Quantum Mechanics

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Lets continue from the first part: Also, the first derivatives of the solution must e continuous at region boundaries:

$$[x = 0] \quad \rightarrow \quad \frac{d\Psi_I(x)}{dx} = \frac{d\Psi_{II}(x)}{dx}$$

$$[x = L] \quad \rightarrow \quad \frac{d\Psi_{II}(x)}{dx} = \frac{d\Psi_{III}(x)}{dx}$$

then we can write:

$$A + b = C + D \quad and \quad Ce^{k_{II}L} + De^{-k_{II}L} = Fe^{k_{III}L}$$

$$-ik_I(A - B) = k_{II}(D - C) \quad and \quad k_{II}(De^{k_{II}L} - Ce^{-k_{II}L}) = -ik_{III}Fe^{k_{III}L}$$

$$A = \frac{C(ik_I + k_{II}) + D(ik_I - k_{II})}{2ik_I} \quad C = \frac{F(ik_I + k_{II})e^{ik_I L}}{2k_{II}e^{k_{II}L}} \quad D = \frac{F(k_{II} - ik}{2k_{II}e^{-}}$$

$$\frac{A}{F} = \frac{e^{ik_I L}}{4ik_I k_{II}}\left[(k_{II}^2 - k_I^2)e^{-k_{II}L} + 2ik_I k_{II} e^{-k_{II}L} - (k_{II}^2 - k_I^2)e^{k_{II}L} + 2ik_I K_{II} e^{k_{II}L}\right]$$

$$\frac{|F|^2}{|A|^2} = \frac{16k_I^2 k_{II}^2}{16k_I^2 k_{II}^2 + [16k_I^2 k_{II}^2 + 4(k_{II}^2 - k_I^2)^2] sinh^2(k_{II}L)}$$

The potential functions are :

$V(x) = 0$ when $x<0$,

$V(x) = V_0$ when $0 \leq x \leq L$ and

$V_x = 0$ when $x>L$

Lets write the general Schrodinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x)$$

As we divided space into three regions lets write Schrodinger equation for every region:

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_I(x)}{dx^2} = E\Psi_I(x), \quad In \quad region \quad I: \quad x<0$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_{II}(x)}{dx^2}+V_0\Psi(x) = E\Psi_{II}(x), \quad In \quad region \quad II: \quad 0 \leq x \leq L$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_{III}(x)}{dx^2} = E\Psi_{III}(x), \quad In \quad region \quad III: \quad x>L$$

Following these relation we found that our wavefunctions are:

$$\Psi_I = Ae^{ik_I x} + Be^{-ik_I x}, \quad k_I = \frac{\sqrt{2mE}}{\hbar}$$

$$\Psi_{II} = Ce^{ik_{II} x} + De^{-ik_{II} x}, \quad k_{II} = \frac{\sqrt{2m(E - V_o)}}{\hbar}$$

$$\Psi_{III} = Fe^{ik_{III} x} + Ge^{-ik_{III} x}, \quad k_{III} = \frac{\sqrt{2mE}}{\hbar}, \quad k_{III} = k_I \quad \boxed{G = 0}$$

It is clear that, the incident wave is $\Psi_{in}(x) = Ae^{ik_I x}$, reflected wave is $\Psi_{ref}(x) = Be^{-ik_I x}$ and transmitter wave is $\Psi_{tra}(x) = Fe^{ik_{III} x}$: The current density: $J = \frac{i\hbar}{2m}(\Psi(x)\frac{d\Psi^*}{dx} - \Psi^*(x)\frac{d\Psi}{dx})$. Then the transmission and reflection probability is $R = \frac{|J_{ref}|}{|J_{in}|}$ and $T = \frac{|J_{tra}|}{|J_{in}|}$.

The transmission probability requires that:

$$T = \frac{|J_{tra}|}{|J_{in}|} = \frac{k_I |F|^2}{k_{III} |A|^2} = \frac{|F|^2}{|A|^2}$$

The reflection probability requires that:

$$R = \frac{|J_{ref}|}{|J_{in}|} = \frac{|B|^2}{|A|^2}$$