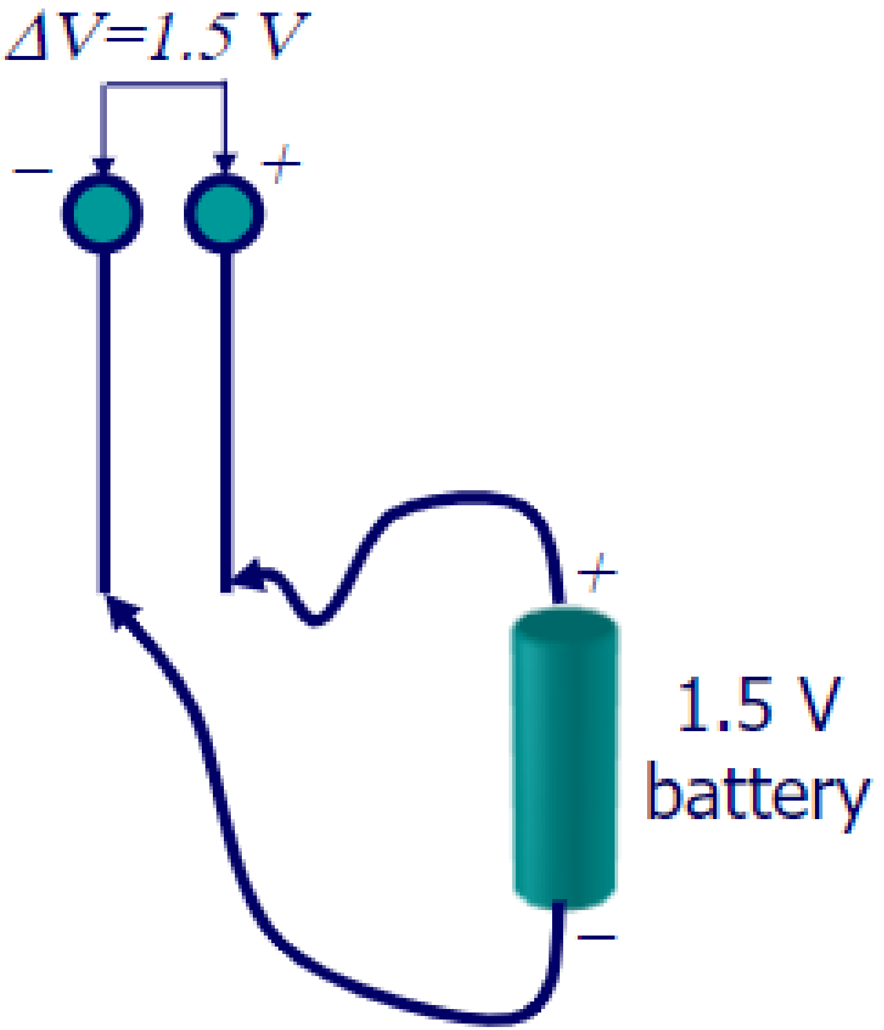
- From the word "capacity," it describes how much charge an arrangement of conductors can hold for a given voltage applied.
- Charges will flow until the right conductor's potential is the same as the + side of the battery, and the left conductor's potential is the same as the – side of the battery.
- How much charge is needed to produce an electric field whose potential difference is 1.5 V?
- Depends on capacitance: $q = CV$



"Charging" the capacitor

definition of capacitance

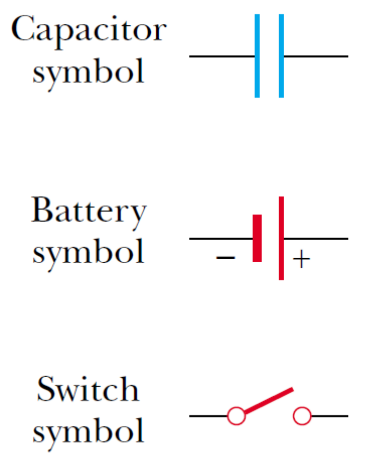
- What happens when the two conductors are moved closer together?
- They are still connected to the battery, so the potential difference cannot change.
- But recall that $V = -\int \vec{E} \cdot d\vec{s}$.
- Since the distance between them decreases, the E field has to increase.
- Charges have to flow to make that happen, so now these two conductors can hold more charge. I.e. the capacitance increases.



26.2 Calculating Capacitance

The capacitance of a pair of conductors can be parallel-plates, concentric cylinders and concentric spheres. Let us assume now that the charged conductors are separated by vacuum.

