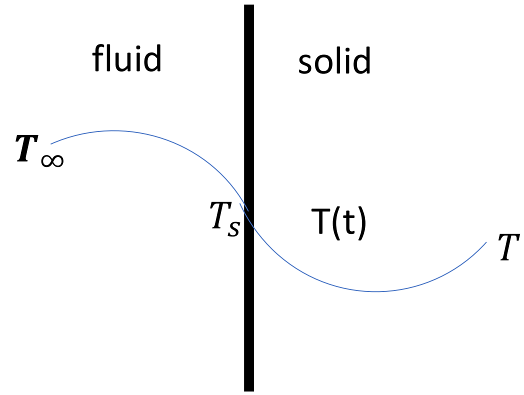
CEN 3311 HEAT TRANSFER

UNSTEADY-STATE CONDUCTION CONVECTION BOUNDARY CONDITIONS (Heat flow with variable surface temperature)

Semi-infinite solid



- $T_a \rightarrow initial\ surface\ temperature$
- $T_\infty \rightarrow enviroment\ temperature$
- $T_s \rightarrow surface\ temperature$
- $\boldsymbol{T_s \neq T_\infty}$

- The energy equation for this system (no heat generation, one- dimensional flow, unsteady-state condition)

$$\frac{\partial T}{\partial t} = \propto \frac{\partial^2 T}{\partial x^2} \quad \ldots\ldots \quad (1)$$

- There is no accumulation of energy at the interface, so the heat flux to the surface equals the flux into the solid.
- Heat convected to surface = heat conducted into surface.

$$hA(T_\infty - T)\Big|_{x=0} = -kA \frac{\partial T}{\partial t}\Big|_{x=0}$$

$$x = o \; T = T_s$$

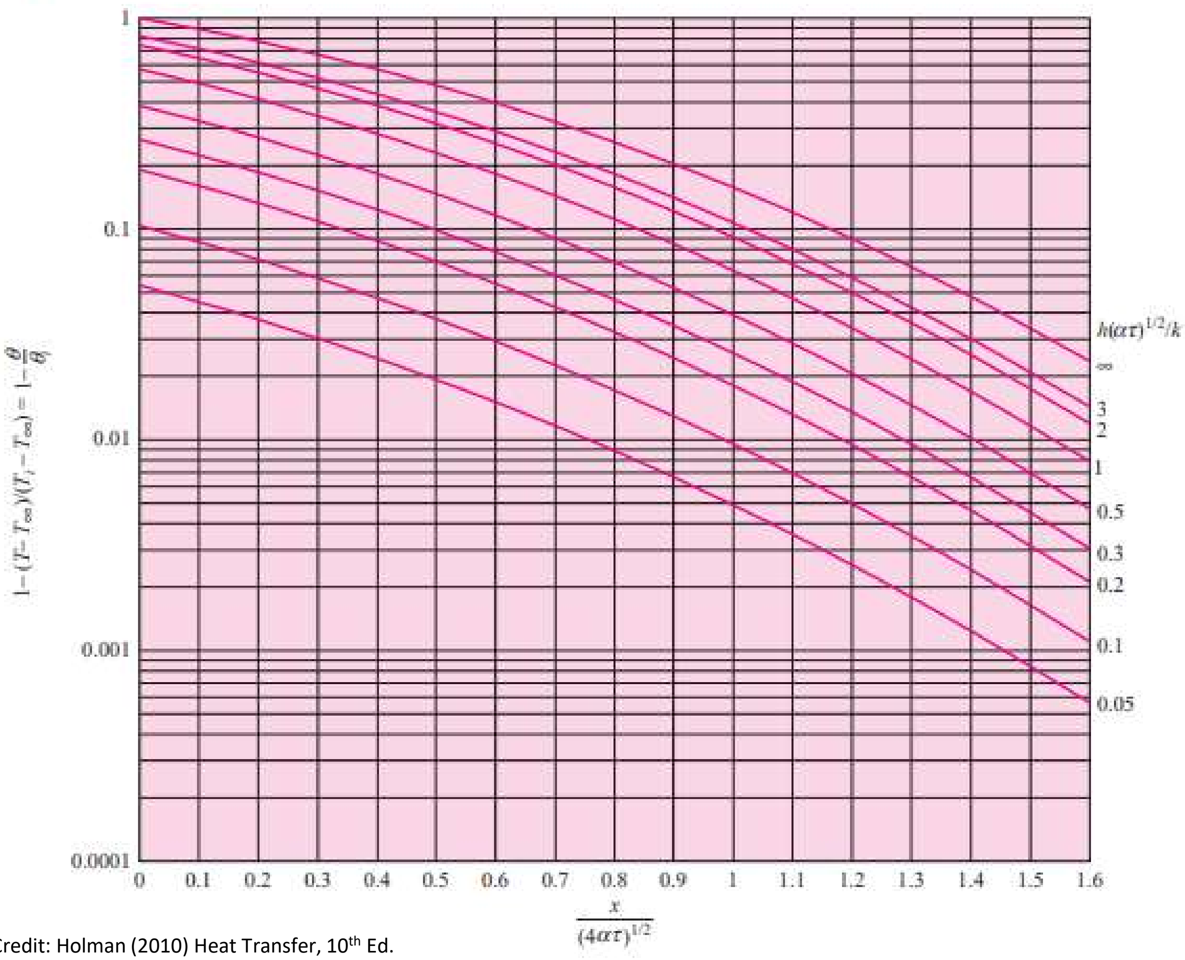
$$hA(T_\infty - T) = -kA \frac{\partial T}{\partial t}\Big|_{x=0} \quad \ldots\ldots \quad (2)$$

