ENE 327 – Pumps and Compressors

WEEK 2: FUNDAMENTAL RELATIONS FOR THE FLOW THROUGH AN ARBITRARY TURBOMACHINE

FUNDAMENTAL RELATIONS FOR THE FLOW THROUGH AN ARBITRARY TURBOMACHINE:

1-D Approximation

- The thickness of the blades are assumed to be zero.
- The number of blades is infinity, axisymmetric flow assumption.
- The flow is assumed to be uniform over each cross section.

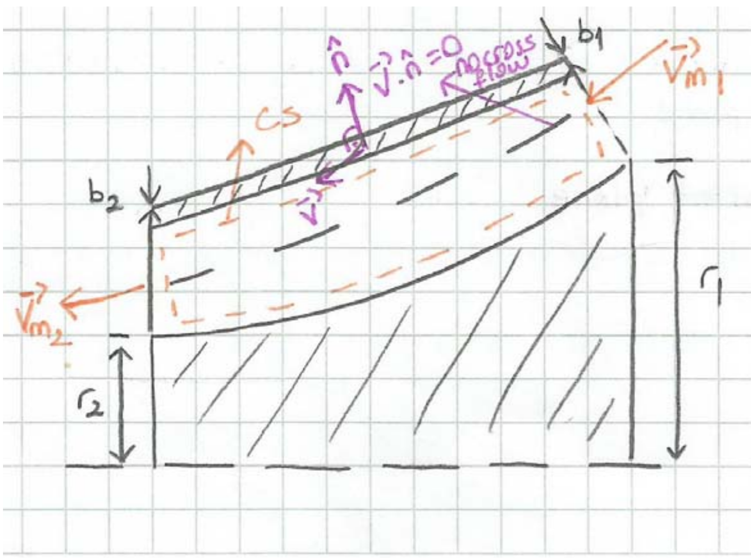

Figure1. Flow through meridional plane

a) Continuity Equation [1]:

The fluid flows in through area, A_1 of the control surface and flows out through area, A_2 of the control surface. No flow can take place through other surfaces of the control volume, since they are formed by streamlines and $\vec{v}.\vec{n} = 0$on these surfaces. Therefore,

$$\int_{A_1} \rho_1 \,.\, (\overrightarrow{v_1}.\overrightarrow{n_1}).\, dA + \int_{A_2} \rho_2 \,.\, (\overrightarrow{v_2}.\overrightarrow{n_2}).\, dA = 0$$

1-D assumption assumes that areas A_1 and A_2 are perpendicular to velocities $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$, respectively; then

$$\vec{v}_1.\hat{n}_1 = -v_{m_1}$$

$$\vec{v}_2.\hat{n}_2 = +v_{m_2}$$

Hence,

$$-\int_{A_1} \rho_1 \,.\, \overrightarrow{v_{m_1}}.\, dA + \int_{A_2} \rho_2 \,.\, \overrightarrow{v_{m_2}}.\, dA = 0$$

For a 1-D flow, the properties are uniform over each cross section, then

$$\dot{m} = \rho_1 . v_{m_1} . A_1 = \rho_2 . v_{m_2} . A_2$$

Noting that $A_1 = 2\pi . r_1 . b_1$and $A_2 = 2\pi . r_2 . b_2$with $b_1$and $b_2$are the width of the blades of the inlet and outlet, respectively.

$$\dot{m} = 2\pi . r_1 . b_1 . v_{m_1} = 2\pi . r_2 . b_2 . v_{m_2} = constant$$

b) Conservation of Angular Momentum [1]:

For a steady flow, the tangential component of the angular momentum is

$$T_Q = \int_{A_1} \rho_1 \,.\, r_1 . v_{Q_1} . (\overrightarrow{v_1}.\overrightarrow{n_1}).\, dA + \int_{A_2} \rho_2 \,.\, r_2 . v_{Q_2} . (\overrightarrow{v_2}.\overrightarrow{n_2}).\, dA$$

which can be rearranged to yield

$$T_Q = \int_{A_2} \rho_2 \,.\, r_2 . v_{Q_2} . v_{m_2} . dA - \int_{A_1} \rho_1 \,.\, r_1 . v_{Q_1} . v_{Q_1} . dA$$

