Chapter 4: Measures of Image Quality

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Diagnostic Radiology Physics:
A Handbook for Teachers and Students

Objective:

To familiarise the student with methods of quantifying image quality.

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following initial work by
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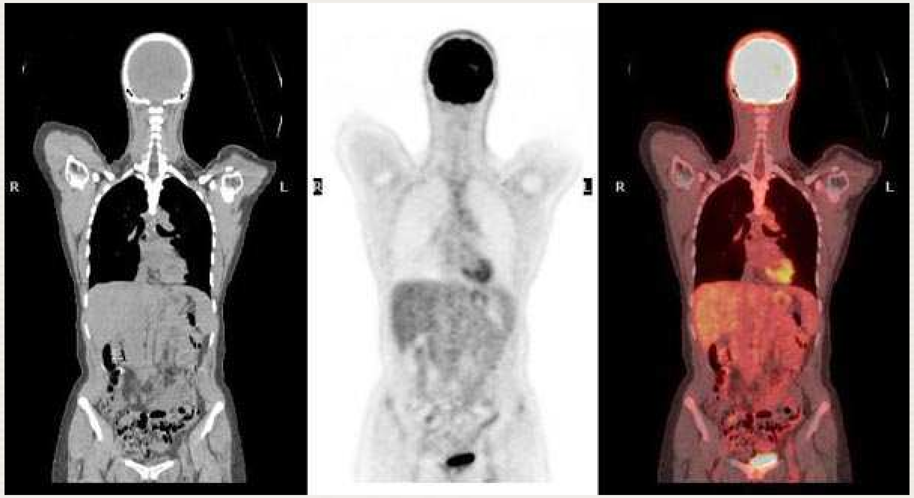
4.1 INTRODUCTION

A medical image is a **Pictorial Representation** of a measurement of an object or function of the body


AN ULTRASOUND OF A KIDNEY


A chest X-ray

CT PET CT & PET Images superimposed



Many different ways exist to acquire medical image data

4.1 INTRODUCTION

Knowledge of image quality allows for comparison of imaging system designs:

- **Within** a modality, and
- Across **Different** imaging modalities

This information can be acquired in 1-3 spatial dimensions

It can be **Static** or **Dynamic**, meaning that it can be measured also as a function of time

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4.1 INTRODUCTION

Fundamental properties associated with these data:

- No image can **Exactly** represent the object or function; at best, one has a measurement with an associated error equal to the difference between the true object and the measured image
- No two images will be **Identical**, even if acquired with the same imaging system of the same anatomic region

variability generally referred to as **Noise**

4.1 INTRODUCTION

There are many **Different** ways to acquire medical image data

Regardless of the method, one must be able to judge the **Fidelity** of the image in an attempt to answer the question:

How Accurately Does the Image Portray the Body or the Bodily Function?

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4.1 INTRODUCTION

This judgment falls under the rubric of

Image Quality

Methods of **Quantifying** image quality are described in this chapter

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4.1 INTRODUCTION

Knowledge of image quality allows **Comparison** of:

- Various imaging system designs for a given modality and
- Information contained in images acquired by different imaging modalities

The impact of image quality on an imaging task, such as **Detection** of a lesion in an organ, can also be determined

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4.1 INTRODUCTION

Various imaging tasks require **Differing Levels** of image quality

An image may be of sufficient quality for **One** task, but inadequate for **Another** task

4.1 INTRODUCTION

The metrics introduced here are much used in the following chapters in this Handbook as the

- Design
- Performance, and
- Quality Control

of different imaging systems are discussed

First, however, one needs to learn the meaning of:

High Image Quality

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4.2.1 Linear Systems Theory

In all imaging systems the output, **g**, is a function of the input, **f**

The function, H, is usually called the **Transfer Function** or **System Response Function**

For a continuous 2D imaging system, this relationship can be written as:

$$g(x,y) = H\{f(x,y)\}$$

The simple concept implies that we can predict the output of an imaging system

if we know the **Input** and the **Characteristics** of the system

That is, **g** is the **Image** of the **Scene f**

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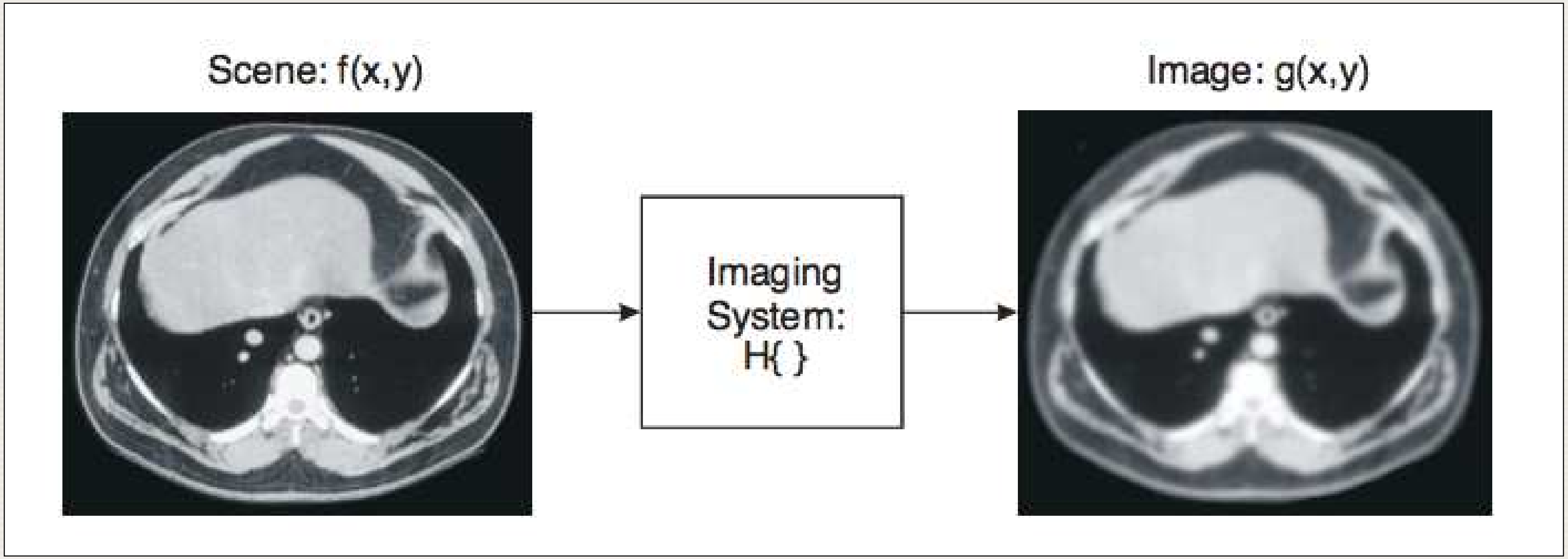
In this chapter, functions are expressed with **Two** dependent variables to represent a 2D image

This **Convention** is chosen to ensure consistency through the chapter, however the imaging problem can be treated in **Any** number of dimensions

4.2.1 Linear Systems Theory

The image, **g(x,y)**, portrays a cross-section of the thorax, **f(x,y)**, blurred by the transfer function, H, of the imaging system:



$$g(x,y) = H\{f(x,y)\}$$

Unfortunately, this general approach to image analysis is very difficult to use

It is necessary to compute the transfer function at **Each Location** in the image for each unique object or scene

This analysis is greatly simplified when two fundamental assumptions can be made:

- **Linearity** and
- **Shift-Invariance**

abbreviated jointly as **LSI**

Linearity

A linear system is one in which the output of the system can be expressed as a **Weighted Sum** of the input constituents

Thus, if a system presented with input $\mathbf{f_1}$ results in output:

$$g_1(x,y) = H\{f_1(x,y)\}$$

and input $\mathbf{f_2}$ results in output:

$$g_2(x,y) = H\{f_2(x,y)\}$$

then:

$$H\{af_1(x,y) + bf_2(x,y)\} = H\{af_1(x,y)\} + H\{bf_2(x.y)\}$$

$$= ag_1(x,y) + bg_2(x,y)$$

Linearity

In general, most imaging systems are either

- Approximately linear or
- Can be linearized or
- Can be treated as being linear over a small range

The **Assumption of Linearity** lets us formulate the transfer function as an integral of the form

$$g(x,y) = \iint f(x',y')H(x,y;x',y')dx'dy'$$

Linearity

However, most modern imaging systems are **Digital**

As a result, images consist of measurement made at specific locations in a **Regular Grid**

With **Digital** systems, these measurements are represented as an array of **Discrete** values

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Linearity

In the **Discrete case**,

our expression can be reformulated as multiplication of a matrix **H**

where the input scene and output image are given as **Vectors** (for 1D images) or **Matrices** (for higher dimension images):

$$\mathbf{g} = \mathbf{H}\mathbf{f}$$

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Linearity

In this formulation, each element in **g** is called a **Pixel** or **Picture Element**

Each element in **f** is called a **Del** or **Detector Element**

A pixel represents the smallest region which can uniquely encode a single value in the image

By similar reasoning, the term **Voxel** or **Volume Element** is used in 3D imaging

Linearity

In the expression for the imaging system:

g is expressed as a weighted sum, **H**, of the source signals, **f**

It is important to note that H or **H** is still quite complicated

If **g** and **f** have **m** x **n** elements

then **H** has **(mn)**^{2} elements

that is, there is a **Unique** transfer function for each pixel in the image because the value of each pixel arises from a different weighted sum of the dels

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Shift Invariance

A system is shift invariant if the system response function, H, does not change as a function of position in the image

By further adding the stipulation of shift-invariance, it is possible to formulate the transfer function without reference to a specific point of origin

This allows us to write the integration in our expression as a convolution:

$$g(x,y) = \iint f(x',y')h(x-x',y-y')dx'dy'$$

where h is now a function of 2 variables while H was a function of 4 variables in the case of a 2D imaging system

Shift Invariance

In the discrete formulation of a shift invariant system, the matrix **H** now has a unique property; it is **Toeplitz**

As a practical measure, we often use a circulant approximation of the Toeplitz matrix

This approximation is valid **Provided** the PSF is small compared to the size of the detector

Shift Invariance

The discrete Fourier transform of the circulant approximation of **H** is a diagonal matrix

This property has particular appeal in analysing LSI systems, as we have gone from a formulation in which H has:

as many as $(\mathbf{mn})^2$ non-zero elements to one that

has exactly **mn** distinct elements

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Shift Invariance

As a result, it is possible to construct a new matrix, ***h***, from **H** such that our expression can now be rewritten

$$\boldsymbol{g} = \boldsymbol{h} * \boldsymbol{f}$$

where * is the circulant convolution operator

In the case of 2D detectors and images ***f***, ***g***, and ***h*** are each matrices with **m** x **n** distinct elements which are cyclically extended in each direction

Shift Invariance

The assumptions of **Linearity** and **Shift-Invariance** are key to making most imaging problems tractable

as there is now a **common** Transfer Function, h, that applies to each pixel in the image

Shift Invariance

Recalling that for Fourier transform pairs the **Convolution** in one domain corresponds to **Multiplication** in the other domain, we can now rewrite the last expression as:

$$\tilde{g} = \tilde{h}\tilde{f}$$

where the tilde (~) denotes the discrete Fourier transform

This implies that an object with a given spatial frequency referenced at the plane of the detector will result in an image with exactly the same spatial frequency

although the **Phase** and **Amplitude** may change

Shift Invariance

With few exceptions most systems are **not** truly shift-invariant

For Example

Consider a simple system in which a pixel in the image is equal to the **Average** of the matching del in the scene and the eight immediate neighbouring dels

The transfer function will be **Identical** for all interior pixels

However, pixels on the 4 **Edges** and 4 **Corners** of the image will have different transfer functions, because they do not have a **Full Complement** of neighbouring pixels upon which to calculate this average

Shift Invariance

That said, most systems can be treated as shift invariant (with regard to this **Boundary Problem**), provided the blurring (or correlation) between pixels is small compared to the size of the image

A **Second** strategy to ensure shift invariance is to consider the transfer function locally, rather than globally

This strategy allows one to ignore differences in the detector physics across the full-field of the detector, such as the oblique incidence of X-rays

4.2.2 Stochastic Properties

In all real imaging systems, it is necessary to consider the degradation of the image from both:

- **Blurring**, given by the transfer characteristics, and the
- Presence of **Noise**

Noise can arise from a number of sources, including the:

- **Generation** of the signal carriers,
- **Propagation** and **Transformation** of these carriers through the imaging process, and
- Addition of **Extraneous Noise** from various sources such as the imaging electronics

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Thus, it is necessary to modify the image transfer equation to include a term for the noise, **n**

Noise is generated from a **Random** process

As a result, the noise recorded in each image will be **Unique**

Any given image $\dot{g}$ will include a single realization of the noise, $\dot{n}$ so that

$$\dot{g} = Hf + \dot{n}$$

Strictly speaking, some noise (e.g. X ray quantum noise) will be generated in the process of forming the scene, **f**, and hence

will be acted upon by the transfer function, **H**

while other noise (e.g. electronic readout noise)

will not have been acted upon by the transfer function

Equation Ignores This Distinction

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Also, strictly speaking, all quanta do not necessarily experience the same transfer function