Parallel-Plate Capacitors



Two parallel, metallic plates of equal area *A* are separated by a distance *d* as shown in Figure 26.2. One plate carries a charge +*Q* , and the other carries a charge -*Q.*
The surface charge density on each plate is:

$$\sigma = \frac{Q}{A}$$

If the plates are very close together (in comparison with their length and width), we can assume the electric field is uniform between the plates and zero elsewhere.
Electric field between the plates is: $$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The potential difference between the plates is: $$\Delta V = Ed = \frac{Qd}{\varepsilon_0 A}$$

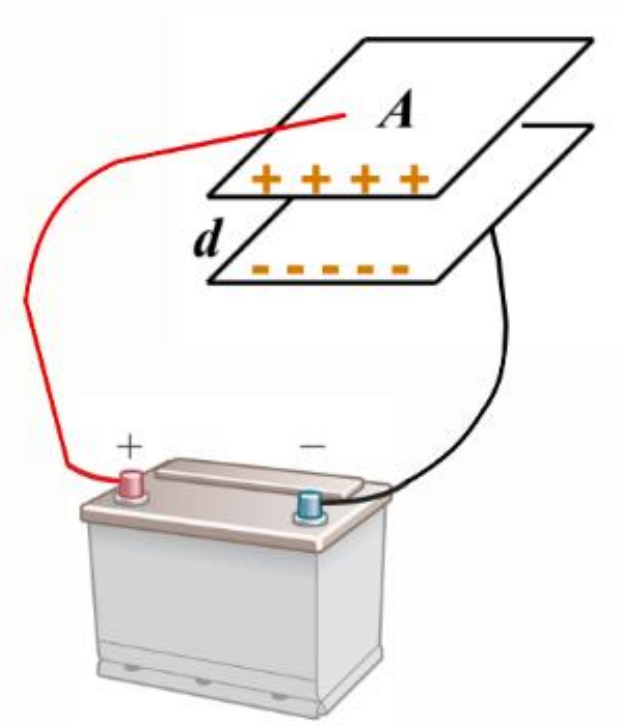


Figure 26.2

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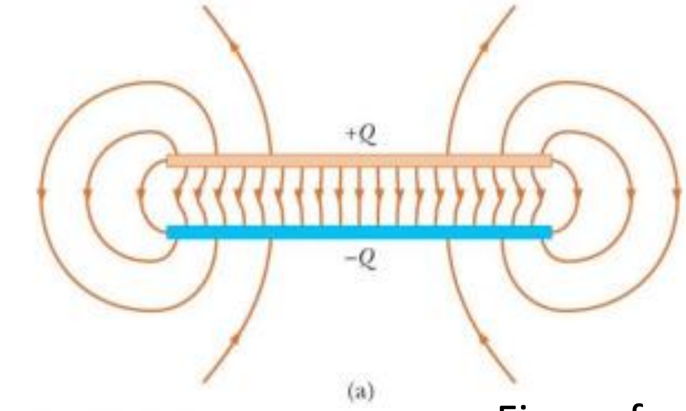
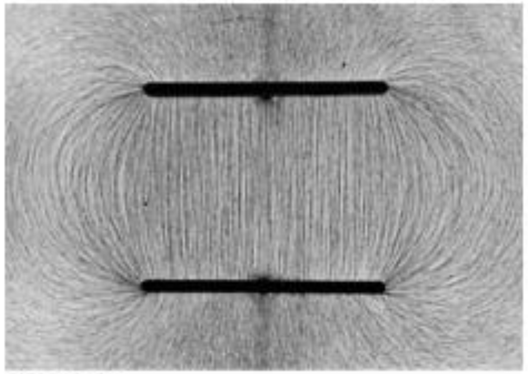
We find that the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{\varepsilon_0 A}} = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$

That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.