- The solution of Eq(1) using Eq(2) as a boundary condition gives:

$$\frac{T - T_a}{T_\infty - T_a} = 1 - erfx - \left[exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\right] x \left[1 - erf\left(X + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

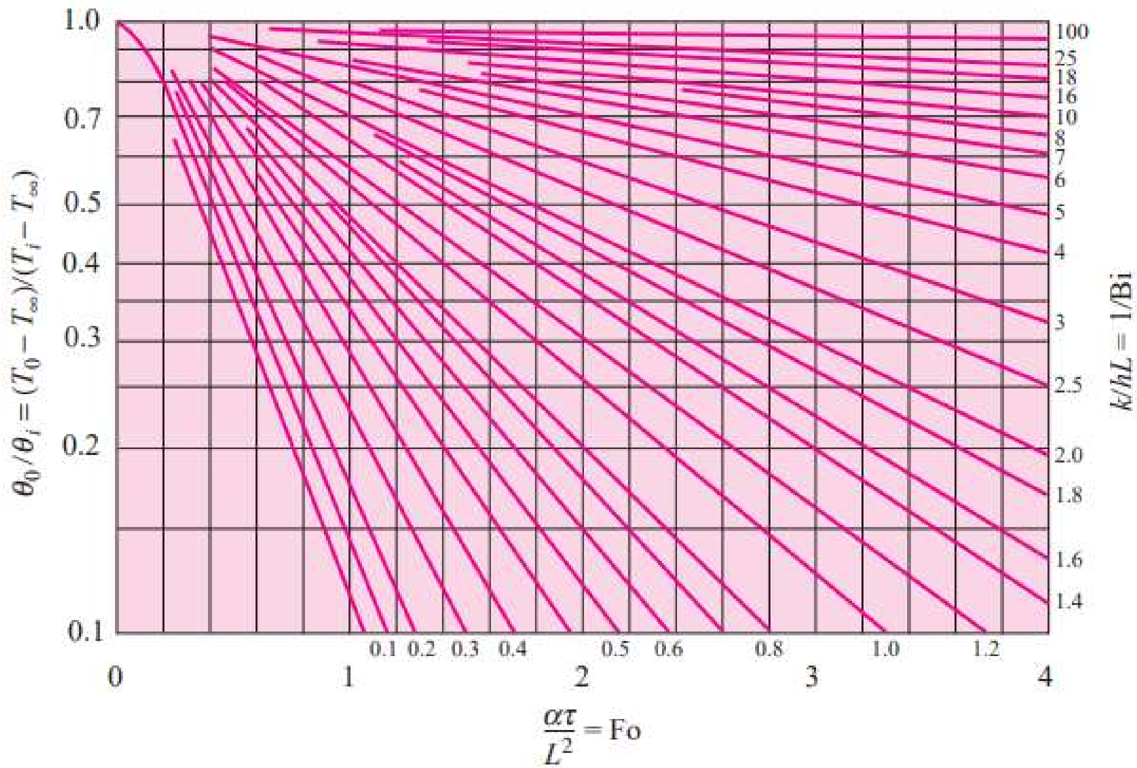
$$Here\colon\ X = \frac{x}{2\sqrt{\propto t}}$$

Figure 4-5 | Temperature distribution in the semi-infinite solid with convection boundary condition.



Credit: Holman (2010) Heat Transfer, 10th Ed.

