Extended Work-Energy Theorem

- We denote the total mechanical energy by

$$E = KE + PE + PE_s$$

- Since $$(KE + PE + PE_s)_f = (KE + PE + PE_s)_i$$

- The total mechanical energy is conserved and remains the same at all times

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2$$

A block projected up a incline

- A 0.5-kg block rests on a horizontal, frictionless surface. The block is pressed back against a spring having a constant of $k = 625$ N/m, compressing the spring by 10.0 cm to point A. Then the block is released.
- (a) Find the maximum distance d the block travels up the frictionless incline if $\theta = 30^\circ$.
- (b) How fast is the block going when halfway to its maximum height?

A block projected up a incline

- Point A (initial state): $v_i = 0, y_i = 0, x_i = -10cm = -0.1m$
- Point B (final state): $v_f = 0, y_f = h = d\sin\theta, x_f = 0$

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}kx_i^2 = mgy_f = mgd\sin\theta$$

$$d = \frac{\frac{1}{2}kx_i^2}{mg\sin\theta} = \frac{0.5(625N/m)(-0.1m)^2}{(0.5kg)(9.8m/s^2)\sin 30^\circ}$$

$$= 1.28m$$

A block projected up a incline

- Point A (initial state): $v_i = 0, y_i = 0, x_i = -10cm = -0.1m$
- Point B (final state): $v_f = ?, y_f = h/2 = d\sin\theta/2, x_f = 0$

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mg(\frac{h}{2})$$

$$\frac{k}{m}x_i^2 = v_f^2 + gh$$

$$h = d\sin\theta = (1.28m)\sin 30^\circ = 0.64m$$

$$v_f = \sqrt{\frac{k}{m}x_i^2 - gh}$$

$$= \ldots\ldots = 2.5m/s$$

Types of Forces

- Conservative forces
  - Work and energy associated with the force can be recovered
  - Examples: Gravity, Spring Force, EM forces
- Nonconservative forces
  - Power is often dispersed and work done against it cannot easily be recovered
  - Examples: Kinetic friction, air drag forces, normal forces, tension forces, applied forces …

Conservative Forces

- A force is the work of an object moving between two points, if the object is independent of the path between the points
  - The work depends only on the starting and ending positions of the object
  - Any conservative force can have a potential energy function associated with it
  - Work done by gravity $W_g = PE_i - PE_f = mgy_i - mgy_f$
  - Work done by spring force

$$W_s = PE_{si} - PE_{sf} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Nonconservative Forces

- If the work you do on an object depends on the path between the final and the starting points of the object, then a power is not conserved.
  - Work depends on the path of motion
  - For a non-conservative force, the potential energy can not be identified Work done by a nonconservative force

$$W_{nc} = \sum \vec{F} \cdot \vec{d} = -f_k d + \sum W_{otherforces}$$

  - It is generally dissipative. The dispersal of energy takes the form of heat or sound

Figure 8.10 Physics for Scientists and Engineers 6th Edition, Thomson Brooks/Cole © 2004; Chapter 8

Extended Work-Energy Theorem

- The work-energy theorem can be written as:

$$W_{net} = KE_f - KE_i = \Delta KE$$

$$W_{net} = W_{nc} + W_c$$

  - W_{nc} represents the work done by nonconservative forces
  - W_c represents the work done by conservative forces
- Any work done by conservative forces can be accounted for by changes in potential energy $W_c = PE_i - PE_f$
  - Gravity work $W_g = PE_i - PE_f = mgy_i - mgy_f$
  - Spring force work $W_s = PE_i - PE_f = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Extended Work-Energy Theorem

