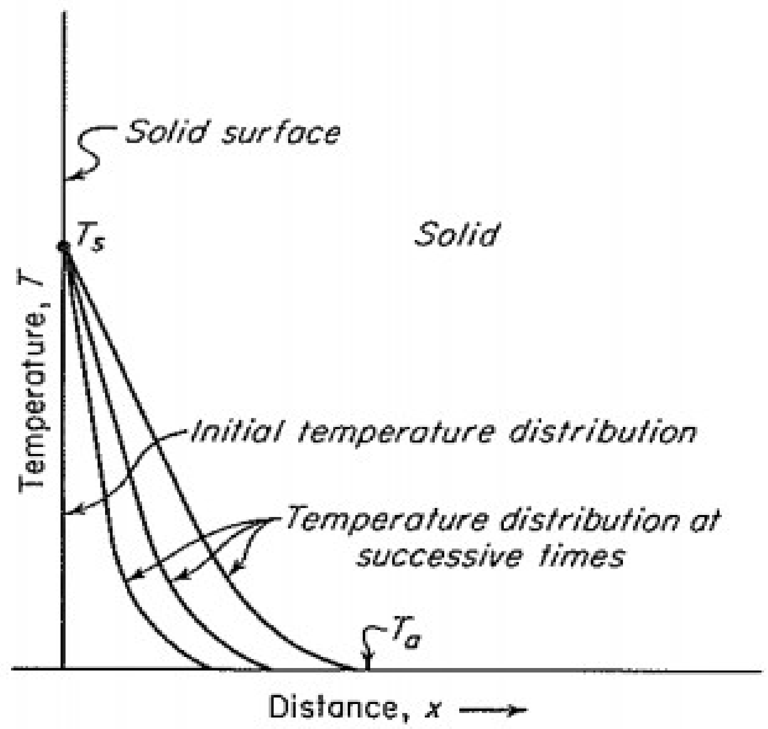
CEN 3311 HEAT TRANSFER

Semi-infinite solid

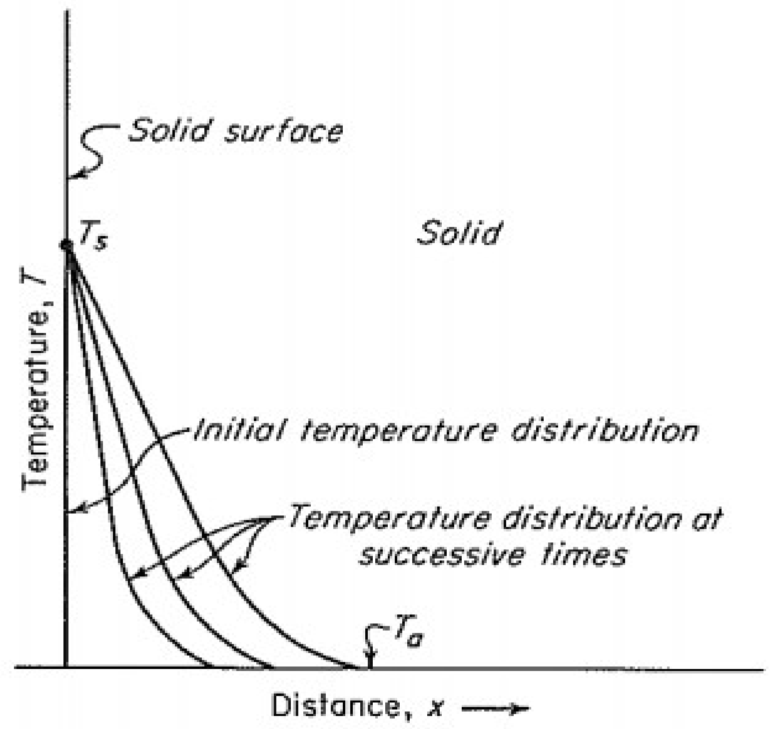
Consider semi-infinite solid given in the figure maintained at the initial temperature of $\boldsymbol{T_a}$.

The surface temperature is suddenly increased and maintained at a temperature of $\boldsymbol{T_s}$.

An expression for the temperature distibution in a solid as a function of time will be derived.



