PHY 431
ADVANCED QUANTUM MECHANICS I

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Course notes are prepared from the book Quantum Mechanics-Concepts and Applications by Zettili.

MATHEMATICAL TOOLS IN QUANTUM MECHANICS

- Hilbert Space and Wave Functions
- Dirac Notation
- Operators –General Definition
- Hermitian Adjoint
- Hermitian Operators
- Commutator Algebra

Hilbert Space and wave functions

A linear vector space consists of two sets of elements and two algebraic rules:

- a set of vectors ψ, ϕ, χ,… and a set of scalars a, b, c, ….
- a rule for vector addition and a rule for scalar multiplication.
- Addition rule: The addition rule has the properties and structure of an abelian group
- Multiplication rule:The product of a scalar with a vector gives another vector.

Dirac Notation

- The physical state of a system is represented in quantum mechanics by elements of a Hilbert space; these elements are called state vectors. We can represent the state vectors in different bases by means of function expansions.
- To free state vectors from coordinate meaning, Dirac introduced what was to become an invaluable notation in quantum mechanics; it allows one to manipulate the formalism of quantum mechanics with ease and clarity.

- Kets: elements of a vector space: Dirac denoted the state vector ψ by the symbol |ψ>, which he called a ket vector, or simply a ket. Kets belong to the Hilbert (vector) space H, or, in short, to the ket-space.
- Bras: elements of a dual space: Dirac denoted the elements of a dual space by the symbol <|, which he called a bra vector, or simply a bra; for instance, the element <ψ| represents a bra.
- Bra-ket: Dirac notation for the scalar product:
- Dirac denoted the scalar (inner) product by the symbol <|>, which he called a a bra-ket. For instance, the scalar product ψ and ϕ is denoted by the bra-ket <ψ|ϕ>.

Operators

- General definition: An operatör A is a mathematical rule that when applied to a ket |ψ> transforms it into another ket |ψ'> of the same space and when it acts on a bra <ϕ| transforms it into another bra : <ϕ'|

$$\hat{A} \mid \psi \rangle = \mid \psi' \rangle, \qquad \langle \phi \mid \hat{A} = \langle \phi' \mid .$$

Hermitian Adjoint

The Hermitian adjoint or conjugate[2], $\alpha^\dagger$, of a complex number α is the complex conjugate of this number: $\alpha^\dagger = \alpha^*$. The Hermitian adjoint, or simply the adjoint, $\hat{A}^\dagger$, of an operator $\hat{A}$ is defined by this relation:

$$\langle \psi \mid \hat{A}^\dagger \mid \phi \rangle = \langle \phi \mid \hat{A} \mid \psi \rangle^*.$$

Following these rules, we can write

$$
\begin{aligned}
(\hat{A}^\dagger)^\dagger &= \hat{A}, \\
(a\hat{A})^\dagger &= a^*\hat{A}^\dagger, \\
(\hat{A}^n)^\dagger &= (\hat{A}^\dagger)^n, \\
(\hat{A}+\hat{B}+\hat{C}+\hat{D})^\dagger &= \hat{A}^\dagger+\hat{B}^\dagger+\hat{C}^\dagger+\hat{D}^\dagger, \\
(\hat{A}\hat{B}\hat{C}\hat{D})^\dagger &= \hat{D}^\dagger\hat{C}^\dagger\hat{B}^\dagger\hat{A}^\dagger, \\
(\hat{A}\hat{B}\hat{C}\hat{D}\mid\psi\rangle)^\dagger &= \langle\psi\mid D^\dagger C^\dagger B^\dagger A^\dagger.
\end{aligned}
$$

- Further discussions about the mathematical concepts of Quantum Mechanics are discussed from the book Quantum Mechanics-Concepts and Applications by Zettili.