Variability in the transfer of individual quanta leads to the **well-known** Swank and Lubberts' effects

4.2.2 Stochastic Properties

The introduction of noise in images means that imaging systems have to be evaluated **Statistically**

The exact treatment of the images is dependent upon **Both** the nature of the noise present when the image is recorded and the imaging system

System linearity (or **Linearizability**) will help to make the treatment of images in the presence of noise tractable

In general, however, we also need to assume that the noise is **Stationary**

A stochastic noise process is **Stationary** if the process does not change when shifted either in time or in space

That is, the **Moments** of a stationary process will not change based upon the time when observations begin

An **Example** is X ray quantum noise, because the probability of generating an X ray does not depend upon when the previous or subsequent X ray quanta are created

Similarly, in a shift-invariant imaging system, it does not matter which point on the detector is used to calculate the moments of a stationary process, as each point is nominally the same

A **Wide-Sense Stationary** (WSS) process is one in which only the mean and covariance are stationary

Since a Poisson process is fully characterized by the **Mean** and a Gaussian process is fully characterized by the **Mean and Variance**, it is typical to only require an imaging process to be WSS

It is, in fact, common to treat the noise as being Gaussian and having **Zero** mean

In practice, this is sufficient for **Almost All** imaging systems

4.2 IMAGE THEORY FUNDAMENTALS

4.2.2 Stochastic Properties

It should be noted that digital images consisting of **Discrete** arrays of pixels or volume elements (voxels) are not strictly stationary

Shifts of the origin that are not commensurate with the pixel spacing will potentially result in different images being acquired

However,a system is said to be **Cyclostationary** if the statistical properties are unchanged by shifts in the origin of specific amounts

i.e. multiples of the pixel or voxel pitch

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A system is **Wide-Sense Cyclo-stationary** if the mean and covariance are unchanged by specific shifts in the origin

In general, we can assume most digital imaging systems are wide-sense cyclo-stationary, at least **Locally**

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To measure the signal in a pixel, exclusive of the noise, we may simply **Average** the value in that pixel over many images to minimize the influence of the noise on the measurement

In a similar fashion, we can estimate the noise in a pixel by calculating the **Standard Deviation** of the value of that pixel over many images of the same scene

Calculations which involve a large number of images are clearly **Time-Consuming** to acquire and process in order to estimate the mean and standard deviation with **Sufficient Accuracy**

However this problem is tremendously simplified if one can additionally assume **Ergodicity**

Ergodic Process: one in which the statistical properties of the ensemble can be obtained by analysing a single realization of the process

For example, X ray quantum noise is frequently referred to as **White Noise**, implying that:

- In different realizations all spatial frequencies are equally represented, or **equivalently** that
- The noise from individual quanta are uncorrelated

White Noise is Ergodic

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This means, for example, that we can calculate the average fluence of an X ray beam either by averaging over a **Region** or averaging over **Multiple Images**

When an appropriate imaging system is used to image an ergodic process (such as a uniform scene imaged with X rays), calculations performed from a number of sample images can be replaced by calculations from **One Image**

For Example, the noise in a particular pixel that was originally measured from image samples can now be measured from a region of a single image

With few exceptions (notably screen-film radiography), modern imaging systems are **Digital**

A digital image is only defined as discrete points in space, called sampling points

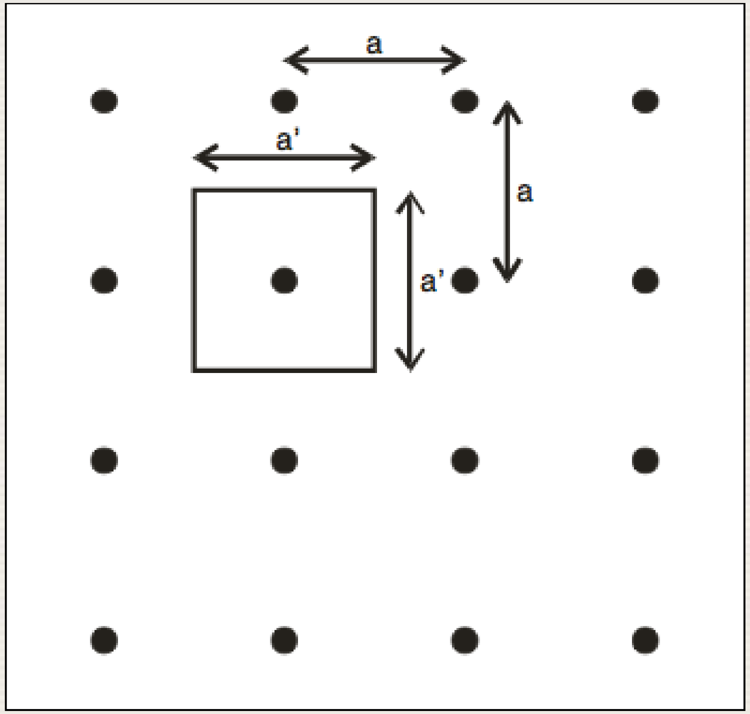
The process of sampling by a detector element (del) generally involves the integration of continuous signal values over a finite region of space around the sampling point

The **Shape** of these regions is defined by the **Sampling Aperture**

Distance between sampling points is called the **Sampling Pitch**

In an idealized 2D detector, the sampling aperture of each del is represented by a square of dimension, **a′**

Such dels are repeated with pitch **a** to cover the entire detector:



Rectangular array of dels in which a single del with a square aperture of dimensions **a′** x **a′** is shown centred upon a series of sampling points with pitch **a** in orthogonal directions

It is not strictly necessary for the aperture and pitch to have the same size, nor to be square

For example, active matrix X ray detectors can have regions which are not radiation sensitive such as the data and control lines and del readout electronics

The **Fill Factor** of an active matrix detector is typically defined as the ratio $\mathbf{a'^2/a^2}$

The Fill Factor is Commonly <1

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It is also possible for the del aperture to be **$>a^2$**

For Example, in CR, the scanning laser will typically stimulate fluorescence from a circular region having a diameter greater than the sampling pitch

As discussed later, this has benefit in **Reducing Aliasing**

The process of sampling a continuous signal *f* by a single del is given by:

$$f(x_i, y_j) = \iint f(x, y) A(x - x_i a, y - y_j a) dx dy$$

where **A** is the aperture function and (x_i, y_i) are **Integer** indices of the del

In practice, the aperture function is **Non-Zero** over a limited area thus providing finite limits to the integral in this equation

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It is clear from this expression that if one were to shift the sampling points by a non-integer amount (i.e. incommensurate with the pixel pitch), the recorded image would vary

It is for this reason that digital systems are only

Cyclo-Stationary

In general these changes are small – especially for objects which are **Large** relative to the sampling pitch

However, for **small** objects, these changes can be significant

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Sampling a continuous signal **f(x, y)** on a regular grid with grid spacing **a**, is equivalent to multiplying **f** by a **comb** function, **comb**$_a$

The **comb** function is an infinite sum of Dirac delta functions centred at the sampling points

Multiplication by the comb function in the image domain is equivalent to **Convolution** by the Fourier transform (FT) of the comb function in the Fourier domain

The FT of the comb function is also a comb function, but with grid spacing **1/a**

This convolution has the form:

$$\left(\tilde{f} * \mathrm{comb}_{1/a}\right)(u,v) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \tilde{f}\left(u - \frac{j}{a}, v - \frac{k}{a}\right)$$

This implies that the FT of **f** is replicated at each point on a grid with a spacing **1/a**, and an infinite sum of all the replicates is taken

The frequency **1/a** is called the **Sampling Rate**

The **Nyquist-Shannon** sampling theorem provides **Guidance** in determining the value of a needed for a specific imaging task:

Ideally, the Fourier spectrum of **f** should not have components above the frequency **1/2a**

This frequency is called the

Nyquist Frequency

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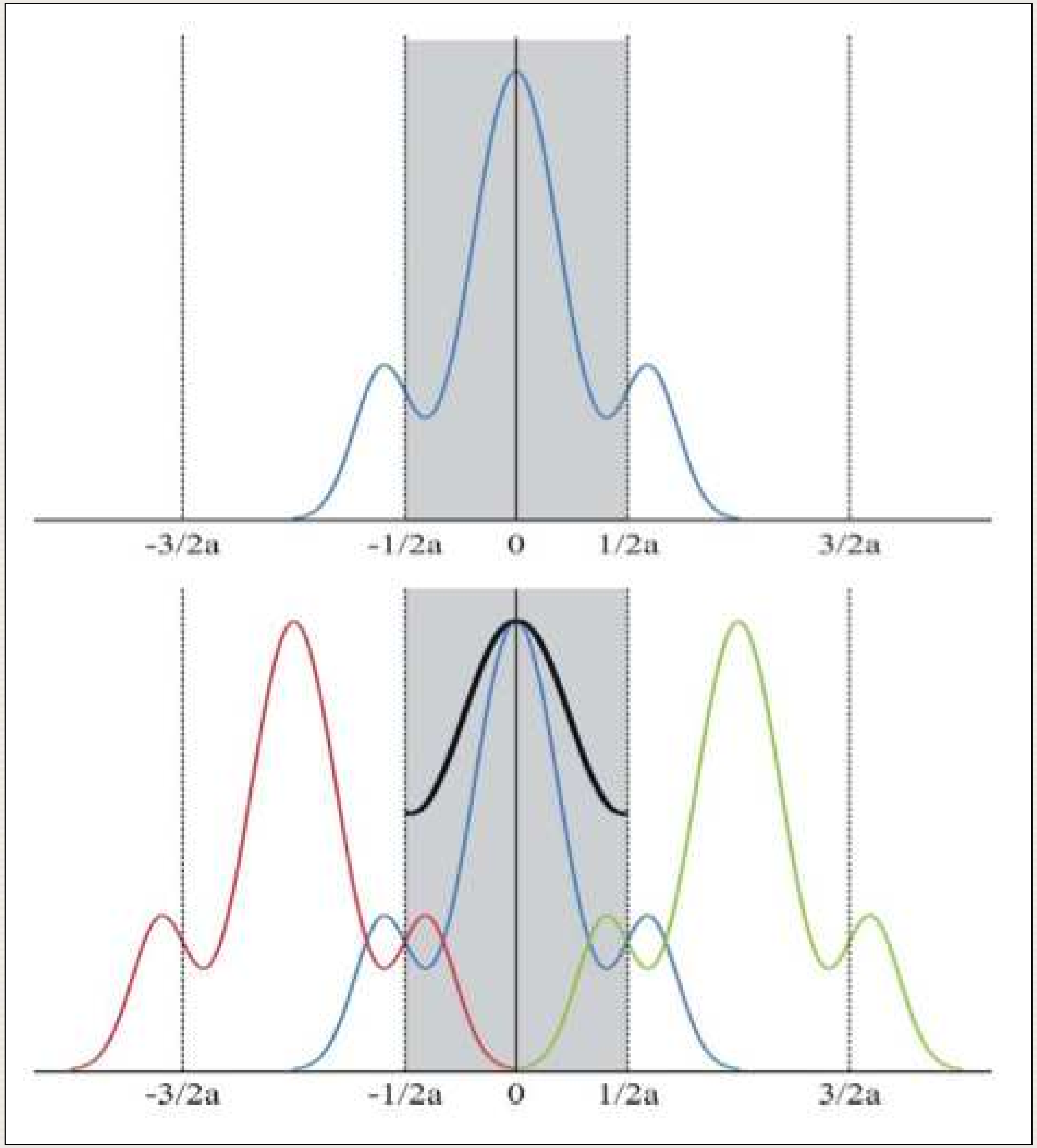
When this condition is not met, the Fourier spectra will contain components with spatial frequencies which **Exceed** the Nyquist frequency, and the infinite sum of spectra will overlap



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This overlap between the superimposed spectra will result in **Aliasing**

Aliasing degrades the sampled image because it **incorrectly** portrays high-frequency information present in the scene as lower-frequency information in the image

Black Curve in figure

To avoid aliasing, the Nyquist frequency must be greater than or equal to the maximum frequency in the image prior to sampling

In many system designs, it is **Impossible** to avoid aliasing

4.3 CONTRAST
4.3.1 Definition

Contrast is **defined** as the ratio of the signal difference to the average signal

The rationale behind this is that a small difference is negligible if the average signal is **Large**, while the same small difference is readily visible if the average signal is **Small**

In general, in medical imaging, we will want to achieve the **highest** contrast possible to best visualize disease features

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4.3 CONTRAST

4.3.1 Definition

There are **two** common definitions of contrast in medical imaging

The **Weber Contrast**, or the Local Contrast, is defined as:

$$C = \frac{f_f - f_b}{f_b}$$

where f_f and f_b represent the signal of the feature and the background, respectively

Note: Contrast is defined in terms of the scene **f**

As we will see, it is equally acceptable to consider the contrast:

- Of the image **g**, or
- Measured at other points in the image chain

  such as the contrast of a feature displayed on a computer monitor

The **Weber Contrast** is commonly used in cases where small features are present on a large uniform background

4.3 CONTRAST

4.3.1 Definition

The Modulation or **Michelson Contrast** is commonly used for patterns where both bright and dark features take up similar fractions of the image

The **Modulation Contrast** is defined as:

$$C_M = \frac{f_{max} - f_{min}}{f_{max} + f_{min}}$$

where $\mathbf{f_{max}}$ and $\mathbf{f_{min}}$ represent the highest and lowest signals

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The Modulation Contrast has particular interest in the Fourier analysis of medical images

Consider a signal of the form:

$$f(x,y) = A + B\sin(2\pi ux)$$

Substituting into the **Modulation Contrast** gives:

$$C_M = \frac{A+B-(A-B)}{A+B+A-B} = \frac{B}{A}$$

Thus, we see that the **Numerator** expresses the amplitude or difference in the signal B = $(f_{max}-f_{min})/2$, while the **Denominator** expresses the average signal A = $(f_{max}+f_{min})/2$

4.3 CONTRAST
4.3.1 Definition

Care should be taken as to which definition of contrast is used

The correct choice is situation dependent

In general, the **Local Contrast** is used when a small object is presented on a uniform background, such as in simple observer experiments (e.g., 2-AFC experiments)

The **Modulation Contrast** has relevance in the Fourier analysis of imaging systems

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In medical imaging, the **Subject Contrast** is defined as the contrast (whether local or modulation) of the object in the scene being imaged

For example:

- In **X ray Imaging**, the subject contrast depends upon the X ray spectrum, and the attenuation of the object and background
- In **Radionuclide Imaging**, the subject contrast depends upon radiopharmaceutical uptake by the lesion and background, the pharmacokinetics, and the attenuation of the gamma rays by the patient

Similarly, one can define the subject contrast for CT, MRI and ultrasound

The **Image Contrast** depends upon the subject contrast and the characteristics of the imaging detector

For example:

In **Radiographic Imaging**, the image contrast is affected by

- the **X Ray Spectrum** incident upon the X ray converter (e.g. the phosphor or semiconductor material of the X ray detector)
- the converter **Composition** and **Thickness**, and
- the **Grayscale Characteristics** of the convertor, whether analogue or digital

The **Display Contrast** is the contrast of the image as displayed for final viewing by an observer

The Display Contrast is dependent upon:

- the **Image Contrast** and
- the **Grayscale Characteristics** of the display device and
- any **Image Processing** that occurs prior to or during display

In the absence of blurring, the ratio of the image contrast to the display contrast is defined as the **Transfer Function** of the imaging system

The grayscale response of **Film** is non-linear

Thus, to stay within the framework of LSI systems analysis, it is necessary to **Linearize** the response of the film

This is typically done using a small-signals model in which the low-contrast variations in the scene recorded in the X-ray beam, **$\Delta I/I_0$**, produce linear changes in the film density, **ΔD**, such that

$$\Delta D = \frac{\gamma \log_{10}(e) \Delta I}{I_0}$$

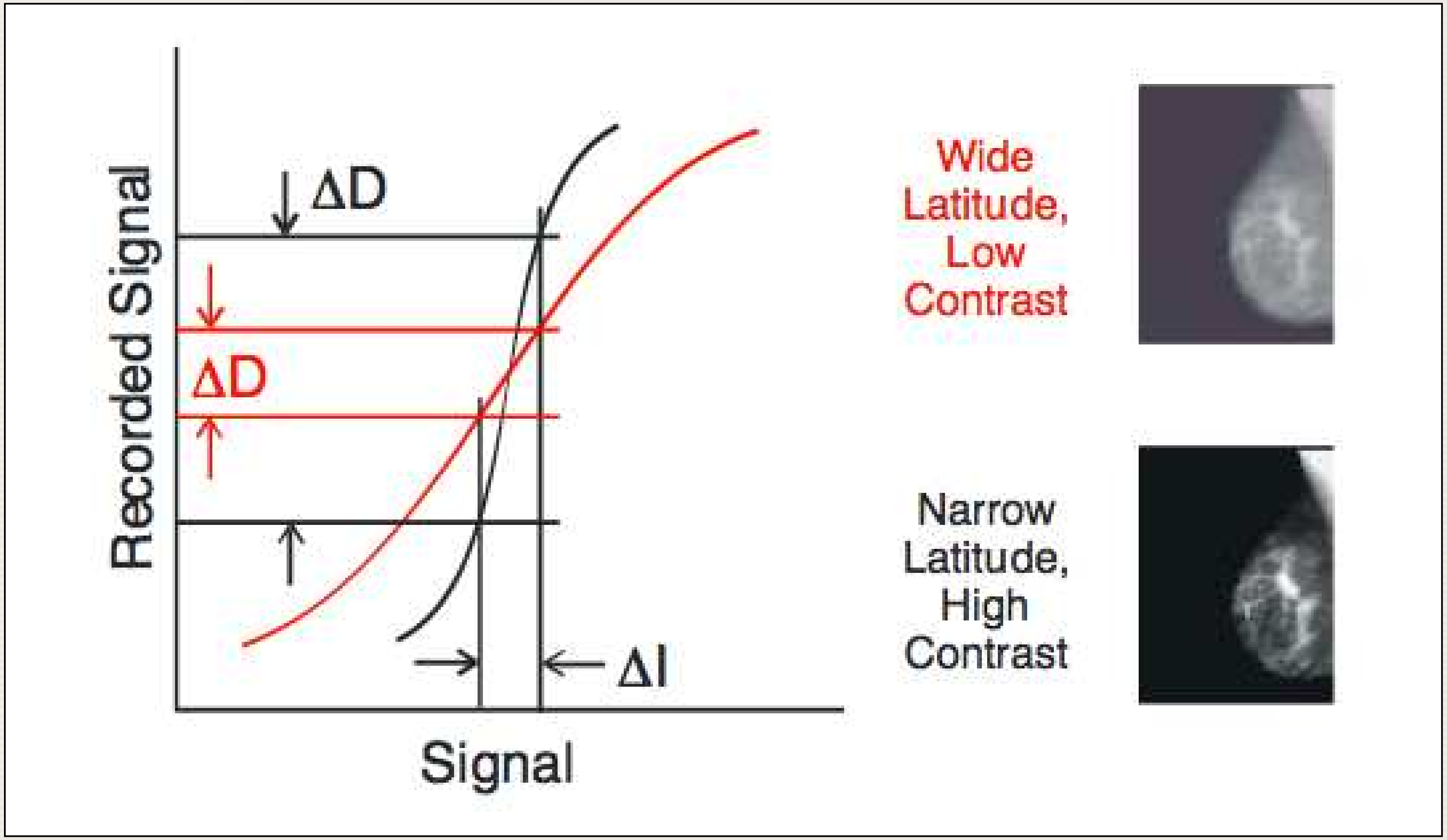
where γ is called the **Film Gamma**

and typically has a value of between 2.5 and 4.5

4.3 CONTRAST

4.3.3 Grayscale Characteristics

Two grayscale response functions are shown:



The **Grayscale Characteristic**, Γ, can now be calculated as:

$$\Gamma = \frac{\Delta D}{\Delta I} = \frac{\gamma \log_{10} e}{I_0}$$

In a similar fashion, the grayscale characteristic of a digital system with a **Digital Display** can be defined

In general, digital displays have a non-linear response with a **Gamma** of between 1.7 and 2.3

It should be noted that Γ does **not** consider the spatial distribution of the signals

In this sense, we can treat Γ as the response of a detector which records the incident X ray quanta, but does not record the **Location** of the X ray quanta

Equivalently, we can consider it as the **DC** (static) response of the imaging system

Given that the Fourier transform of a constant is equal to a Delta Function at zero spatial frequency

we can also consider this response to be the

Zero Spatial Frequency Response

of the imaging system

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4.4 UNSHARPNESS

In the preceding discussion of contrast, we considered **Large Objects** in the **Absence** of blurring

However, in general, we cannot ignore either assumption

When viewed from the spatial domain, blurring reduces contrast of small objects

The effect of blurring is to spread the signal laterally, so that a focused point is now a **Diffuse** point

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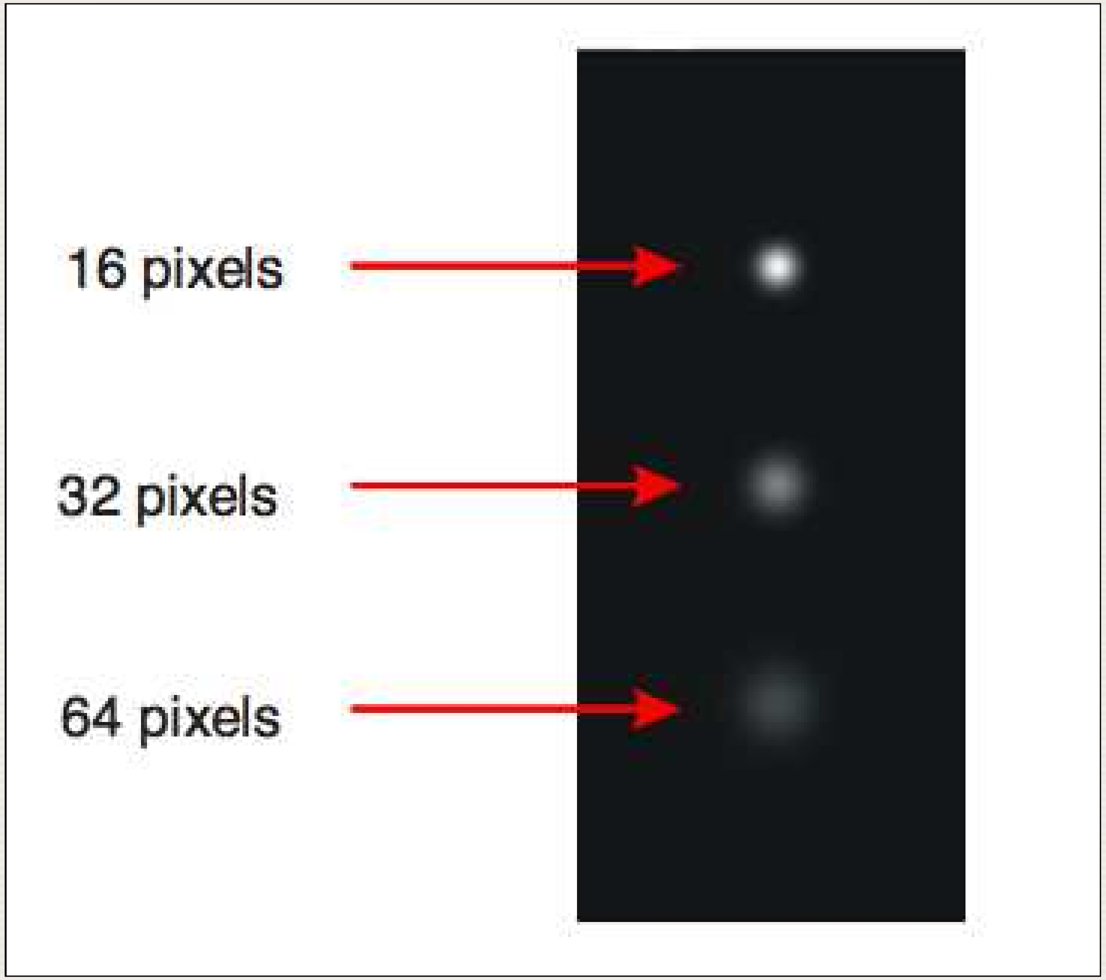
4.4 UNSHARPNESS

One fundamental property of blurring is that the more the signal is spread out, the lower its intensity, and thus the lower the contrast



An image of a point is shown blurred by convolution with a Gaussian kernel of diameter 16, 32 and 64 pixels

