Quantum Mechanics

Prof. Dr. Handan Olgar

Ankara University

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The continuity condition requires that:

$$\Psi_I(0) = \Psi_{II}(0), \quad boundary \quad between \quad regionsI \quad and \quad II$$

$$\Psi_{II}(L) = \Psi_{III}(L), \quad boundary \quad between \quad regionsII \quad and \quad III$$

Also, the first derivatives of the solution must e continuous at region boundaries:

$$[x = 0] \quad \rightarrow \quad \frac{d\Psi_I(x)}{dx} = \frac{d\Psi_{II}(x)}{dx}$$

$$[x = L] \quad \rightarrow \quad \frac{d\Psi_{II}(x)}{dx} = \frac{d\Psi_{III}(x)}{dx}$$

then we can write:

$$A + B = C + D \quad and \quad Ce^{ik_{II}L} + De^{-ik_{II}L} = Fe^{ik_{III}L}$$

$$-ik_I(A - B) = ik_{II}(D - C) \quad and \quad ik_{II}(De^{ik_{II}L} - Ce^{-ik_{II}L}) = -ik_{III}Fe^{k_{III}L}$$

from all those relations we find :

$$F = 4k_I k_{II} A e^{-ik_I L} \Big[4k_I k_{II} cos(k_{II}L) Big - 2i(k_I^2 + k_{II}^2) sin(k_2 a)]$$

$$\frac{F}{A} = \frac{4k_I k_{II}}{\sqrt{16k_I^2 k_{II}^2 cos^2(k_{II} L) + 4k_I^2 k_{II}^2 sin^2(k_{II} L)}}$$

$$T = \Big[1 + \frac{1}{4}\frac{(k_I - \Big(k_{II})^2}{k_I k_{II}}\Big)^2 sin^2(k_{II} a) Big]^{-1}$$