CEN 3311 HEAT TRANSFER

Heat flow through a cylinder: (Radial system):

- Consider a long thich-walled cylinder. (hollow cylinder)

r_i → inside Radius

r_o → outside Radius

L → length

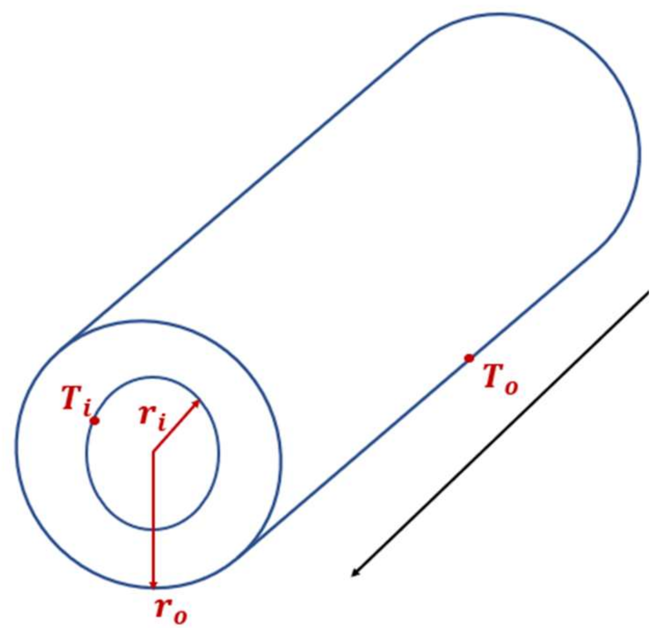
T_i → temperature of the inside surface

T_o → temperature of the outside surface

$\frac{L}{D} \gg$ → It may be assumed that heat flows only in a radial direction

''one-dimensional heat flow''

steady-state conditions.



Fourier's law:

$$\frac{q}{A_r} = -k\frac{dT}{dr} \qquad (q \rightarrow q_r)$$

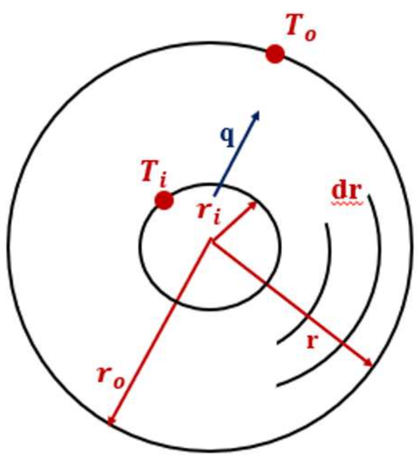
The area for heat flow in the cylindrical system is

$$A_r = 2\pi rL$$

(The area normal to the direction of heat transfer)

Therefore;

$$\boldsymbol{q_r = -2\pi rLk\frac{dT}{dr}}$$



Circular cross section

Electrical analog

This equation can be put in a more convenient form as follows:

$$\mathbf{q} = \frac{\boldsymbol{k\bar{A}_L(T_i - T_o)}}{\boldsymbol{r_o - r_i}}$$

This is similar to the the equation used for cartesian coordinates, i.e. For flat walls. Here A_L is defined as:

$$\bar{A}_L = \frac{2\pi L(r_0 - r_i)}{\ln({}^{r_o}/_{r_i})}$$

$$\bar{A}_L = 2\pi L\bar{r}_L$$

$$\bar{r}_L = \frac{(r_0 - r_i)}{\ln({}^{r_o}/_{r_i})}$$

Logaritmic mean area

Logaritmic mean radius

Composite Cylinder Wall

Let's consider a composite wall system and draw its electrical analogue.

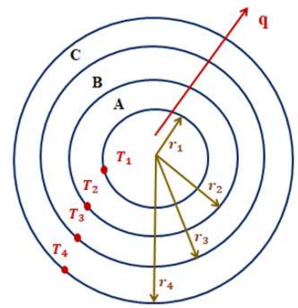
- The centers of the cylinders are common.
- A typical example of this system is a heat exchanger having vapour inside.

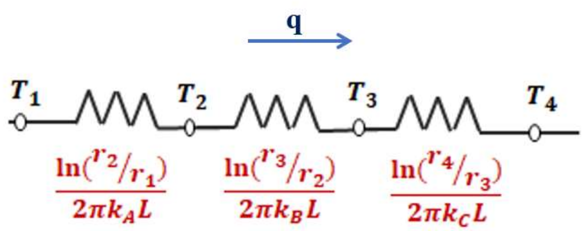
The cyclinder is insulated by three layers of insulators.

$L/D \gg 1$ → the heat flow is only in the direction of r

steay-state conditions → the heat flow rate (q) is the same through all insulators.

q= constant

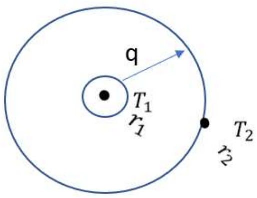




Spheres:

- Spherical systems may be treated as one-dimensional when the temperature is a function of radius only. For steady-state heat flow through a sphere:
- $q = -kA_r \frac{dT}{dr}$
- $A_r = 4\pi r^2 (heat\ transfer\ area)$
- $q = -4\pi r^2 k \frac{dT}{dr}$
- $q \int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi k \int_{T_1}^{T_2} dT$
- $q \left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 4\pi k (T_1 - T_2)$
- $q = \frac{4\pi k (T_1 - T_2)}{\left(\frac{r_2 - r_1}{r_1 . r_2}\right)}$
- Heat transfer area is variable. It is a function of (r).

The area perpendicular to the direction of heat flow.



Summary

- **One dimensional steady-state heat flow:**
- **Through a wall:** $q = kA\frac{(T_1 - T_2)}{(X_2 - X_1)}$
- **Through a cylinder:** $q = \frac{2\pi kL(T_i - T_0)}{\ln({r_o}/{r_i})}$
- **Through a sphere:**$q = \frac{4\pi k(T_1 - T_2)}{\left(\frac{r_2 - r_1}{r_1 . r_2}\right)}$