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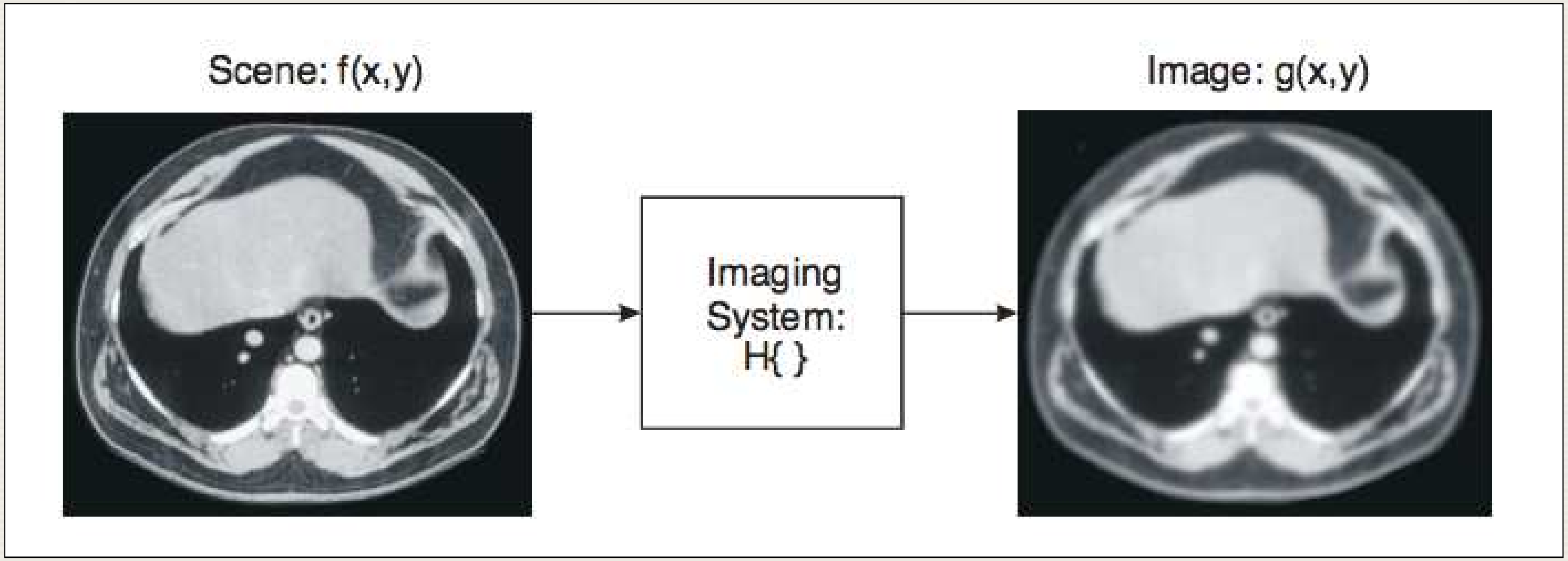
This also means that the peak signal is only degraded if the size of the object is **smaller** than the width of the blurring function

The contrast of larger objects will not be affected:



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Consider the operation of an **Impulse Function** on an imaging system

If an imaging system is characterized by a LSI response function h(x-x', y-y'), then this response can be measured by providing a **Delta Function** as input to the system

Setting f(x, y) = ∂(x, y) gives:

$$g(x,y) = \iint \delta(x,y)h(x-x',y-y')dx'dy' = h(x,y)$$

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We refer to the system transfer function as the point spread function, **PSF**, when specified in the spatial domain

In fact, the blurring of a point object, seen in images, is a pictorial display of the PSF

It is common to consider the PSF either as being:

- **Separable**

  or

- **Circular Symmetric**

$$h(x,y) = h(x)h(y)$$

$$h(r) = h(x,y)$$

where

$$r = \sqrt{x^2 + y^2}$$

depending upon the imaging system

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While it is possible to calculate the blurring of any object in the spatial domain via convolution with the LSI system transfer function, h, the problem is generally better approached in the **Fourier Domain**

To this end, it is informative to consider the effect of blurring on **Modulation Contrast**

Consider a sinusoidal modulation given by:

$$f(x,y) = A + B\sin(2\pi(ux + vy))$$

4.4 UNSHARPNESS

4.4.1 Quantifying Unsharpness

The recorded signal will be **degraded** by the system transfer function $\tilde{h}(u,v)$

such that

$$g(x,y) = A\tilde{h}(0,0) + B\left|\tilde{h}(u,v)\right| \sin(2\pi(ux + vy))$$

Here, any **Phase Shift** of the image relative to the scene is ignored for simplicity

We see, therefore, that the modulation contrast of object *f* is

$$C_f = \frac{B}{A}$$

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4.4.1 Quantifying Unsharpness

The Modulation Contrast of the image, g, is:

$$C_g = \frac{B|\tilde{h}(u,v)|}{A\tilde{h}(0,0)}$$

We can now define a new function, T, called the **Modulation Transfer Function** (MTF), which is defined as the absolute value ratio of C_g/C_f at a given spatial frequency (u, v)

$$T(u,v) = \frac{|\tilde{h}(u,v)|}{\tilde{h}(0,0)}$$

4.4 UNSHARPNESS

4.4.1 Quantifying Unsharpness

The MTF quantifies the degradation of the contrast of a system as a function of spatial frequency

By definition, the modulation at zero spatial frequency, **T(0,0)=1**

In the majority of imaging systems, and in the absence of image processing, the MTF is bounded by **$0 \leq T \leq 1$**

In addition, it should also be noted that based on the same derivation, the Grayscale Characteristic

$$\Gamma = \tilde{h}(0,0)$$

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4.4 UNSHARPNESS
4.4.1 Quantifying Unsharpness

The measurement of the 2D PSF for projection or cross-sectional imaging systems or 3D PSF for volumetric imaging systems (and hence the corresponding 2D or 3D MTF) requires that the imaging system be presented with an **Impulse Function**

In practice, this can be accomplished by imaging a pinhole in radiography, a wire in cross-section in axial CT, or a single scatterer in ultrasound

The knowledge of the MTF in 2D or 3D is useful in calculations in **Signal Detection Theory**

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4.4.1 Quantifying Unsharpness

It is more common, however, to measure the MTF in a **Single** dimension

In the case of **Radiography**, a practical method to measure the 1D MTF is to image a slit formed by two metal bars spaced closely together

Such a slit can be used to measure the **LSF**

Among other benefits, imaging a slit will provide better resilience to quantum noise, and multiple slit camera images can be superimposed (**Boot-Strapped**) to better define the tails of the LSF

The LSF is, in fact, an **Integral Representation** of the 2D PSF

For example, consider a slit aligned vertically in an image

which here we assume corresponds to the y-axis

Then the LSF, h(x) is given by:

$$h(x) = \int h(x,y)dy$$

The integral can be simplified if we assume that the PSF is **Separable**:

$$\int h(x)\mathrm{h}(y)dy = h(x)$$

as in video-based imaging systems

It should be clear from this that the LSF and the 1D MTF are Fourier transform **pairs**

If we assume a **Rotationally Symmetric** PSF, as might be found in a phosphor-based detector, the PSF is related to the LSF by the **Abel transform**:

$$h(x) = 2\int_x^{\infty} \frac{h(r)r}{\sqrt{x^2-r^2}}\,dr$$

and

$$h(r) = -\frac{1}{\pi}\frac{d}{dr}\left(\int_r^{\infty} \frac{h(x)r\,dx}{x\sqrt{x^2-r^2}}\right)$$

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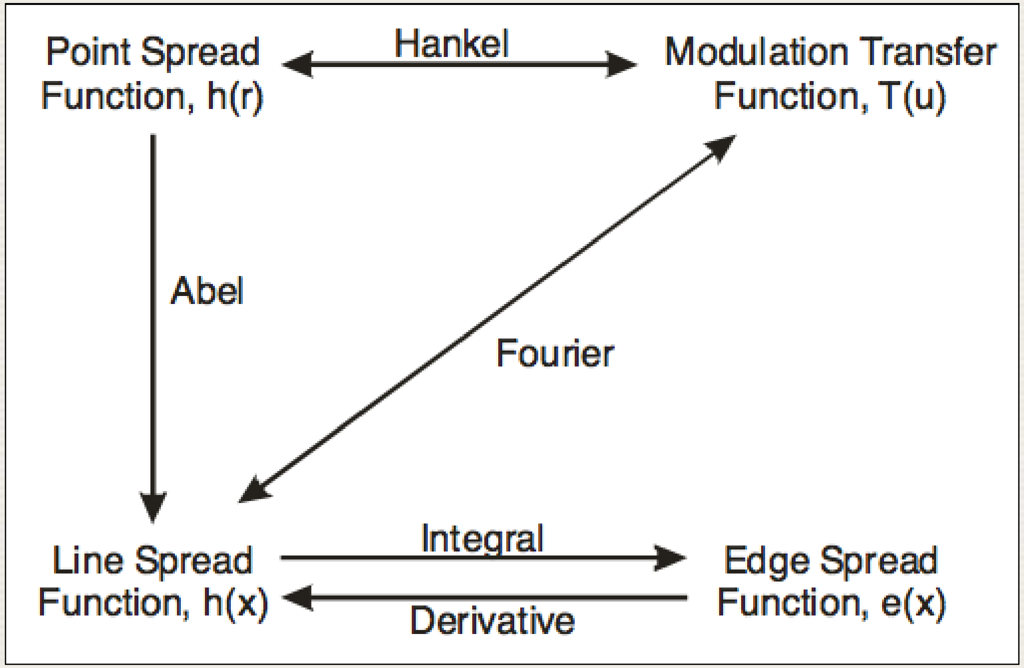
Note that while the forward Abel transform is **Tractable**, the inverse transform is not

However, the inverse transform can be calculated by first applying the Fourier transform and then the **Hankel** transform:

The 1D forms of the system response function are shown, together with the functional relationship



A further **Simplification** is to image an **Edge**, rather than a line

The **Edge Spread Function** (ESF) is simply an integral representation of the LSF, so that:

$$e(x) = \int_{-\infty}^{x} h(x)dx$$

and

$$h(x) = \frac{d}{dx} e(x)$$

Today, the ESF is the **Preferred Method** for measuring the system response function of radiographic systems

There are **Two** clear benefits:

- an edge is **Easy** to produce for almost any imaging system, although issues such as the position of the edge need to be carefully considered
- the ESF is amenable to measuring the **Pre-Sampled MTF** of digital systems

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Limiting Spatial Resolution

The spatial resolution is a metric to quantify the ability of an imaging system to display two unique objects closely separated in space

The limiting spatial resolution is typically defined as the maximum spatial frequency for which modulation is preserved without distortion or aliasing

Limiting Spatial Resolution

The limiting resolution can be measured by:

- Imaging line patterns or star patterns in radiography and
- Arrays of cylinders imaged in cross-section in cross-sectional imaging systems such as CT and ultrasound

All of these methods use high-contrast, sharp-edged objects

Limiting Spatial Resolution

As such the limiting spatial resolution is typically measured in **Line Pairs** per unit length

This suggests that the basis functions in such patterns are **Rect Functions**

By contrast, the MTF is specified in terms of **Sinusoids**

This is specified in terms of spatial frequencies in **Cycles** per unit length

Limiting Spatial Resolution

There is no strict relationship between a particular MTF value and the limiting spatial resolution of an imaging system

The **Coltman Transform** can be used to relate the:

Square Wave Response

measured with a bar or star pattern

and

the Sinusoidal Response measured by the **MTF**

Limiting Spatial Resolution

Ultimately, however, the ability to detect an object (and hence resolve it from its neighbour) is related to the **Signal to Noise Ratio** of the object

As a **Rule of Thumb**, the limit of resolution for most imaging systems for high-contrast objects (e.g., a bar pattern) occurs at the spatial frequency where the

$$\mathbf{MTF \approx 0.05\ (5\%)}$$

Modulation Transfer Function (MTF)

In practice, it is difficult to measure the MTF of an analogue system (such as film) **without** first digitizing the analogue image

As such, it is important that the digitization process satisfies the **Nyquist-Shannon** sampling theorem to avoid aliasing

This is possible in some instances, such as digitizing a film, where the digitizer optics can be designed to eliminate **Aliasing**

Modulation Transfer Function (MTF)

In this instance, however, the MTF that is measured is not the MTF of the film but rather is given by:

$$T_m = T_a T_d$$

where $\mathbf{T_m}$ is the measured MTF, $\mathbf{T_a}$ is the MTF of the analogue system, and $\mathbf{T_d}$ is the MTF of the digitizer

With this equation, it is possible to recover $\mathbf{T_a}$ provided $\mathbf{T_d} > 0$ over the range of frequencies of interest

Modulation Transfer Function (MTF)

In many systems, however, it is **not** possible to avoid aliasing

For example, in a DR detector that consists of an a-Se photoconductor coupled to a TFT array

The Selenium has Very High Limiting Spatial Resolution

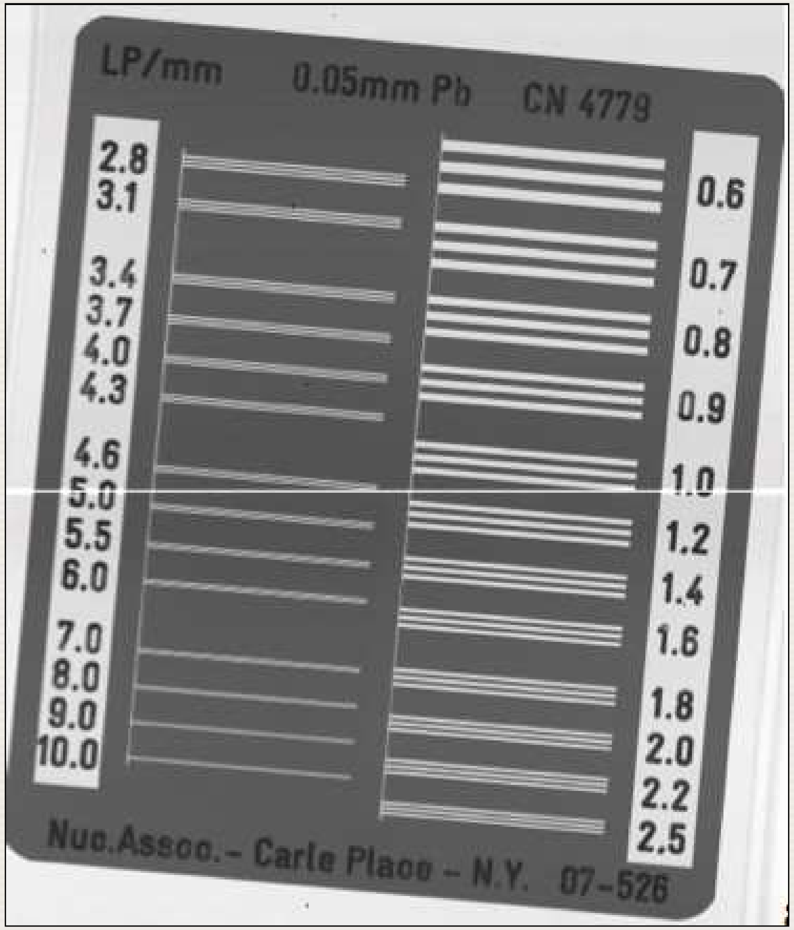
much higher than can be supported by the pixel pitch of the detector