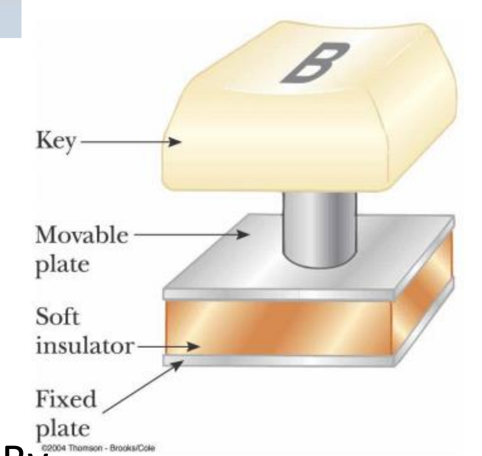


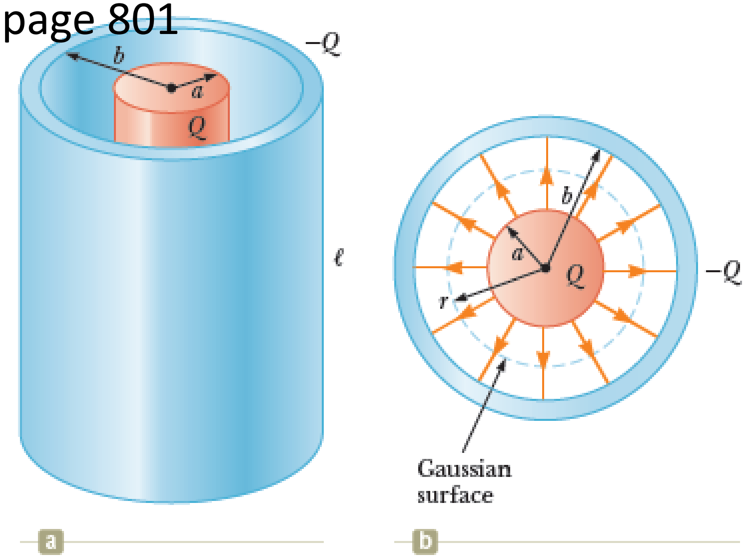
Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett, page 799

- Assumption that electric field is uniform is valid in the central region, but not at the ends of the plates
- If separation between plates is small compared with their length, effect of non-uniform field can be ignored

Example 26.1 The Cylindrical Capacitor

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of neglible thickness, radius $b>a$, and charge $-Q$ (Figure 26.4a). Find the capacitance of this cylindrical capacitor if its length is l.

Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett, page 801



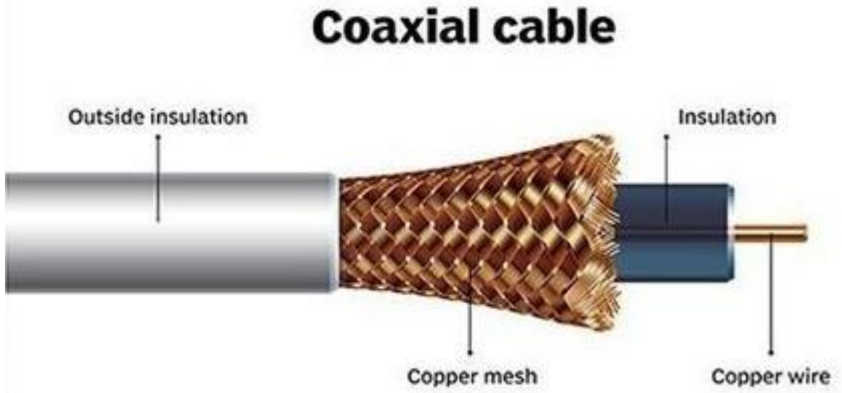
$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$V_b - V_a = -\int_a^b E_r dr = -2k_e \lambda \int_a^b \frac{dr}{r} = -2k_e \lambda \ln\left(\frac{b}{a}\right)$$

$$\lambda = Q/\ell$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\left(2k_e \frac{Q}{l}\right)\ln\left(\frac{b}{a}\right)} = \frac{l}{2k_e \ln(b/a)}$$

An example of this type of geometric arrangement is a *coaxial cable.*



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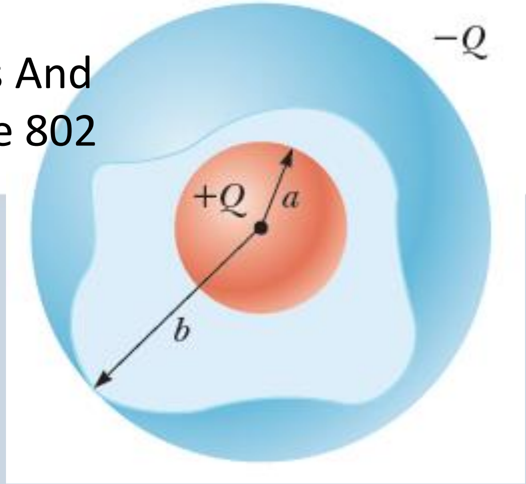
Example 26.2 The Spherical Capacitor

A spherical capacitor consists of a spherical conducting shell of radius *b* and charge -Q concentric with a smaller conducting sphere of radius *a* and charge *Q*. Find the capacitance of this device.

Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett, page 802



The electric field between the spheres is,

$$E = k_e Q / r^2$$

The potential difference between the conductors is,

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$V_b - V_a = -k_e Q \int_a^b \frac{dr}{r^2}$$

$$V_b - V_a = k_e Q \left[\frac{1}{r}\right]_a^b$$

$$V_b - V_a = k_e Q \left(\frac{1}{b} - \frac{1}{a}\right) = k_e Q \frac{a-b}{ab}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{|V_b - V_a|} = \frac{ab}{k_e (b-a)}$$

If the radius *b* of the outer sphere approaches infinity, what does the capacitance become?

$$C = \lim_{b->\infty} \frac{ab}{k_e (b-a)}$$

$$C = \frac{ab}{k_e b} = \frac{a}{k_e} = 4\pi\varepsilon_0 a$$

The capacitance of an isolated spherical conductor

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Capacitance Summary

- Parallel Plate Capacitor $$C = \frac{\varepsilon_0 A}{d}$$
- Cylindrical (nested cylinder) Capacitor $$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$
- Spherical (nested sphere) Capacitor $$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$
- Capacitance for isolated Sphere $$C = 4\pi\varepsilon_0 R$$
- Units: $\varepsilon_0 \times$length = C^2/Nm = F (farad), named after Michael Faraday. [note: ε_0 = 8.85 pF/m]

26.3 Combination of Capacitors

Parallel Combination

Two capacitors connected as shown in Figure 26.7a are known as a **parallel combination** of capacitors. Figure 26.7b shows a circuit diagram for this combination of capacitors. The left plates of the capacitors are connected to the positive terminal of the battery by a conducting wire and are therefore both at the same electric potential as the positive terminal. Likewise, the right plates are at the same potential as the negative terminal. Therefore,

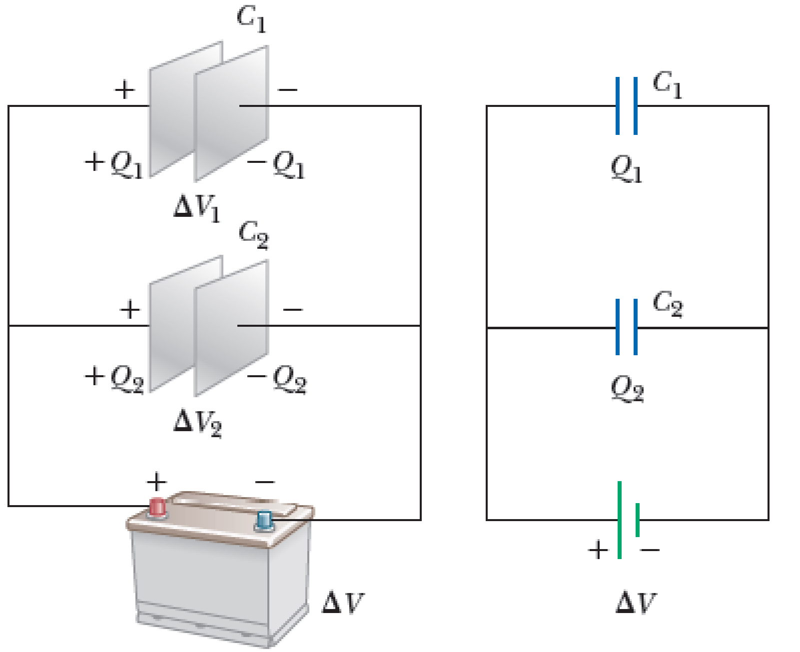
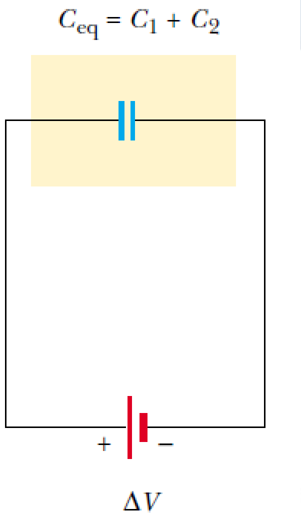


Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett, page 803



$$\Delta V_1 = \Delta V_2 = \Delta V$$

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

The *total charge Q* stored by the two capacitors is

$$Q_{tot} = Q_1 + Q_2$$

$$C_{eq}\Delta V = C_1\Delta V + C_2\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \text{(parallel combination)}$$

For three or more capacitors connected in parallel,

$$C_{eq} = C_1 + C_2 + C_3 + C_4 + \ldots$$

The equivalent capacitance is greater than any of the individual capacitances.

Series Combination

Two capacitors connected as shown in Figure 26.10a and the equivalent circuit diagram in Figure 26.10b are known as a series combination of capacitors. The left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. When the battery is connected, all the right plates end up with a charge +Q , and all the left plates end up with a charge –Q. Therefore, the charges on capacitors connected in series are the same.