- Any work done by conservative forces can be accounted for by changes in potential energy

$$W_c = PE_i - PE_f = -(PE_f - PE_i) = -\Delta PE$$

$$W_{nc} = \Delta KE + \Delta PE = (KE_f - KE_i) + (PE_f - PE_i)$$

$$W_{nc} = (KE_f + PE_f) - (KE_i + PE_i)$$

- Mechanical energy include kinetic and potential energy

$$E = KE + PE = KE + PE_g + PE_s = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

$$W_{nc} = E_f - E_i$$

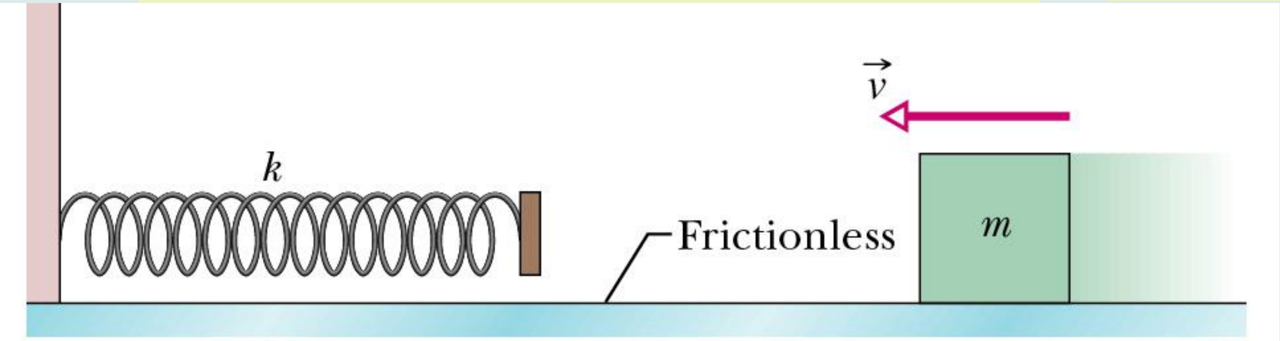
Conservation of Mechanical Energy

- A block of mass *m = 0.80 kg* slides across a horizontal frictionless counter with a speed of *v = 0.50 m/s*. It runs into and compresses a spring of spring constant *k = 1500 N/m*. When the block is momentarily stopped by the spring, by what distance ***d*** is the spring compressed?

$$W_{nc} = (KE_f + PE_f) - (KE_i + PE_i)$$

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

$$0 + 0 + \frac{1}{2}kd^2 = \frac{1}{2}mv^2 + 0 + 0$$



$$d = \sqrt{\frac{m}{k}v^2} = 1.15cm$$

Changes in Mechanical Energy for conservative forces

- A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate stats from rest at the top. The surface friction can be negligible. Use energy methods to determine the speed of the crate at the bottom of the ramp.

$$-fd + \sum W_{otherforces} = (\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2) - (\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2)$$

$$(\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2) = (\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2)$$

$$d = 1m, y_i = d\sin 30^\circ = 0.5m, v_i = 0$$

$$y_f = 0, v_f = ?$$

$$(\frac{1}{2}mv_f^2 + 0 + 0) = (0 + mgy_i + 0)$$

$$v_f = \sqrt{2gy_i} = 3.1m/s$$

Figure 8.11
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Changes in Mechanical Energy for Non-conservative forces

- A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate stats from rest at the top. The surface in contact have a coefficient of kinetic friction of 0.15. Use energy methods to determine the speed of the crate at the bottom of the ramp.

$$-fd+\sum W_{otherforces}=(\frac{1}{2}mv_f^2+mgy_f+\frac{1}{2}kx_f^2)-(\frac{1}{2}mv_i^2+mgy_i+\frac{1}{2}kx_i^2)$$

$$-\mu_k Nd+0=(\frac{1}{2}mv_f^2+0+0)-(0+mgy_i+0)$$

$$\mu_k=0.15, d=1m, y_i=d\sin 30^\circ=0.5m, N=?$$

$$N-mg\cos\theta=0$$

$$-\mu_k dmg\cos\theta=\frac{1}{2}mv_f^2-mgy_i$$

$$v_f=\sqrt{2g(y_i-\mu_k d\cos\theta)}=2.7m/s$$

Figure 8.11
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Changes in Mechanical Energy for Non-conservative forces

- A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate stats from rest at the top. The surface in contact have a coefficient of kinetic friction of 0.15. How far does the crate slide on the horizontal floor if it continues to experience a friction force.