Sometimes, only one surface of a solid subjected to high (or low) temperature and the heat penetrates inside the solid.



The solid is considered semi-infinite if it extents to infinity in our one direction

For constant properties, the differential equation for the temperature distribution:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\propto}\frac{\partial T}{\partial t}$$

x → distance from surface

The boundary and initial conditions:

IC:	$t = 0$	$0 \leq x \leq \infty$	$T = T_a$
BC1:	$t > 0$	$x = 0$	$T = T_s$
BC2:	$t > 0$	$x \rightarrow \infty$	$T = T_a$

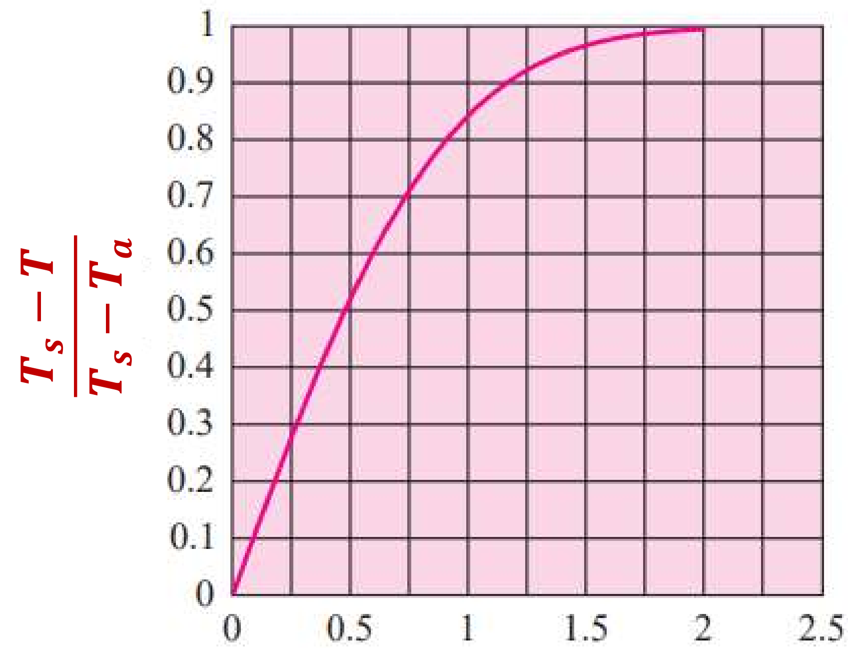
This is a problem that may be solved by *the Laplace-transform technique*.

The solution is:

$$\frac{T_s - T}{T_s - T_a} = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2}\, d\eta$$

$$\eta = \frac{x}{2\sqrt{\propto t}}$$

The function in the equation is known as «Gauss Error Integral» or «Probability Integral». This equation is plotted in the figure.



Credit: Holman (2010) Heat Transfer, 10th Ed.

$$\boldsymbol{\eta = \frac{x}{2\sqrt{\propto t}}}$$

Response of semi-infinite solid to sudden change in surface temperature

$\eta = \frac{x}{2\sqrt{\propto t}}$ ⟶ dimensionless

$\propto$ ⟶ Thermal diffusivity

x ⟶ Distance from surface

t ⟶ Time after change in surface temperature

T ⟶ Temperature at any point a distance x from the hot surface

Table: The error function

$\frac{x}{2\sqrt{\propto t}}$	$erf\,\frac{x}{2\sqrt{\propto t}}$
-	-
w	erf w
-	-

The solution is:

1

$$\frac{T_s - T}{T_s - T_a} = erf\,\frac{x}{2\sqrt{\propto t}}$$

2 The Gauss function is defined on:

$$erf\,\frac{x}{2\sqrt{\propto t}} = \frac{2}{\sqrt{\pi}}\int^{x/2\sqrt{\propto t}} e^{-\eta^2}\,d\eta$$

3 Therefore: $$\frac{T_s - T}{T_s - T_a} = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2}\, d\eta$$ $$\eta = \frac{x}{2\sqrt{\propto t}}$$

<u>Solution of this equation</u>:

If we know the $\boldsymbol{erf}\,\frac{\boldsymbol{x}}{\mathbf{2}\sqrt{\propto \boldsymbol{t}}}$; we can calculate T from the table,