Modulation Transfer Function (MTF)

This resolution pattern is made with such a system:

A digital radiograph of a bar pattern is shown

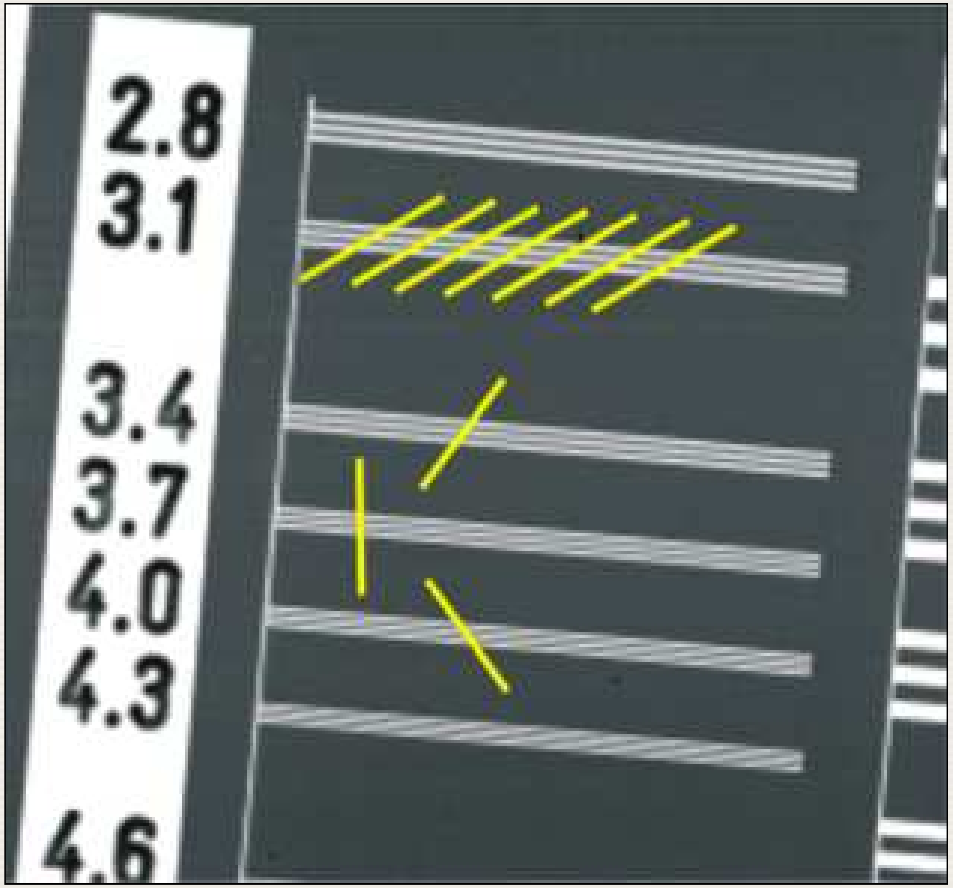
Each group in the pattern (e.g. 0.6 lp/mm) contains **three** equally spaced elements



Modulation Transfer Function (MTF)

A **magnified** region of the pattern is shown:

Here, we can deduce that the limiting resolution is **3.4 lp/mm**



Higher frequencies are **aliased** as shown by the reversal of the bands (highlighted in yellow) which arise from the digital sampling process

Modulation Transfer Function (MTF)

In such instances, there are some important facts to understand

First, aliasing will occur with such a system, as can be seen

It is Unavoidable

This means, that predicting the exact image recorded by a system requires knowledge of the:

- **Location** of the objects in the scene relative to the detector matrix with sub-pixel precision, as well as the
- **Blurring** of the system prior to sampling

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Modulation Transfer Function (MTF)

The latter can be determined by measuring what is known as the **Pre-Sampling MTF**

The pre-sampling MTF is measured using a high sampling frequency so that **No Aliasing** is present in the measurement

It is important to realise that in spite of its name, the pre-sampling MTF does include the blurring effects of the sampling aperture

Modulation Transfer Function (MTF)

The pre-sampling MTF measurement starts with imaging a well-defined edge placed at a small angle (1.5°- 3°) to the pixel matrix/array

From this digital image, the exact angle of the edge is detected and the distance of individual pixels to the edge is computed to construct a **Super-Sampled** (SS) edge spread function (ESF)

Differentiation of the SS-ESF generates a LSF, whose FT gives the MTF

This is the Preferred Method for Measuring the MTF Today

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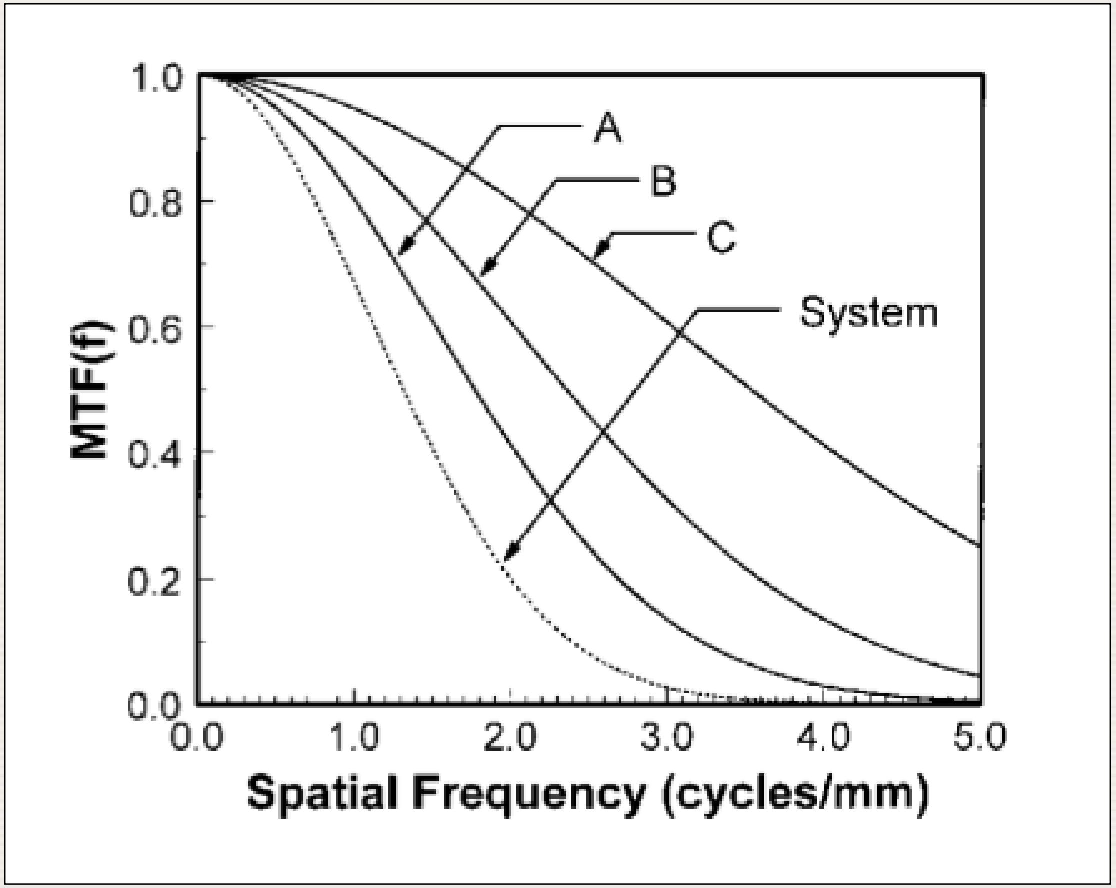
4.4.3 Resolution of a Cascaded Imaging System

In the previous section, we dealt with the special situation in which an analogue image, such as film, is digitized by a device such as a scanning photometer

In this situation, the measured MTF is the **Product** of the film MTF and the MTF of the scanning system

This principle can be extended to more generic imaging systems which are composed of a **Series** of individual components

Example of how a system MTF is the product of its components:



The overall or system MTF is the product of the MTFs of the three components A, B and C

A **Classic Example** is to compare the blurring of the focal spot and imaging geometry with that of the detector

Another classic example is of a video fluoroscopic detector containing an X ray image intensifier

In this instance, the MTF of the image is determined by the MTFs of the:

- Image intensifier
- Video camera
- Optical coupling

This is true because the image passes sequentially through each of the components, and each successive component sees an increasingly blurred image

The **One Caveat** to this concept is that aliasing must be addressed very carefully once sampling has occurred

The principle of **Cascaded Systems Analysis** is frequently used, as it:

- Allows one to determine the impact of each component on spatial resolution, and
- Provides a useful tool for analysing how a system design can be improved

4.5 NOISE

The Greek philosopher **Heraclitus** (c. 535 B.C.) is claimed to have said that:

“You cannot step twice into the same river"

It can similarly be asserted that one can never acquire the same image twice

There Lies the Fundamental Nature of Image Noise

4.5 NOISE

Noise arises as **Random** variations in the recorded signal (e.g. the number of X-ray quanta detected) from pixel-to-pixel

Noise is Not Related to Anatomy

Rather, it arises from the **random** generation of the image signal

Note, however, that noise is related for example to the number of X ray quanta; thus, highly attenuating structures (like bones) will look noisier than less attenuating structures

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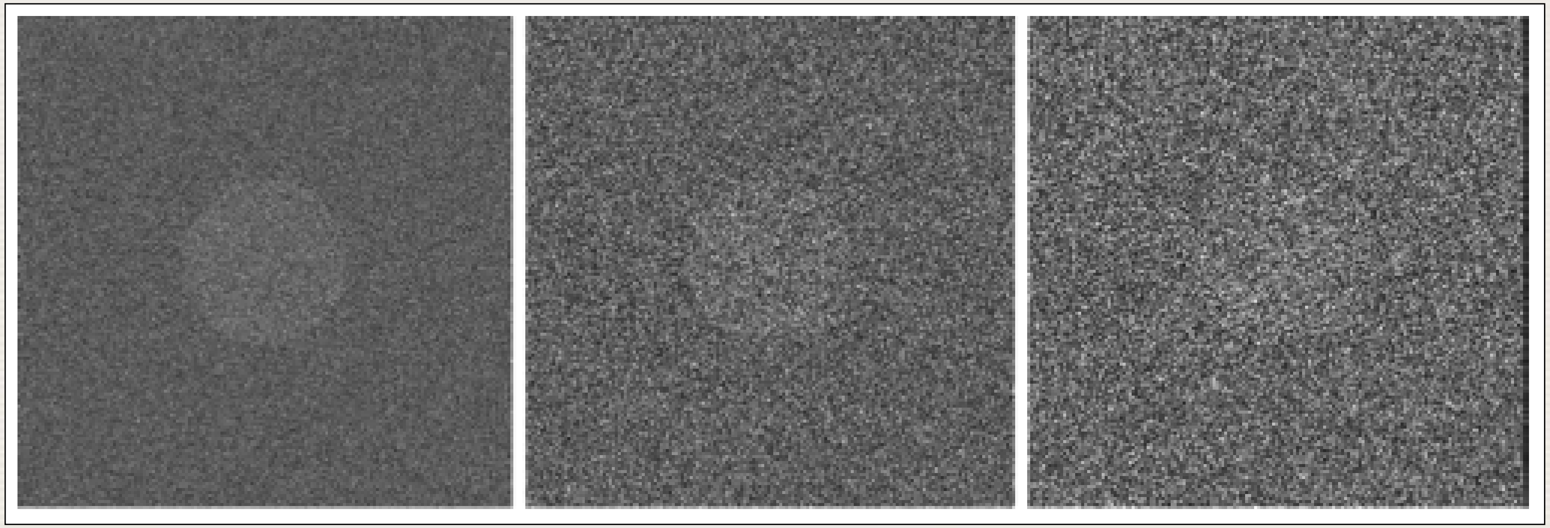
4.5 NOISE