For the uniform flow over each cross-section

$$T_Q = \rho_2 . r_2 . v_{Q_2} . v_{m_2} . A_2 - \rho_1 . r_1 . v_{Q_1} . v_{m_1} . A_1$$

Now, using the continuity equation;

$$T_Q = \dot{m} . r_2 . v_{Q_2} - \dot{m} . r_1 . v_{Q_1}$$

rearrange this equation;

$$T_Q = \dot{m} . \left[r_2 . v_{Q_2} - r_1 . v_{Q_1} \right]$$

which is known Euler's turbine equation.

- The torque developed is equal to the rate of change of angular momentum
- The torque exerted in a fluid element in angular motion is equal to mass flow rate times the change in $r.V_Q$. For a flow in which the torque is zero $r.V_Q = constant$. This is called a free vortex flow.
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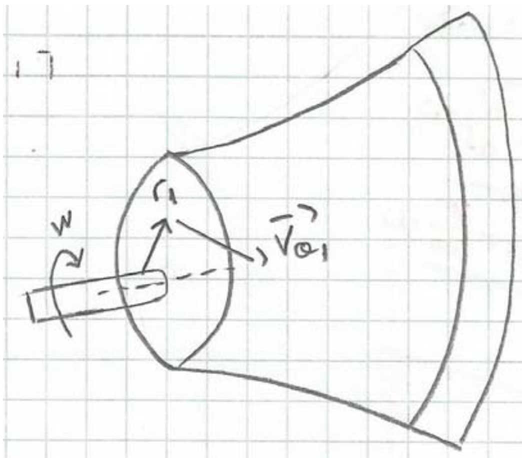

Figure 2: Swirl in a turbomachinery motor

The velocity component at the inlet and outlet of a pump and a turbine are shown, respectively.

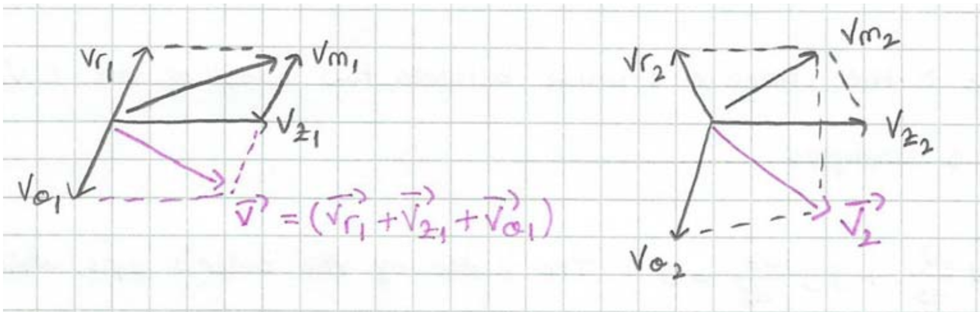

Figure 3. Velocity components

$$\dot{m}.r_2.v_{Q_2}$$
$$\dot{m}.r_1.v_{Q_1}$$
$$\dot{m}.r_2.v_{Q_2} > \dot{m}.r_1.v_{Q_1}$$
$$T_Q > 0(pump)$$
$$\dot{m}.r_2.v_{Q_2} < \dot{m}.r_1.v_{Q_1}$$
$$T_Q < 0(turbine)$$

REFERENCES

1. Aksel, M.H., 2016, “Notes on Fluids Mechanics”, Vol. 1, METU Publications
2. DOUGLAS, J. F., GASIOREK, J. M. and SWAFFIELD, J. A., *Fluid Mechanics,* 3rd ed., Prentice Hall, Inc., New Jersey, 2003.
3. FOX, R. W. and MCDONALD, A. T., *Introduction to Fluid Mechanics,* 6th ed., John Wiley and Sons, Inc., New York, 2005.
4. ÜÇER, A. Ş., *Turbomachinery,* Middle East Technical University, Ankara,Turkey, 1982.