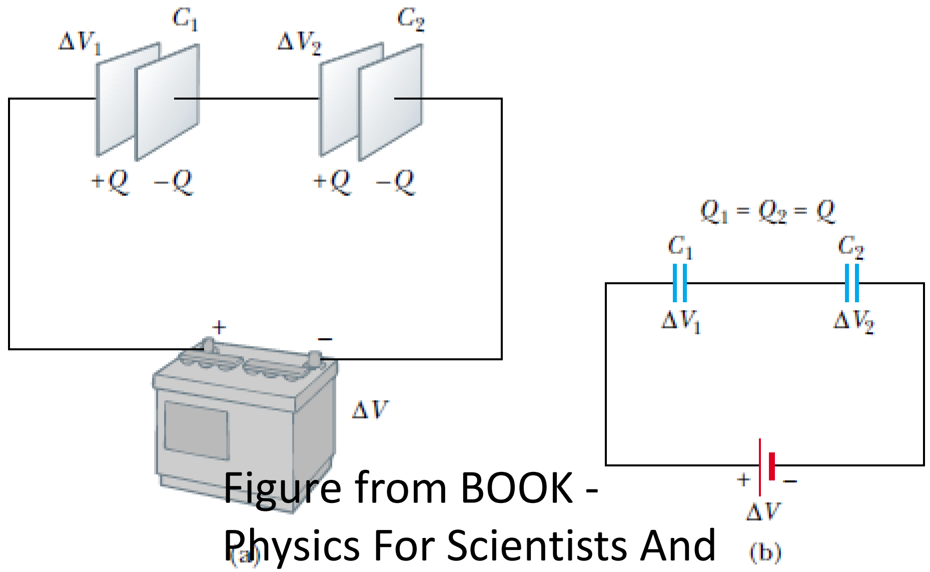


Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett, page 804

$$Q_1 = Q_2 = Q$$

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2$$

$$\Delta V_{tot} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\Delta V_{tot} = \frac{Q}{C_{eq}}$$

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(series combination)}$$

For three or more capacitors connected in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \ldots$$

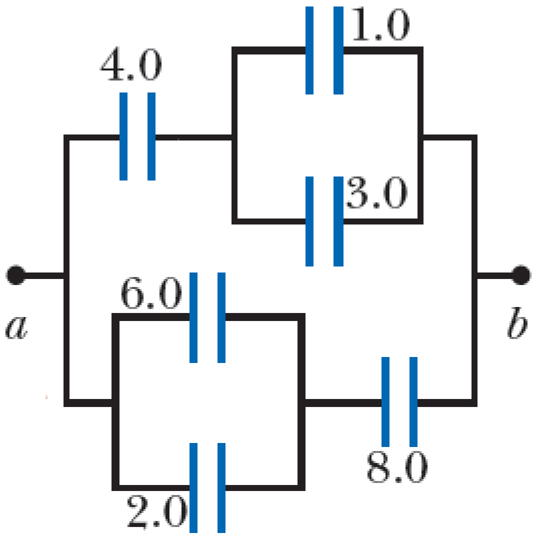
The equivalent capacitance is less than any of the individual capacitances.

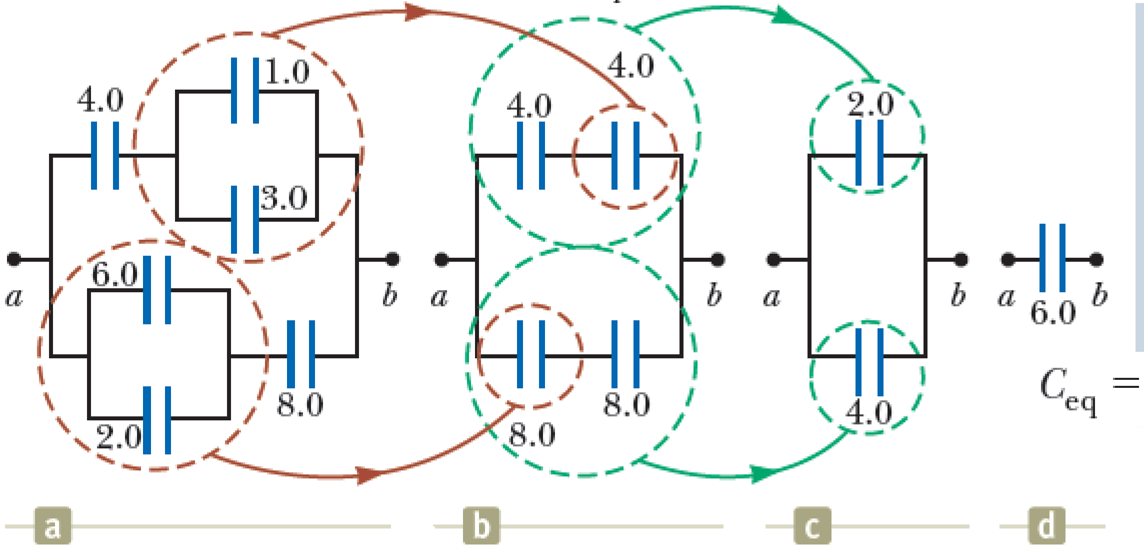
Example 26.3 Equivalent Capacitance

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Figure 26.9a. All capacitances are in microfarads.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = 2.0\ \mu F$$