$$-fd+\sum W_{otherforces}=(\frac{1}{2}mv_f^2+mgy_f+\frac{1}{2}kx_f^2)-(\frac{1}{2}mv_i^2+mgy_i+\frac{1}{2}kx_i^2)$$

$$-\mu_k Nx+0=(0+0+0)-(\frac{1}{2}mv_i^2+0+0)$$

$$\mu_k=0.15, v_i=2.7m/s, N=?$$

$$N-mg=0$$

$$-\mu_k mgx=-\frac{1}{2}mv_i^2$$

$$x=\frac{v_i^2}{2\mu_k g}=2.5m$$

Figure 8.11
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Block-Spring Collision

- A block having a mass of 0.8 kg is given an initial velocity v_A = 1.2 m/s to the right and collides with a spring whose mass is negligible and whose force constant is k = 50 N/m as shown in figure. Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

$$\frac{1}{2}mv_{max}^2 + 0 + 0 = \frac{1}{2}mv_A^2 + 0 + 0$$

$$x_{max} = \sqrt{\frac{m}{k}}v_A = \sqrt{\frac{0.8kg}{50N/m}}(1.2m/s) = 0.15m$$

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Block-Spring Collision

- A block having a mass of 0.8 kg is given an initial velocity v_A = 1.2 m/s to the right and collides with a spring whose mass is negligible and whose force constant is k = 50 N/m as shown in figure. **Suppose a constant force of kinetic friction acts between the block and the surface**, with μ_k = 0.5, what is the maximum compression x_c in the spring.

$$-fd+\sum W_{otherforces}=(\frac{1}{2}mv_f^2+mgy_f+\frac{1}{2}kx_f^2)-(\frac{1}{2}mv_i^2+mgy_i+\frac{1}{2}kx_i^2)$$

$$-\mu_k Nd+0=(0+0+\frac{1}{2}kx_c^2)-(\frac{1}{2}mv_A^2+0+0)$$

$$N=mg \quad \text{and} \quad d=x_c$$

$$\frac{1}{2}kx_c^2-\frac{1}{2}mv_A^2=-\mu_k mgx_c$$

$$25x_c^2+3.9x_c-0.58=0 \qquad x_c=0.093m$$

Energy Review

- Kinetic Energy
  - Associated with movement of members of a system
- Potential Energy
  - Determined by the configuration of the system
  - Gravitational and Elastic
- Internal Energy
  - Related to the temperature of the system

Conservation of Energy

- ***Energy is conserved***
  - This means that energy cannot be created nor destroyed
  - If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer

Practical Case

- $\Delta E = \Delta K + \Delta U = 0$
- The total amount of energy in the system is constant.

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

Practical Case

- $\Delta K + \Delta U + \Delta E_{int} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$
- The Work-Kinetic Energy theorem is a special case of Conservation of Energy $\Delta K + \Delta U = W$

Ways to Transfer Energy Into or Out of A System

- ***Work*** – transfers by applying a force and causing a displacement of the point of application of the force
- ***Mechanical Waves*** – allow a disturbance to propagate through a medium
- ***Heat*** – is driven by a temperature difference between two regions in space
- ***Matter Transfer*** – matter physically crosses the boundary of the system, carrying energy with it
- ***Electrical Transmission*** – transfer is by electric current
- ***Electromagnetic Radiation*** – energy is transferred by electromagnetic waves

Connected Blocks in Motion

- Two blocks are connected by a light string that passes over a frictionless pulley. The block of mass m1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m2 fall a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m1 and the surface.

$$-fd + \sum W_{otherforces} = \Delta KE + \Delta PE$$

$$\Delta PE = \Delta PE_g + \Delta PE_s = (0 - m_2 gh) + (\frac{1}{2}kx^2 - 0)$$

$$-\mu_k Nx + 0 = -m_2 gh + \frac{1}{2}kx^2$$

$$N = mg \quad \text{and} \quad x = h$$

$$-\mu_k m_1 gh = -m_2 gh + \frac{1}{2}kh^2$$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

Figure 8.15
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