Mechanical properties of solids

Poisson's ratio

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu \frac{\Delta l_1}{l_1}$$

- Fig 3.3 Solid State Physics: An Introduction, by Philip Hofmann Wiley-VCH Berlin.

ν≤0.5

This means that the volume of the solid always increases under tensile stress

Poisson's ratio

- Fig 3.3 Solid State Physics: An Introduction, by Philip Hofmann Wiley-VCH Berlin.

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu \frac{\Delta l_1}{l_1}$$

the volume is (assume the extensions are small)

$$(l_1 + \Delta l_1)(l_2 + \Delta l_2)(l_3 + \Delta l_3) \approx l_1 l_2 l_3 + \Delta l_1 l_2 l_3 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3$$

change in volume

$$\Delta l_1 l_2 l_3 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3 = \Delta l_1 l_2 l_3 + l_1(-\nu \frac{\Delta l_1}{l_1} l_2) l_3 + l_1 l_2(-\nu \frac{\Delta l_1}{l_1} l_3)$$

$$= (1 - 2\nu)\Delta l_1 l_2 l_3$$

and since $\Delta l_1 > 0$ it follows that $\nu \leq 0.5$

Poisson's ratio

- There is also a lower limit to the Poisson ratio. We get

$$-1 < \nu \leq 0.5$$

Some examples: volume change for cube is $= (1 - 2\nu)\Delta l_1 l_2 l_3$

ν	what happens?
ν >0.5	~~tensile stress: volume decrease, compressive stress: volume increase~~
ν = 0.5	no volume change, incompressible solid
0< ν<0.5	"normal" case for most materials, volume increase upon tens. stress, volume decrease upon compr. stress
-1< ν<0	volume increase upon tens. stress, volume decrease upon compr. stress; wires get thicker as you pull them!

Poisson ratio: examples

material	v
diamond	0.21
Al	0.33
Cu	0.35
Pb	0.4
Steel	0.29
rubber	close to 0.5
cork	close to 0

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu \frac{\Delta l_1}{l_1}$$

This is why it is possible to get a cork back into a wine bottle!

Foams with a negative poisson ratio

from: Exploring the nano-world with LEGO bricks
http://mrsec.wisc.edu/Edetc/LEGO/index.html

Elastic deformation: macroscopic

other deformations:

- shear stress: twisting of the sample
- hydrostatic pressure: compression
- torsion stress: torsion (not discussed here)

Shear stress / modulus of rigidity

$$A$$

shear stress:
tangential force
on an object per area

$$\tau = \frac{F}{A}$$

unit: Pa

modulus of rigidity

$$G = \frac{\tau}{\alpha}$$

unit: Pa

Hydrostatic pressure / bulk modulus

exposure to hydrostatic pressure

bulk modulus

$$K = -\Delta P \frac{V}{\Delta V}$$

unit: Pa

Relation between elastic constants

- in a more formal treatment, the quantities are related:

$$G = \frac{Y}{2(1+\nu)}$$

(online note philiphofmann.net)

$$K = \frac{Y}{3(1-2\nu)}$$

problem 3.1 in book

modulus of rigidity and bulk modulus as a function of Young's modulus and Poisson ratio.

Why is the force linear?

- Fig 1.1 Solid State Physics: An Introduction, by Philip Hofmann Wiley-VCH Berlin.

$$-\mathrm{grad}\phi(\mathbf{r})$$

a

$$\phi(x) = \phi(a) + \frac{d\phi(a)}{dx}(x-a) + \frac{1}{2}\frac{d^2\phi(a)}{dx^2}(x-a)^2 + \frac{1}{6}\frac{d^3\phi(a)}{dx^3}(x-a)^3 + \ldots$$

energy offset

=0

harmonic potential linear force.

- Fig 3.1 Solid State Physics: An Introduction, by Philip Hofmann Wiley-VCH Berlin.

Pb:

cohesive energy: 2.3 eV / atom

interatomic distance: 3.43 Å

W:

cohesive energy: 8.9 eV / atom

interatomic distance: 2.73 Å

a

$$\phi(x) = \phi(a) + \frac{d\phi(a)}{dx}(x-a) + \frac{1}{2}\frac{d^2\phi(a)}{dx^2}(x-a)^2 + \frac{1}{6}\frac{d^3\phi(a)}{dx^3}(x-a)^3 + \dots$$

linear restoring force:

$$F = -K(x-a) \qquad K = \frac{d^2\phi(a)}{dx^2}$$

curvature of the potential, not depth. So why is it related to the cohesive energy?

stress/strain curve for a ductile metal

$$\sigma = \frac{F}{A}$$

$$\epsilon = \frac{\Delta l}{l}$$

- Fig 3.2 Solid State Physics: An Introduction, by Philip Hofmann Wiley-VCH Berlin.

Shear stress / modulus of rigidity

$$A$$

shear stress:
tangential force
on an object per area

$$\tau = \frac{F}{A}$$

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→

modulus of rigidity

$$G = \frac{\tau}{\alpha}$$