Figure 4-7 | (*Continued*). (*b*) expanded scale for 0 < Fo < 4, from Reference 2.



Credit: Holman (2010) Heat Transfer, 10th Ed.

Midplane temperature (To) for an infinie plate of thickness of 2s
(scale for 0<Fo>4)

Cylinder

Figure 4-8 | Axis temperature for an infinite cylinder of radius r_0: (*a*) full scale.

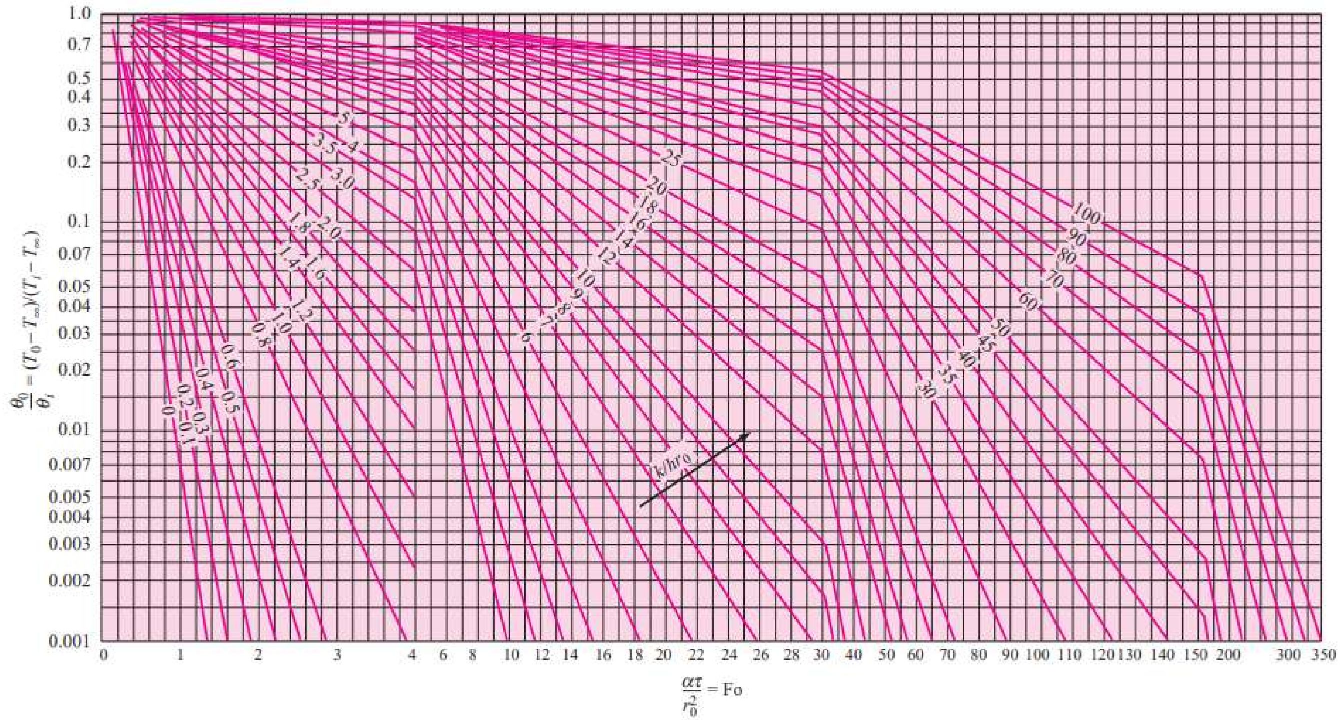
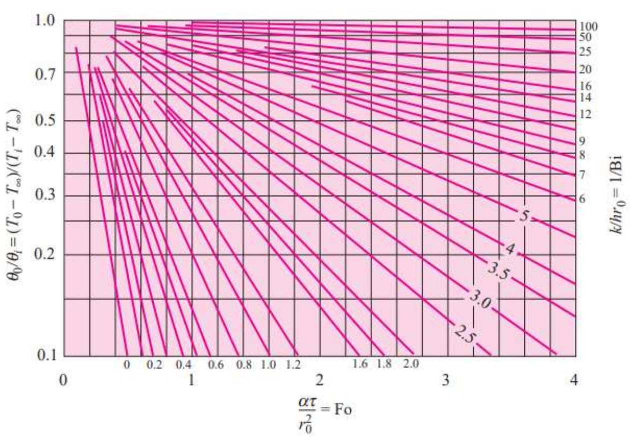


Figure 4-8 | (*Continued*). (*b*) expanded scale for $0 < \text{Fo} < 4$, from Reference 2.



