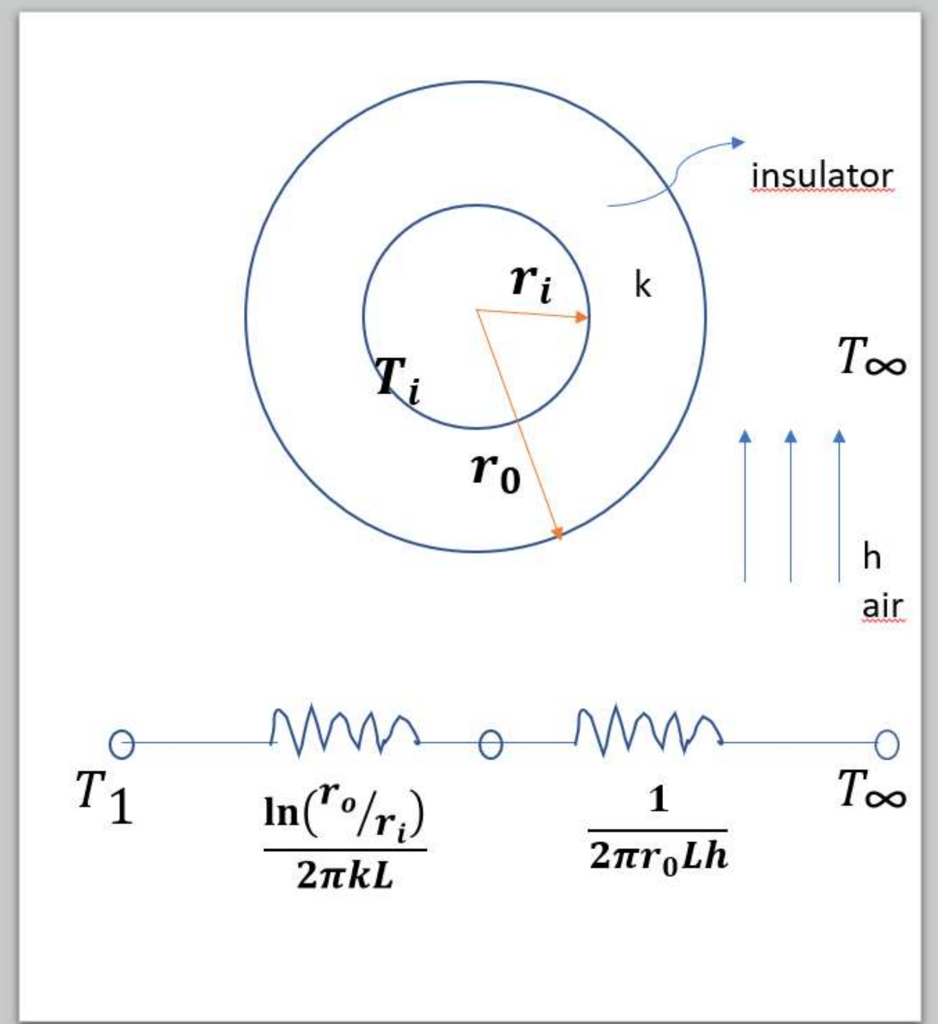
CEN 3311 HEAT TRANSFER

Critical Thickness of Insulation

- Let us consider a layer of insulation which mighty be installed around a circular pipe(see figure)
- T_i= The inner temperature od the insulation(fixed)
- T_∞ =The environment temperature(air bulk temperature)
- From the thermal network the heat transfer is: $q = \frac{2\pi L-(T_i-T_\infty)}{\frac{\ln(\frac{r_0}{r_i})}{k}+\frac{1}{hr_0}}$



According to this equation:

- When the outside radius($\boldsymbol{r_0}$) increases, then in the denominator, the first term increases but the second term decreases.
- Thus, there must be a critical radius, $\boldsymbol{r_c}$,which will allow maximum rate of heat transfer, q.
- The critical radius, $\boldsymbol{r_c}$, can be obtained by differentiating and setting the resulting equation equal to zero.

- Heat transfer through the insulation by conduction :

$$q = \frac{(T_i - T)}{\frac{\ln(\frac{r_0}{r_i})}{2\pi kL}}$$

- Heat transfer from the insulator to the air (heat loss)is

$$q = h2\pi \boldsymbol{r_0} L - (T - T_\infty)$$

$$\frac{dq}{dr_0} = \frac{-2\pi L(T_1 - T_\infty)\left[\frac{1}{kr_0} - \frac{1}{hr_0^2}\right]}{\left[\frac{ln\left({}^{r_0}/{r_i}\right)}{k} + \frac{1}{r_0 h}\right]^2} = 0$$

$$\frac{1}{kr_0} = \frac{1}{hr_0^2} \rightarrow hr_0^2 = kr_0$$

$$r_0 = \frac{k}{h}$$

$$r_c = \frac{k}{h}$$

- $r_c = \frac{k}{h}$

- To determine whether this result maximizes or minimizes the rate of heat flow, q ; the second derivative must be evaluated.

- The second derivative at

- $r_0 = \frac{k}{h}$

gives

$$\frac{d^2 q}{dr_0^2} < 0$$

- Since this result is always negative, r_c is the insulation radius for which the heat flow rate is a maximum.
- Critical insulation Radius maximizes heat transfer.
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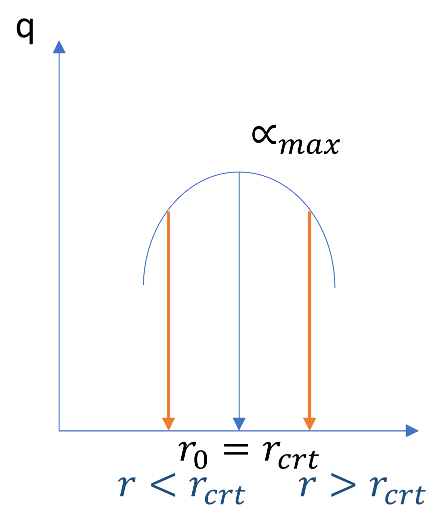
- Below r_c , heat flow increases with increasing r_0
- Above r_c , heat flow decreases with increasing r_0
- Low thermal conductivity(k) and high heat transfer coefficient(h) decreases critical thickness of insulation (r_c)
- $Critical\ insulator\ thickness \rightarrow (r_{crt} - r_i)$
- $Critical\ insulation\ radius \rightarrow r_{cr}$



q
$\propto_{max}$
$r_0 = r_{crt}$
$r < r_{crt}$
$r > r_{crt}$