In a well-designed X-ray imaging system, X-ray quantum noise will be the **Limiting Factor** in the detection of objects



The ability to detect an object is dependent upon both the contrast of the object and the noise in the image

As illustrated, the ability to discern the disk is degraded as the magnitude of the noise is increased

4.5 NOISE

The optimal radiation dose is just sufficient to visualize the anatomy or disease of interest, thus minimizing the potential for harm

In a seminal work, **Albert Rose** showed that the ability to detect an object is related to the ratio of the signal to noise

We shall return to this important result

However, first we must learn the **Fundamentals** of image noise

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4.5 NOISE

4.5.1 Poisson Nature of Photons

The process of generating X-ray quanta is **Random**

The intrinsic fluctuation in the number of X-ray quanta is called X-ray **Quantum Noise**

X-ray quantum noise is **Poisson** distributed

In particular, the probability of observing **n** photons given **α**, the mean number of photons, is

$$P(n, \alpha) = \alpha^n e^{-\alpha} / n!$$

where **α** can be any positive number and **n** must be an integer

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4.5 NOISE

4.5.1 Poisson Nature of Photons

A fundamental principle of the Poisson distribution is that the variance, σ^2, is **Equal** to the mean value, α

When dealing with large mean numbers, most distributions become approximately **Gaussian**

This applies to the Poisson distribution when a large number of X-ray quanta (e.g. >50 per del) are detected

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The **Mean-Variance Equality** for X-ray quantum noise limited systems is useful experimentally

For Example, it is useful to test whether the images recorded by a system are limited by the X-ray quantum noise

Such systems are said to be X-ray quantum noise limited, and the X-ray absorber is called the **Primary Quantum Sink**

to imply that the **Primary** determinant of the image noise is the **Number** of X-ray quanta recorded

In the mean-variance experiment, one measures the mean and standard deviation **Parametrically** as a function of dose

When plotted **Log-Log**, the slope of this curve should be **1/2**

When performed for digital X-ray detectors, including CT systems, this helps to determine the range of air kerma or detector dose over which the system is X-ray **Quantum Noise Limited**

Image noise is said to be **Uncorrelated** if the value in each pixel is independent of the values in neighbouring pixels

If this is true and the system is **Stationary** and **Ergodic**, then it is trivial to achieve a complete characterization of the system noise

One simply needs to calculate the **Variance** (or Standard Deviation) of the image on a per-pixel basis

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4.5.2 Measures of Variance & Correlation/Co-variance

Uncorrelated noise is called **White Noise** because all spatial frequencies are represented in equal amounts

All X-ray noise in images starts as white noise, since the production of X-ray quanta is **Uncorrelated** both in time and in space

Thus, the probability of creating an X-ray at any point in time and any particular direction does not depend on the previous quanta which were generated, nor any subsequent quanta

4.5.2 Measures of Variance & Correlation/Co-variance

Unfortunately, it is **Rare** to find an imaging system in which the resultant images are uncorrelated in space

This arises from the fact that each X-ray will create multiple **Secondary Carriers** which are necessarily correlated, and these carriers diffuse from a single point of creation

Thus the signal recorded from a single X-ray is often **Spread** among several pixels

As a result the pixel variance is reduced and neighbouring pixel values are **Correlated**

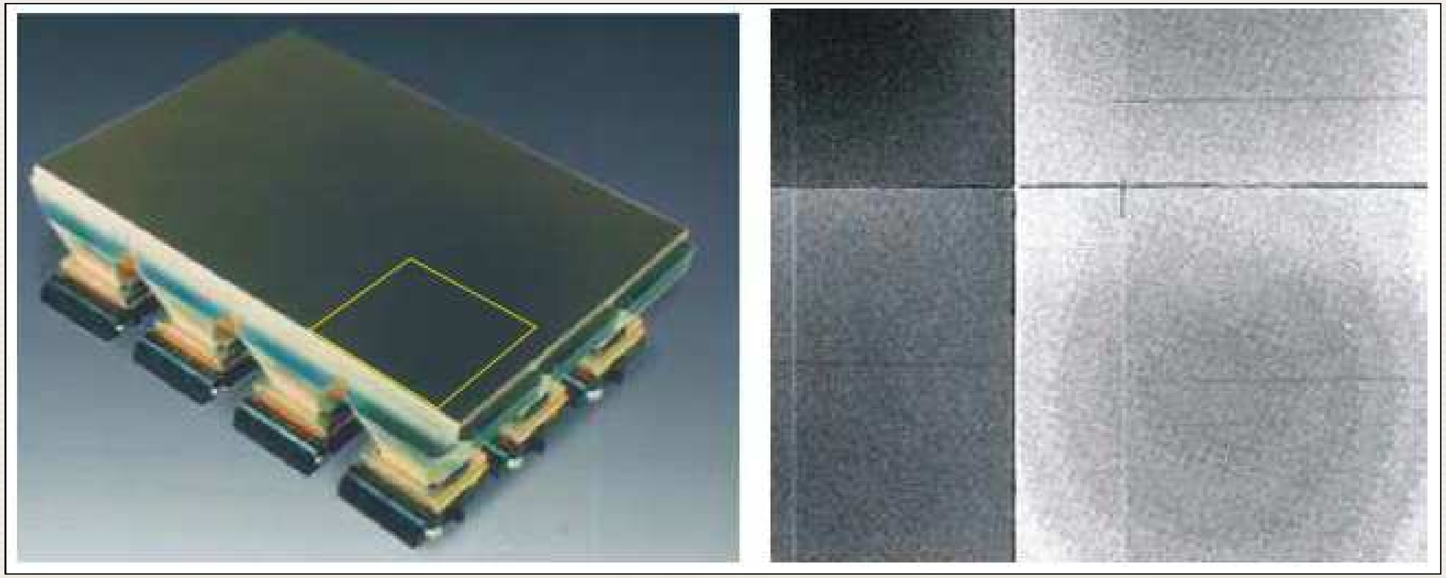
Noise can also be correlated by spatial non-uniformity in the imaging system - that is, **Non-Stationarity**

In most real imaging systems, the condition of stationarity is only **Partially** met

One is often placed in a situation where it must be decided if the stationarity condition is sufficiently met to treat the system as **Shift Invariant**

An Example:

An early digital X-ray detector prototype is shown which consisted of a phosphor screen coupled to an array of fibre-optic tapers and CCD cameras



The image on the right is a measurement of the per-pixel variance on a small region indicated by yellow on the detector face

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4.5.2 Measures of Variance & Correlation/Co-variance

The image on the right is a **Variance Image**, obtained by estimating the variance in each pixel using multiple images

i.e. multiple realizations from the ensemble

The image shows that there are strong spatial variations in the variance due to:

- Differences in the **Coupling Efficiency** of the fibre optics and the
- **Sensitivity** differences of the CCDs

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4.5.2 Measures of Variance & Correlation/Co-variance

Noise can be characterized by the **Auto Correlation** at each point in the image, calculated as the ensemble average:

$$R(x,y,x+\Delta x,y+\Delta y) = \langle \dot{g}(x,y)\dot{g}(x+\Delta x,y+\Delta y) \rangle$$

Here, we use the notation **ġ** to denote that **g** is a random variable

Correlations about the mean

$$\Delta\dot{g}(x,y) = \dot{g}(x,y) - \langle \dot{g}(x,y) \rangle$$

are given by the **Autocovariance Function**

$$K(x,y,x+\Delta x,y+\Delta y) = \langle \Delta\dot{g}(x,y)\Delta\dot{g}(x+\Delta x,y+\Delta y) \rangle$$

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4.5.2 Measures of Variance & Correlation/Co-variance

Based on the assumption of **Stationarity**,

$$\langle \dot{\mathbf{g}}(x, y) \rangle = \mathbf{g},$$

is a constant independent of position

If the random process is **Wide-Sense Stationary**, then both the autocorrelation and the autocovariance are independent of position (x, y) and only dependent upon displacement

$$R(\Delta x, \Delta y) = R(x, y, x + \Delta x, y + \Delta y)$$

$$K(\Delta x, \Delta y) = K(x, y, x + \Delta x, y + \Delta y)$$

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If the random process is **Ergodic**, then the ensemble average can be replaced by a spatial average

Considering a digital image of a stationary ergodic process, such as incident X-ray quanta, the autocovariance forms a matrix

$$\boldsymbol{K}(\Delta x, \Delta y) = \frac{1}{2X}\frac{1}{2Y}\sum_{-X}^{+X}\sum_{-Y}^{+Y}\boldsymbol{g}(\mathrm{x},\mathrm{y})\boldsymbol{g}(x+\Delta \mathrm{x}, y+\Delta \mathrm{y})$$

where the region over which the calculation is applied is **2X × 2Y** pixels

The value of the **Autocovariance** at the origin is equal to the variance:

$$\mathbf{K}(0,0) = \langle \Delta \dot{\boldsymbol{g}}(x,y) \Delta \dot{\boldsymbol{g}}(x,y) \rangle = \sigma_A^2$$

where the subscript **A** denotes that the calculation is performed over an aperture of area **A**, typically the pixel aperture

The correlation of noise can be determined in either the:

- **Spatial Domain** using autocorrelation (as we have seen in the previous section) or
- **Spatial Frequency Domain** using Noise Power Spectra (**NPS**)

also known as **Wiener Spectra**

There are a number of requirements which must be met for the NPS of an imaging system to be tractable

These include: **Linearity**, **Shift-Invariance**, **Ergodicity** and **Wide-Sense Stationarity**

In the case of digital devices the latter requirement is replaced by Wide-Sense **Cyclo-Stationarity**

If the above criteria are met, then the NPS completely describes the noise properties of an imaging system

4.5 NOISE

4.5.3 Noise Power Spectra

In point of fact, it is **Impossible** to meet all of these criteria **Exactly**

For Example, all practical detectors have finite size and thus are not strictly stationary

However, in spite of these limitations, it is generally possible to calculate the **Local** NPS

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By **Definition**, the NPS is the ensemble average of the square of the Fourier transform of the spatial density fluctuations

$$W(u,v) = \langle \lim_{X,Y\to\infty} \frac{1}{2X}\frac{1}{2Y}\left|\int_{-X}^{+X}\int_{-Y}^{+Y} \Delta\dot{g}(x,y)e^{-2\pi i(ux+vy)}dxdy\right|^2 \rangle$$

The NPS and the autocovariance function form a **Fourier Transform Pair**

This can be seen by taking the Fourier transform of the autocovariance function and applying the convolution theorem

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The NPS of a discrete random process, such as when measured with a **Digital** X-ray detector, is:

$$\boldsymbol{W}(u,v) = \left\langle \lim_{N_x N_y \to \infty} \frac{x_0}{N_x} \frac{y_0}{N_y} \left| \sum_{x,y} \Delta\dot{\boldsymbol{g}}(x,y) e^{-2\pi i(ux+vy)} \right|^2 \right\rangle$$

This equation requires that we perform the summation over **all** space

In practice, this is impossible as we are dealing with detectors of limited extent

By restricting the calculation to a **finite** region, it is possible to determine the Fourier content of the fluctuations in that specific region

We call this simple calculation a **Sample Spectrum**

It represents one possible instantiation of the noise seen by the imaging system, and we denote this by:

$$\dot{W}(u,v) = \frac{x_0}{N_x}\frac{y_0}{N_y}\left|\sum_{m,n} \Delta\dot{g}(x,y)e^{-2\pi i(umx_0+vny_0)}\right|^2$$

An estimate of the true NPS is created by **averaging** the sample spectra from M realizations of the noise

$$\ddot{W}(u,v) = \frac{1}{M}\sum_{i=1}^{M} \dot{W}_i(u,v)$$

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4.5.3 Noise Power Spectra

Ideally, the average should be done by calculating sample spectra from **Multiple Images** over the same region of the detector

However, by assuming Stationarity and Ergodicity, we can take averages over **Multiple Regions** of the detector

significantly reducing the number of images that we need to acquire

4.5 NOISE

4.5.3 Noise Power Spectra

Now, the estimate of the NPS, $\ddot{\mathbf{W}}$, has an accuracy that is determined by the number of samples used to make the estimate

Assuming Gaussian statistics, at frequency (u, v), the error in the estimate $\ddot{\mathbf{W}}(x, y)$ will have a standard error given by:

$$\sqrt{c/_{M}}\,\ddot{W}(u,v)$$

where c=2 for u=0 or v=0, and c=1 otherwise

The values of c arise from the circulant nature of the Fourier transform

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4.5.3 Noise Power Spectra

Typically, **64** x **64** pixel regions are sufficiently large to calculate the NPS

Approximately **1000** such regions are needed for good 2-D spectral estimates

Remembering that the autocorrelation function and the NPS are Fourier transform pairs, it follows from **Parseval's Theorem** that

$$\boldsymbol{K}(0,0) = \frac{1}{x_0 y_0 N_x N_y} \sum_{u,v} \ddot{\boldsymbol{W}}(u,v)$$

This provides a useful and rapid method of **verifying** a NPS calculation

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There are **Many Uses** of the NPS