Credit: Holman (2010) Heat Transfer, 10th Ed.

Sphere

Figure 4-9 | Center temperature for a sphere of radius r_0: (*a*) full scale.

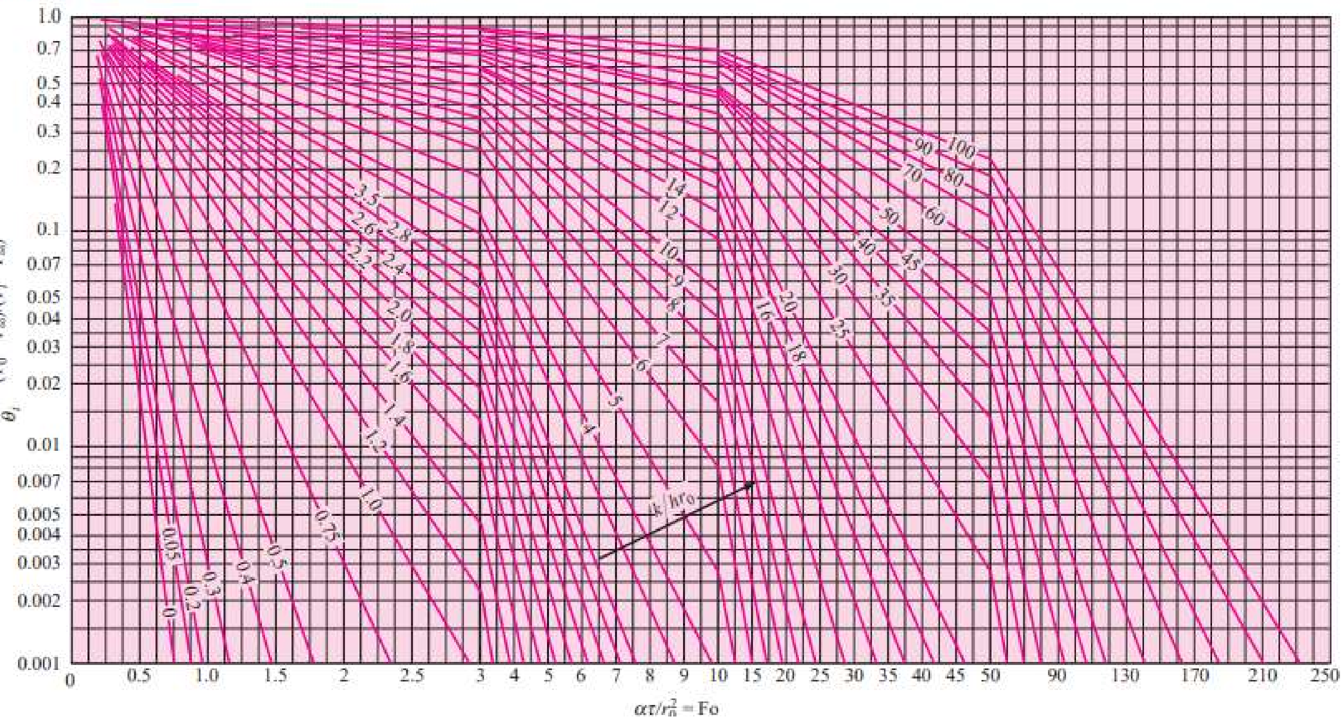
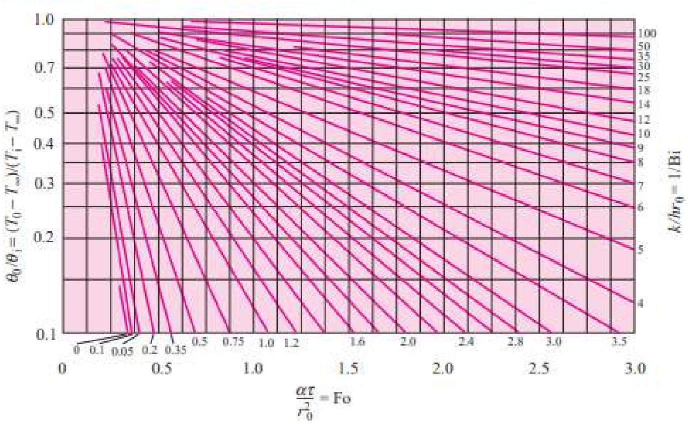


Figure 4-9 | (*Continued*). (*b*) expanded scale for 0 < Fo < 3, from Reference 2.



(*b*)

Credit: Holman (2010) Heat Transfer, 10th Ed.