$$C_{eq} = C_1 + C_2 = 6.0\ \mu F$$

Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett page 806

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8.0\ \mu F} + \frac{1}{8.0\ \mu F} = \frac{1}{4.0\ \mu F}$$

$$C_{eq} = 4.0\ \mu F$$

Find the voltage across and the charge on each capacitor.

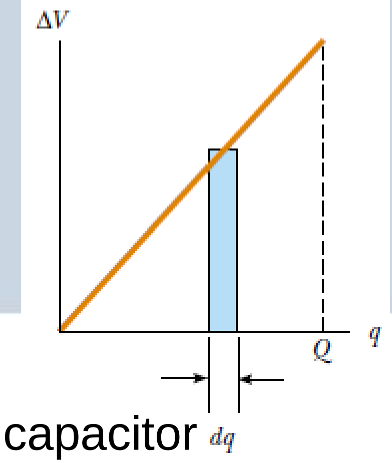
26.4 Energy Stored in a Charged Capacitor

- A capacitor can store energy. When the switch is closed, some of the chemical potential energy in the battery is transformed to electric potential energy associated with the separation of positive and negative charges on the plates.
- Recall the definition of the electric potential V=U/q
- The work necessary to transfer an increment of charge dq from the plate carrying charge +q to the plate carrying charge -q (which is at the higher electric potential) is:

$$dW = \Delta V dq = \frac{q}{C} dq$$

- The total work required to charge the capacitor from $q = 0$ to some final charge $q = Q$ is:

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}$$



- The work done in charging the capacitor appears as electric potential energy U stored in the capacitor is:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

- This result applies to any capacitor, regardless of its geometry.

- We see that for a given capacitance, the stored energy increases as the charge increases and as the potential difference increases.

- In practice, there is a limit to the maximum energy (or charge) that can be stored because, at a sufficiently great value of ΔV, discharge ultimately occurs between the plates.

- For a parallel-plate capacitor, $\Delta V = Ed$ and $C = \varepsilon_0 A/d$, so the the potential energy stored in a charged capacitor becomes:

$$U_E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\left(\varepsilon_0 A d\right)E^2$$

- Because the volume occupied by the electric field is Ad, the energy per unit volume known as the energy density, is:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

- The energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

26.5 Capacitors with Dielectrics

A dielectric is a nonconducting material, such as rubber, glass, or waxed paper. When a dielectric is inserted between the plates of a capacitor, the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor κ, which is called the dielectric constant of the material. The dielectric constant varies from one material to another.

The voltages with and without the dielectric are related by the factor κ as follows:

$$\Delta V = \frac{V_0}{\kappa}$$

$$\kappa > 1$$

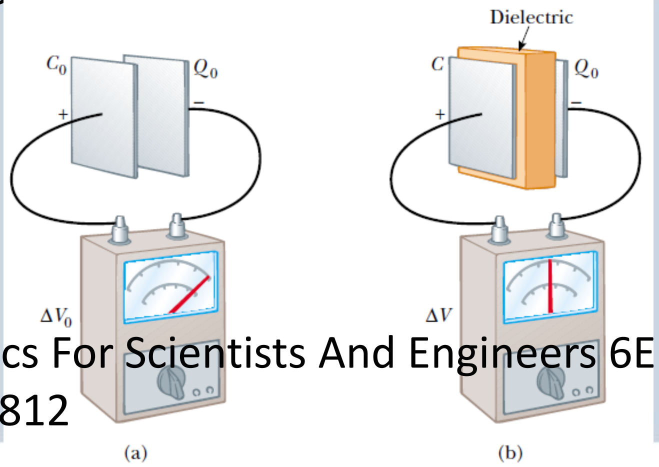


Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett page 812

Because the charge Q_0 on the capacitor does not change, we conclude that the capacitance must change to the value,

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0 / \kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa C_0$$