It is most commonly used in characterizing imaging device **Performance**

In particular, the NPS is exceptionally valuable in investigating **Sources** of detector noise

For Example, poor grounding often causes line-frequency (typically 50 or 60 Hz) noise or its harmonics to be present in the image

NPS facilitates the identification of this noise

In such applications, it is common to calculate

Normalized Noise Power Spectra (NNPS)

since the absolute noise power is less important than the relative noise power

As we shall see, absolute calculations of the NPS are an integral part of **DQE** and **NEQ** measurements, and the NPS is required to calculate the **SNR** in application of signal-detection theory

Unlike the MTF, there is no way to measure the

Pre-Sampling NPS

As a result, high frequency quantum noise (frequencies higher than supported by the sampling grid) will be aliased to lower frequencies in the same way that high frequency signals are aliased to lower frequencies

Radiation detectors with high spatial resolution, such as **a-Se Photoconductors**, will naturally alias high frequency noise

4.5.3 Noise Power Spectra

Radiation detectors based on **Phosphors** naturally blur both the signal and the noise prior to sampling

And thus can be designed so that both signal and noise aliasing are not present

There is **No Consensus** as to whether noise aliasing is beneficial or detrimental

Ultimately, the role of noise aliasing is determined by the imaging task, as we shall see later

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As with the MTF, it is sometimes preferable to display 1D sections through the 2D (or 3D) noise power spectrum or autocovariance

There are two presentations which are used:

the **Central Section** $$W_C(u) = W(u,0)$$

and

the **Integral Form** $$W_I(u) = \sum_v W(u,v)$$

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4.5 NOISE

4.5.3 Noise Power Spectra

Similarly, if the noise is **Rotationally Symmetric**, the noise can be averaged in annular regions and presented radially

The choice of presentation depends upon the intended use

It is most **Common** to present the central section

Regardless, the various 1D presentations are easily related by the central slice theorem, as shown in the next slide

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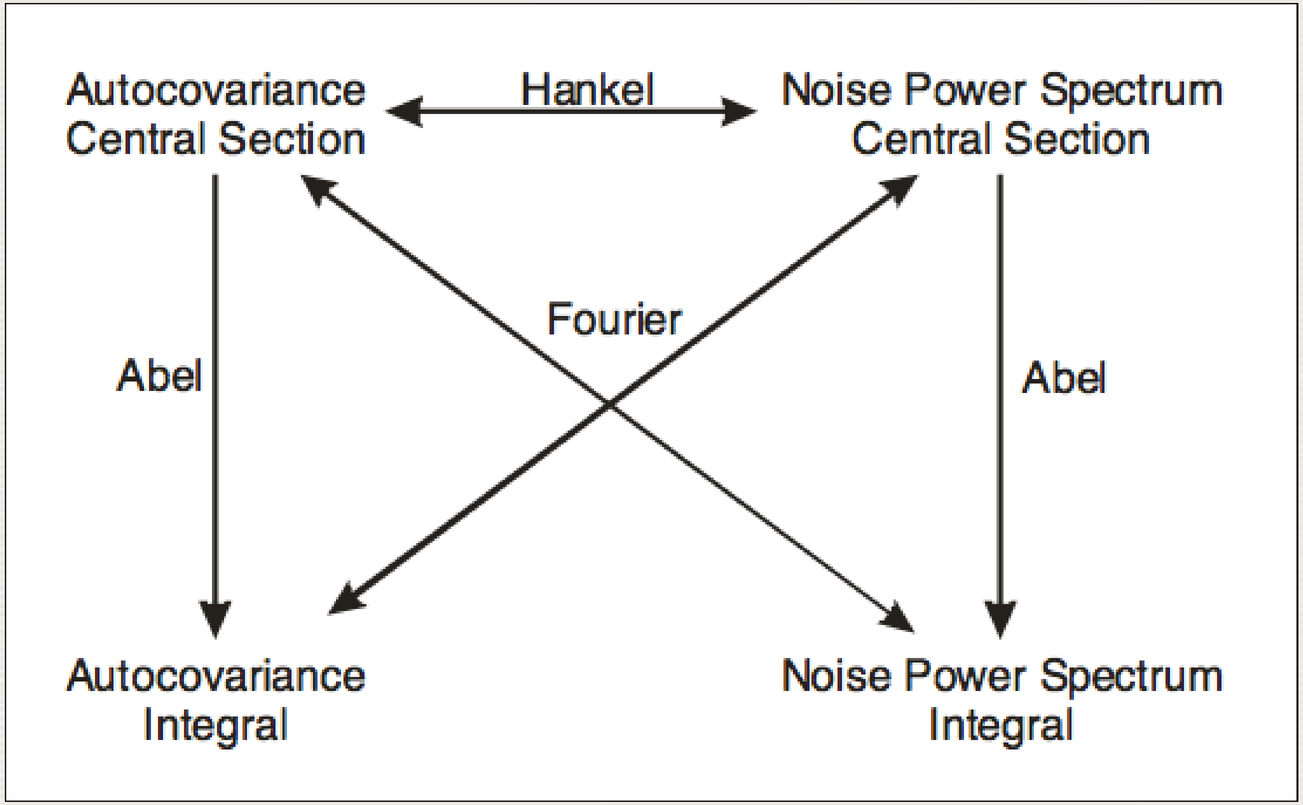
Both 1D integral and central sections of the NPS and autocovariance can be presented

The various presentations are related by integral (or discrete) transformations



Here, the relationships for rotationally symmetric 1D noise power spectra and autocovariance functions are shown

4.5.4 NPS of a Cascaded Imaging System

The propagation or cascade of noise is substantially more complicated than the composition of the MTF

A proper analysis of noise must account for the **Correlation** of the various noise sources

These can be numerous, including:

- The primary X-ray **Quantum Noise**
- The noise arising from the **Transduction** of the primary quanta into secondary quanta (such as light photons in a phosphor or carriers in a semiconductor)
- Various **Additive Noise Sources** such as electronic noise from the readout circuitry of digital detectors

While the general theory of noise propagation is beyond the scope of this work, the **two** simple examples which follow may be illustrative:

- **Image Subtraction**
- **Primary & Secondary Quantum Noise**

Image Subtraction

It is common to **Add** or **Subtract** or otherwise manipulate medical images

A classic example is

Digital Subtraction Angiography (DSA)

In DSA, a projection image with contrast agent is subtracted from a pre-contrast mask image to produce an image that shows the **Difference** in attenuation between the two images which arises from the contrast agent

Image Subtraction

Strictly speaking, the **Logarithms** are subtracted

In the absence of patient motion, the resultant image depicts the

Contrast Enhanced Vascularity

Image Subtraction

The effect of the subtraction is to **Increase** the image noise

This arises because for a given pixel in the image, the pixel values in the mask and the contrast-enhanced images are **Uncorrelated**

As a result, the subtraction incorporates the noise of **Both** images

Image Subtraction

Noise adds in **Quadrature**, thus the noise in the subtracted image is **√2** larger than the noise is in the source images

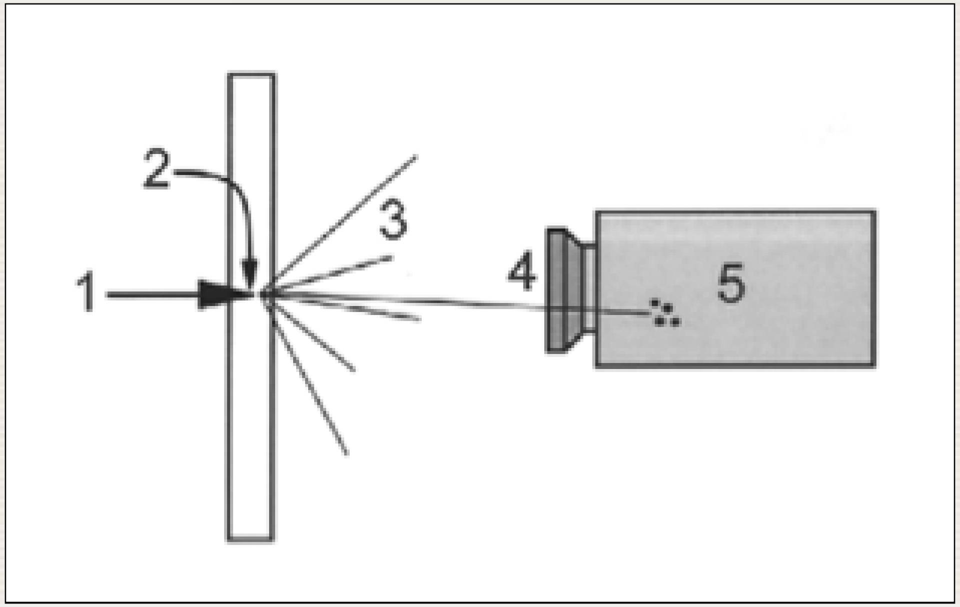
To **Ameliorate** the noise increase in the subtraction image, it is typical to

Acquire the Mask Image at Much Higher Dose

thereby reducing the contribution of the mask noise to the subtraction

Primary & Secondary Quantum Noise

Consider this simple imaging system:



The concept of **Quantum Accounting** is illustrated

A simple X-ray detector is shown

At each stage of the imaging system, the number of quanta per incident X-ray is calculated to determine the **Dominant** noise source

Primary & Secondary Quantum Noise

In this system:

- X-ray quanta are incident on a phosphor screen (**Stage 1**)
- A fraction of those quanta are absorbed to produce light (**Stage 2**)
- A substantial number of light quanta (perhaps 300-3000) are produced per X-ray quantum (**Stage 3**)
- A small fraction of the light quanta are collected by the lens (**Stage 4**)
- A fraction of the collected light quanta produce carriers in the optical image receptor (e.g. a CCD camera) (**Stage 5**)

Primary & Secondary Quantum Noise

The process of producing an electronic image from the source distribution of X-rays will **necessarily** introduce noise

In fact, **each stage** will alter the noise of the resultant image

In this simple model, there are two primary sources of noise:

- X-ray (or **Primary**) quantum noise
- **Secondary** quantum noise

Primary & Secondary Quantum Noise

Secondary Quantum Noise - noise arising from:

- **Production** of light in the phosphor
- **Transmission** of light through the optical system and
- **Transduction** of light into signal carriers in the optical image receptor

Both the light quanta and signal carriers are:

Secondary Quanta

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Primary & Secondary Quantum Noise

Each stage involves a **Random** process

The generation of X-ray quanta is governed by a **Poisson** process

In general, we can treat the generation of light quantum from individual X-ray quanta as being **Gaussian**

Stages 3 - 5 involve the selection of a fraction of the secondary quanta and thus are governed by **Binomial** processes

Primary & Secondary Quantum Noise

The **Cascade** of these processes can be calculated mathematically

However, a simple approach to estimating the dominant noise source in a medical image is to determine the number of quanta at each stage of the imaging cascade:

the Stage with the Minimum Number of Quanta will be the Dominant Noise Source

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4.6 ANALYSIS OF SIGNAL & NOISE

4.6.1 Quantum Signal-to-Noise Ratio

There is a fundamental **Difference** between the high-contrast and low-contrast resolution of an imaging system

In general, the high-contrast resolution is limited by the intrinsic blurring of the imaging system

At some point, the system is unable to resolve two objects that are separated by a short distance, instead portraying them as a single object

However, at **Low Contrast**, objects (even very large objects) may not be discernible because the signal of the object is substantially **Lower** than the noise in the region containing the object

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4.6 ANALYSIS OF SIGNAL & NOISE

4.6.1 Quantum Signal-to-Noise Ratio

Generally, the Signal-to-Noise Ratio (**SNR**) is defined as the inverse of the Coefficient of Variation:

$$SNR = \langle g \rangle / \sigma_g$$

where $\langle g \rangle$ is the **mean** value

σ_g the **standard deviation**

This definition of the SNR requires that a single pixel (or region) be measured repeatedly over various images (of the ensemble), provided that each measurement is **Independent** (i.e. there is no correlation with time)

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4.6 ANALYSIS OF SIGNAL & NOISE

4.6.1 Quantum Signal-to-Noise Ratio

In an **Ergodic** system, the ensemble average can be replaced by an average over a region

This definition is of value for photonic (or quantum) noise because in a uniform X-ray field, X-ray quanta are **Not** spatially correlated

However, most imaging systems do blur the image to some degree, and hence introduce **Correlation** in the noise

As a result, it is **Generally Inappropriate** to calculate pixel noise by analysing pixel values in a region for absolute noise calculations

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4.6 ANALYSIS OF SIGNAL & NOISE

4.6.1 Quantum Signal-to-Noise Ratio

The **Definition** of SNR as

$$SNR = \langle g \rangle / \sigma_g$$

is only useful when the image data are always positive, such as photon counts or luminance

