Quantum Mechanics

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Potential Barrier Problem

- $E<V_0$ The potential functions are :
- $V(x) = 0$ when $x<0$
- $V(x) = V_0$ when $0 \leq x \leq L$ and
- $V_x = 0$ when $x>L$
- Lets write the general Schrodinger equation

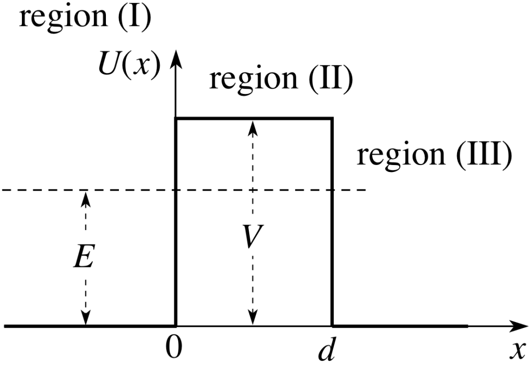
$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x)$$

As we divided space into three regions lets write Schrodinger equation for every region:

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_I(x)}{dx^2} = E\Psi_I(x), \quad In \quad region \quad I: \quad x<0$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_{II}(x)}{dx^2}+V_0\Psi(x) = E\Psi_{II}(x), \quad In \quad region \quad II: \quad 0 \leq x \leq L$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_{III}(x)}{dx^2} = E\Psi_{III}(x), \quad In \quad region \quad III: \quad x>L$$


region (I)
U(x)
region (II)
region (III)
E
V
0
d
x

Following these relation we found that our wavefunctions are:

$$\Psi_I = Ae^{ik_I x} + Be^{-ik_I x}, \quad k_I = \frac{\sqrt{2mE}}{\hbar}$$

$$\Psi_{II} = Ce^{k_{II} x} + De^{-k_{II} x}, \quad k_{II} = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$$\Psi_{III} = Fe^{ik_{III} x} + Ge^{-ik_{III} x}, \quad k_{III} = \frac{\sqrt{2mE}}{\hbar}, \quad k_{III} = k_I \quad \boxed{G = 0}$$

It is clear that, the incident wave is $\Psi_{in}(x) = Ae^{ik_I x}$, reflected wave is $\Psi_{ref}(x) = Be^{-ik_I x}$ and transmitter wave is $\Psi_{tra}(x) = Fe^{ik_{III} x}$: The amplitudes of the waves respectively,

$$|\Psi_{in}(x)|^2 = \Psi_{in}(x)^* \Psi_{in}(x) = (Ae^{ik_I x})^* Ae^{ik_I x} = |A|^2$$

$$|\Psi_{ref}(x)|^2 = \Psi_{ref}(x)^* \Psi_{ref}(x) = (Be^{-ik_I x})^* Be^{-ik_I x} = |B|^2$$

$$|\Psi_{tra}(x)|^2 = \Psi_{tra}(x)^* \Psi_{tra}(x) = (Fe^{ik_{III} x})^* Fe^{ik_{III} x} = |F|^2$$

The transmission probability requires that:

$$T = \frac{|\Psi_{tra}(x)|^2}{|\Psi_{in}(x)|^2} = \frac{|F|^2}{|A|^2}$$

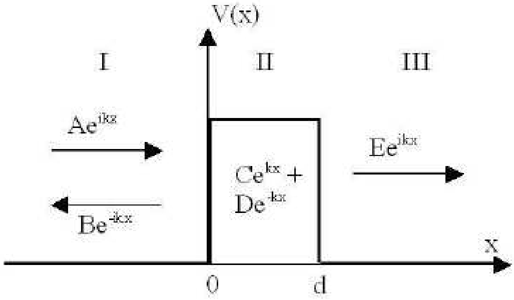
The reflection probability requires that:

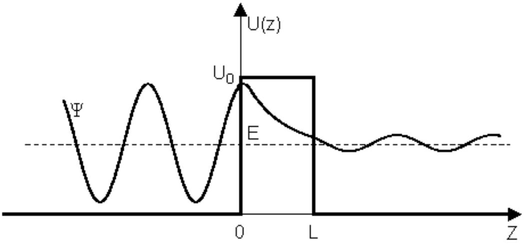
$$R = \frac{|\Psi_{ref}(x)|^2}{|\Psi_{in}(x)|^2} = \frac{|B|^2}{|A|^2}$$

The continuity condition requires that:

$$\Psi_I(0) = \Psi_{II}(0), \quad boundary \quad between \quad regionsI \quad and \quad II$$

$$\Psi_{II}(L) = \Psi_{III}(L), \quad boundary \quad between \quad regionsII \quad and \quad III$$


V(x)
I
II
III
Ae^ikx
Be^-ikx
Ce^kx +
De^-kx
Ee^ikx
x
0
d



U(z)
U_0
Ψ
E
0
L
Z