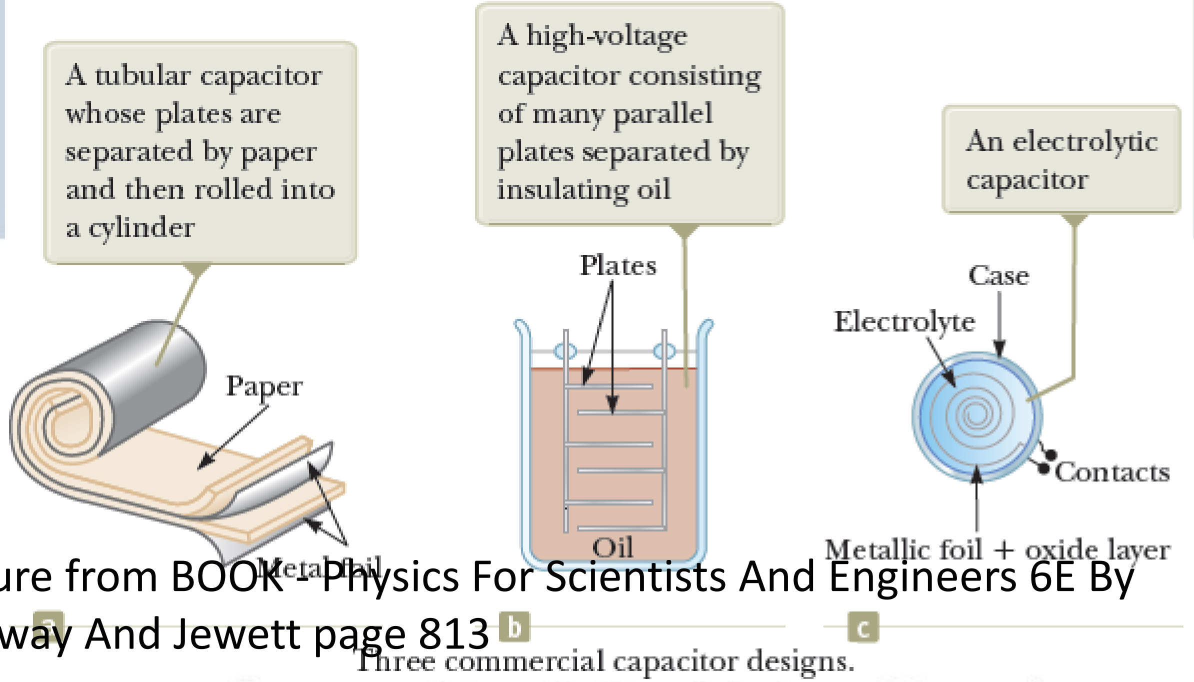
That is, the capacitance *increases* by the factor κ when the dielectric completely fills the region between the plates.

For a parallel-plate capacitor, we can express the capacitance when the capacitor is filled with a dielectric as, $C = \kappa \frac{\varepsilon_0 A}{d}$

In practice, the lowest value of d is limited by the electric discharge that could occur through the dielectric medium separating the plates. For any given separation d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the dielectric strength (maximum electric field) of the dielectric.

We see that a dielectric provides the following advantages:

- Increase in capacitance
- Increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C.



Three commercial capacitor designs.

Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett page 813

Example 26.6 A Paper-Filled Capacitor

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper.

(A) Find its capacitance.

Solution Because $\kappa = 3.7$ for paper (see Table 26.1), we have

$$C = \kappa \frac{\epsilon_0 A}{d}$$
$$= 3.7 \left(\frac{(8.85 \times 10^{-12}\, C^2/N \cdot m^2)(6.0 \times 10^{-4}\, m^2)}{1.0 \times 10^{-3}\, m} \right)$$
$$= 20 \times 10^{-12}\, F = \boxed{20\ pF}$$

(B) What is the maximum charge that can be placed on the capacitor?

$$\Delta V_{max} = E_{max} d = (16 \times 10^6\, V/m)(1.0 \times 10^{-3}\, m)$$
$$= 16 \times 10^3\, V$$

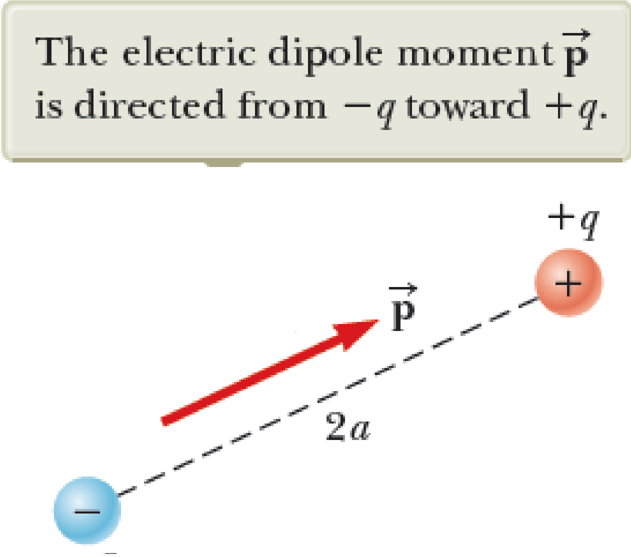
$$Q_{max} = C\, \Delta V_{max} = (20 \times 10^{-12}\, F)(16 \times 10^3\, V)$$
$$= \boxed{0.32\ \mu C}$$