In systems where **Positivity** is not guaranteed, such as an ultrasound system, the SNR is defined as the **Power Ratio**, and is typically expressed in decibels:

$$SNR_{dB} = 10 \log_{10} \frac{P_s}{P_n} = 10 \log_{10} \left(\frac{A_s}{A_n} \right)^2 = 20 \log_{10} \frac{A_s}{A_n}$$

where **P** is the average power and **A** is the root mean square amplitude of the signal, **s**, or noise, **n**

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Based on the work of **Albert Rose**, it is clear that the image quality of X-ray imaging systems is determined by the number of quanta used to produce an image

This leads to the definition of the **DQE**:

A Measure of the Fraction of the Quantum SNR of the Incident Quanta that is Recorded in the Image by an Imaging System

Thus, the DQE is a measure of the **Fidelity** of an imaging system

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It is common to **Define** the DQE as:

$$DQE = \frac{SNR_{out}^2}{SNR_{in}^2}$$

where the SNR^2 of the image is denoted by the subscript **out** and the SNR^2 of the **Incident** X-ray quanta is given by:

$$SNR_{in}^2 = \phi$$

where ϕ is the **Average** number of X-ray quanta incident on the detector

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4.6.2 Detective Quantum Efficiency

Rodney Shaw introduced the concept of the DQE to medical imaging and also introduced the term **Noise-Equivalent Quanta** (NEQ)

The **NEQ** is the effective number of quanta needed to achieve a specific SNR in an ideal detector

We can write: $DQE = \frac{NEQ}{\phi}$ so that $NEQ = SNR_{out}^2$

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In some sense,

the **NEQ** denotes the **Net Worth** of the image data in terms of X-ray quanta

and

the **DQE** defines the **Efficiency** with which an imaging system converts X-ray quanta into image data

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This definition of DQE is the **Zero Spatial Frequency** value of the DQE

Zero spatial frequency refers to a detector that counts X-ray quanta but does not produce a pixelated image

i.e. we only care about the **Efficiency** of counting the X-ray quanta

Thus ϕ is a simple count of the X-ray quanta incident on the detector

By this definition, an imaging system which:

- Perfectly absorbs each X-ray and
- Does not introduce any other noise

will **Perfectly Preserve** the SNR of the X-ray quanta

and hence:

$$\mathbf{NEQ} = \phi$$

and

$$\mathbf{DQE} = 1$$

If we consider an X-ray detector that is **Perfect** in every way **Except** that the quantum detection efficiency **η**<1.0, we observe that while the incident number of quanta is again ϕ, only **η**ϕ quanta are absorbed

As a result:

$$\mathbf{NEQ} = \eta\phi \text{ and } \mathbf{DQE} = \eta$$

Thus, in this special instance, the DQE is equal to the quantum detection efficiency, **η**, the efficiency with which X-ray quanta are absorbed in the detector

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The **DQE** can be more generally expressed in terms of spatial frequencies:

$$DQE(u,v) = \frac{SNR_{out}^2(u,v)}{SNR_{in}^2(u,v)}$$

where we rely upon the property that X-ray quantum noise is **White**, leading to the SNR_{in} being a constant

$$SNR_{in}^2(u,v) = \Phi$$

Here, Φ is the **Photon Fluence**

and has units of **Inverse Area**

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$$DQE(u,v) = \frac{NEQ(u,v)}{\Phi}$$

The **DQE(u, v)** tells us how well the imaging system preserves the SNR_{in} at a specific spatial frequency, (u,v)

In a similar fashion, **NEQ(u, v)** denotes the effective number of quanta that the image is worth at that frequency

The NEQ and DQE can be calculated from **Measurable Quantities** - specifically:

$$DQE(u,v) = \frac{\Phi\Gamma^2 T^2(u,v)}{W(u,v)}$$

and

$$NEQ(u,v) = \frac{\Phi^2\Gamma^2 T^2(u,v)}{W(u,v)}$$

It is clear from these equations that in an ideal system, the NPS is proportional to the MTF squared

$$W(u,v) = T^2(u,v)$$

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Standard measurement conditions for the NEQ and DQE have been specified by the **IEC**

Most typically, an **RQA-5** spectrum is used for radiography and **RQA-M** for mammography

Tabulations of fluence as a function of air kerma are used in conjunction with measurements of kerma to calculate Φ

As we've defined it, the quantum SNR is related to the relative variation of pixel values in a **Uniform Region**

However, it is often necessary to compare the amplitude of a specific signal to the **Background Noise**

An **Alternate Definition** of the SNR is the difference in the means of two regions to the noise in those regions:

$$SNR = \frac{|\langle x_a \rangle - \langle x_b \rangle|}{\sigma}$$

where $\mathbf{x_a}$ and $\mathbf{x_b}$ are the mean values in the region of an object (a) and background (b) and $\boldsymbol{\sigma}$ is the standard deviation of the background

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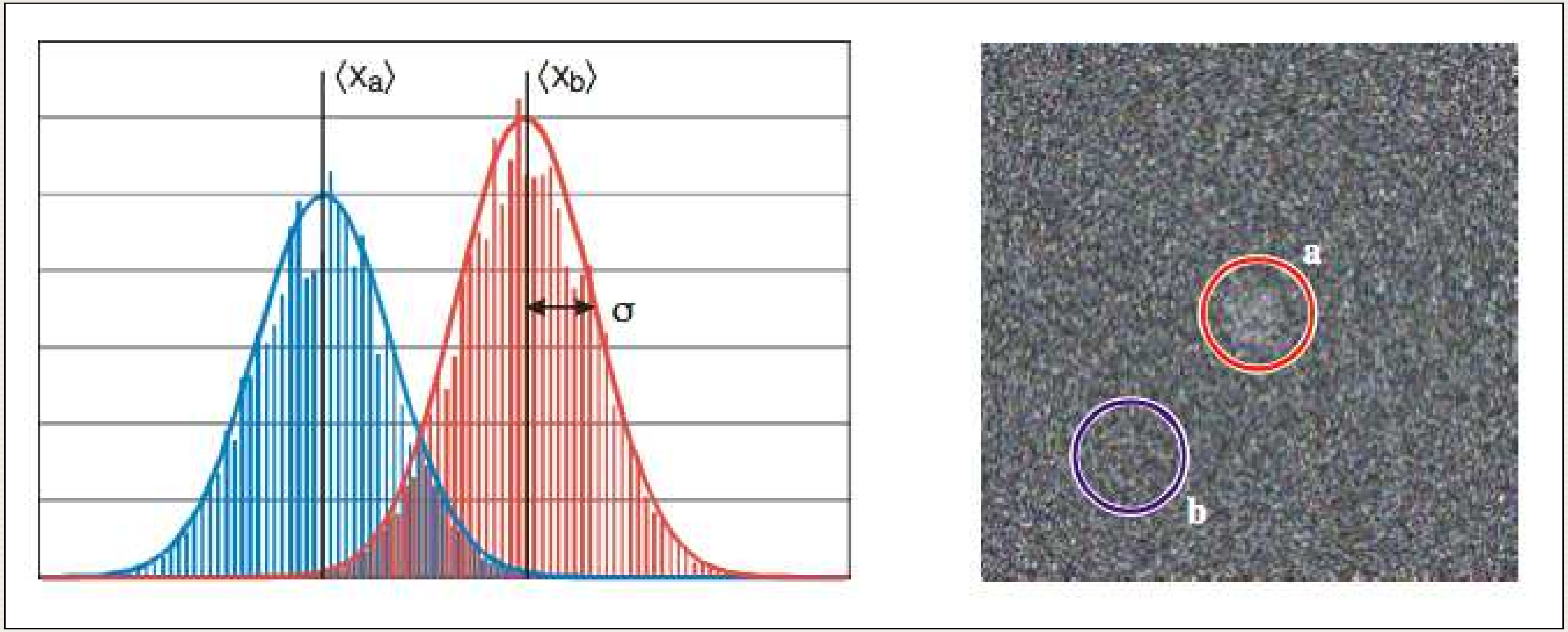

A uniform disk (a) is shown on a uniform background (b) in the presence of X-ray quantum noise

The SNR of the object is calculated as the difference in the mean signals divided by the noise characterized by the standard deviation (**σ**) of the **Background**

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The choice of the background region is important:

the Standard Deviation should be Calculated using the Region that yields a Meaningful Result

For Example, if image processing (such as **Thresholding**) is used to force the background to a uniform value, then SNR as defined will be indefinite

Note that the SNR as defined goes by a number of names, including the signal difference to noise ratio (**SdNR**) and the contrast to noise ratio (**CNR**)

4.6.3 Signal-to-Noise Ratio

The value of the SNR was first explained by **Albert Rose**

who was interested in quantifying the quality of television images

Rose showed that an object is distinguishable from the background if the

$$\mathbf{SNR \geq 5}$$

This can be related to a simple **t-test** in which an error rate of less than 1 in 10^6 occurs when the difference in the means is equal to 5 standard deviations

Today, this is known as the **Rose Criterion** in imaging research

It should be noted that the requirement of **SNR ≥ 5** is actually quite strict

Depending upon the image task, it is possible to successfully operate at lower SNR values

Assumption of the Rose model: the limiting factor in the detection of an object is the radiation **dose** (and hence number of X-ray quanta) used to produce the image

This is True

in an ideal imaging system

In fact, the design of all imaging systems is driven by the goal of being X-ray (primary) quantum noise limited

Robert Wagner has proposed a **Taxonomy** of noise limitations which is worth noting

There are **Four** potential limitations in terms of the detection of objects:

1) **Quantum Noise** limited
2) **Artefact** limited
3) **Anatomy** limited
4) **Observer** limited

X Ray Quantum Noise Limited performance is the **Preferred** mode of operation, because the ability to detect or discriminate an object is determined solely by the radiation dose

This is How All Detectors Should Operate

Ideally

Artefact limitation is the case in which the imaging system introduces artefacts which limit detection

Classic examples include CT and MR where acquisition artefacts can predominate over the signal of interest

Anatomic limited detection occurs when the normal anatomy (e.g. ribs in chest radiography or the breast parenchyma in mammography) mask the detection of objects, thereby reducing observer performance

4.6 ANALYSIS OF SIGNAL & NOISE

4.6.3 Signal-to-Noise Ratio

Finally, there are situations in which the **Observer** is the limiting factor in performance

For Example, a lesion may be readily visible, but the observer is distracted by an obvious benign or normal finding

Thus Detection was Possible but did Not Occur

In this chapter, we deal exclusively with quantum noise limited performance, which can be calculated using **Ideal Observer** methods

In Chapter 18, the **Modelling of Real Observers** is discussed

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Task-Specific

The MTF, NPS, NEQ and DQE are **Frequency** dependent characterizations of the detector

However, these allow us to **Calculate** the image of a scene

In particular, we can now use the SNR to quantify the ability of the detector to be used in:

- Signal Known Exactly (**SKE**),
- Background Known Exactly (**BKE**) tasks

assuming an ideal observer working with Gaussian statistics

Task-Specific

In this scenario, the observer is challenged with the task of deciding between **Two** hypotheses based upon a given set of data

Under the **First Hypothesis**, the expected input signal is present, f_I, and the image g_I is described by an appropriate Gaussian probability distribution

Under **Alternate Hypothesis**, the expected input signal is absent, f_{II}

Task-Specific

The SNR of this task is given by:

$$SNR_I^2 = \langle \Gamma^2 \iint \frac{|\Delta f(u,v)|^2 T(u,v)^2}{W(u,v)} du dv \rangle$$

where

$$\Delta f(u,v) = f_I(u,v) - f_{II}(u,v)$$

is the difference between the signal **Absent** and **Present**

Task-Specific

For a digital detector, T is the presampled MTF, and thus we must account for aliasing by summing over all aliases of the signal:

$$SNR_I^2 = \left\langle \Gamma^2 \iint \frac{\sum_{j,k} \left| \Delta f(u+u_j, v+v_k) \right|^2 T(u+u_j, v+v_k)^2}{W(u,v)} dudv \right\rangle$$

where the indices j, k are used to index the aliases (in 2D)

In this way, we can calculate the SNR of the ideal observer for the detection of any object in an SKE/BKE task

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Task-Specific

In Chapter 18, methods are described that extend this model to include characteristics of **Real Observers**

4.6.4 SNR^2/Dose

The **Ultimate Goal** of radiation safety in medical imaging is to obtain a **Figure of Merit** based on the maximum benefit to the patient for the smallest detriment

We can now calculate the SNR for the detection of a known object (for example a tumour) on a known background

This calculation is based upon parameters of a specific detector so that detectors can be **Compared** or **Optimized**

This calculation can act as a useful surrogate of the benefit, since a disease once detected can be treated

We need therefore to relate this benefit to some **Metric of Risk**

A **Useful** metric:

$$SNR^2/E$$

where E is the **Effective Dose**

4.6.4 SNR^2/Dose

This metric is formulated with the SNR^2 based on the fact that in quantum noise limited imaging the

$$SNR \propto \sqrt{\phi}$$

Thus the ratio is **Invariant** with dose

Other Descriptors of patient dose may also be useful

for example, optimization in terms of skin dose

Using this formation, it is possible, for example, to determine the **Optimal Radiographic Technique** (tube voltage, filtration, etc) for a specific task

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