26.6 Electric Dipole in an Electric Field

The **electric dipole moment** of this configuration is defined as the vector $\vec{p}$ directed from -*q* toward +*q* along the line joining the charges and having magnitude

$$p \equiv 2aq$$

Now suppose that an electric dipole is placed in a uniform electric field ***E***, as shown in Figure 26.22. Let us imagine that the dipole moment makes an angle θ with the field. The net force on the dipole is zero. However, the two forces produce a net torque on the dipole; as a result, the dipole rotates in the direction that brings the dipole moment vector into greater alignment with the field. This force tends to produce a clockwise rotation.



$$\tau = 2Fa\sin\theta$$

Because $F = qE$ and $p = 2aq$, we can express τ as

$$\tau = 2aqE\sin\theta = pE\sin\theta$$

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

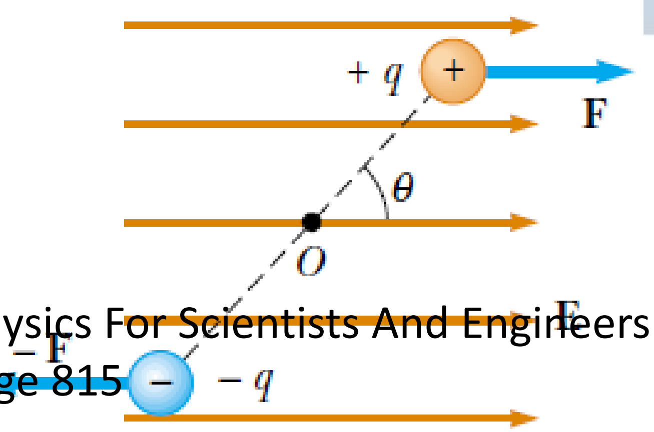


Figure from BOOK - Physics For Scientists And Engineers 6E By Serway And Jewett page 815

Figure 26.22

We can determine the potential energy of the system—an electric dipole in an external electric field. The work dW required to rotate the dipole through an angle $d\theta$ is $dW = \tau d\theta$. Because $\tau = pE\sin\theta$ and because the work results in an increase in the potential energy U, we find that for a rotation from θ_i to θ_f the change in potential energy of the system is,

$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau \, d\theta = \int_{\theta_i}^{\theta_f} pE \sin\theta \, d\theta = pE \int_{\theta_i}^{\theta_f} \sin\theta \, d\theta$$

$$= pE[-\cos\theta]_{\theta_i}^{\theta_f} = pE(\cos\theta_i - \cos\theta_f)$$

The term that contains $\cos\theta_i$ is a constant that depends on the initial orientation of the dipole. It is convenient for us to choose a reference angle of $\theta_i = 90°$, so that $\cos\theta_i = \cos 90° = 0$. Furthermore, let us choose $U_i = 0$ at $\theta_i = 90°$ as our reference of potential energy. Hence, we can express a general value of $U = U_f$ as

$$U = -pE\cos\theta$$

We can write this expression for the potential energy of a dipole in an electric field as the dot product of the vectors p and E:

$$U = -\mathbf{p} \cdot \mathbf{E}$$

Summary

- Capacitance says how much charge is on an arrangement of conductors for a given potential. $q = CV$
- Capacitance depends only on geometry
  - Parallel Plate Capacitor $C = \frac{\varepsilon_0 A}{d}$
  - Cylindrical Capacitor $C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$
  - Spherical Capacitor $C = 4\pi\varepsilon_0 \frac{ab}{b-a}$
  - Isolated Sphere $C = 4\pi\varepsilon_0 R$
- Units, F (farad) = C^2/Nm or C/V (note ε_0 = 8.85 pF/m)
- Capacitors in parallel $C_{eq} = \sum_{j=1}^{n} C_j$ in series $\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$
- Energy and energy density stored by capacitor

  $U = \frac{1}{2}CV^2$ $u = \frac{1}{2}\varepsilon_0 E^2$

- Dielectric constant increases capacitance due to induced, opposing field. κ is a unitless number.
- The torque acting on an electric dipole in a uniform electric field E is: $\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E}$
- The potential energy of the system of an electric dipole in a uniform external electric field $\boldsymbol{E}$ is: $U = -\boldsymbol{pE}$