PHY 431
ADVANCED QUANTUM MECHANICS I

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Course notes are prepared from the book Quantum Mechanics-Concepts and Applications by Zettili.

Postulates of Quantum Mechanics

Basics of Quantum Mechanics-

- Quantum Mechanics is nothing more but linear algebra and Hilbert spaces
- What makes quantum mechanics quantum mechanics is the physical interpretation of the results that are obtained

- First Postulate of Quantum Mechanics -

First postulate of Quantum mechanics:

Every physically-realizable state of the system is described in quantum mechanics by a state function ψ that contains all accessible physical information about the system in that state.

- First Postulate of Quantum Mechanics -

If ψ_1 and ψ_2 represent two physically-realizable states of the system, then the linear combination

$$\psi = c_1\psi_1 + c_2\psi_2$$

where c_1 and c_2 are arbitrary complex constants, represents a third physically realizable state of the system.

Quantum mechanics describes the outcome of an ensemble of measurements, where an ensemble of measurements consists of a very large number of identical experiments performed on identical non-interacting systems, all of which have been identically prepared so as to be in the same state.

Second Postulate of Quantum Mechanics

If a system is in a quantum state represented by a wavefunction ψ, then

$$PdV = |\psi|^2 dV$$

is the probability that in a position measurement at time *t* the particle will be detected in the infinitesimal volume d*V*.

Note:

$|\psi(x,t)|^2$ ➔ position and time probability density

The importance of normalization follows from the Born interpretation of the state function as a position probability amplitude. According to the second postulate of quantum mechanics, the integrated probability density can be interpreted as a probability that in a position measurement at time t, we will find the particle anywhere in space.

Second Postulate of Quantum Mechanics

Therefore, the normalization condition for the wavefunction is:

$$\int P dV = \int \left|\psi(x,y,z)\right|^2 dV = \int \psi^*(x,y,z)\psi(x,y,z) dV = 1$$

Limitations on the wavefunction:

- Only normalizable functions can represent a quantum state and these are called physically admissible functions.
- State function must be continuous and single valued function.
- State function must be a smoothly-varying function (continuous derivative).

Third Postulate of Quantum Mechanics

Every observable in quantum mechanics is represented by an operator which is used to obtain physical information about the observable from the state function. For an observable that is represented in classical physics by a function $Q(x,p)$, the corresponding operator is $Q(\hat{x}, \hat{p})$.

Observable	Operator
Position	$\hat{x}$
Momentum	$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$
Energy	$E = \frac{\hat{p}^2}{2m} + V(\hat{x}) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$

Operators associated to physical quantities

We cannot use functions (otherwise we would end with classical mechanics)

Any physical quantity is associated with an operator.

An operator O is “the recipe to transform Ψ into Ψ' ”

We write: $O\,\Psi = \Psi'$

If $O\,\Psi = o\Psi$ (o is a number, meaning that O does not modify Ψ, just a scaling factor), we say that Ψ is an eigenfunction of O and o is the eigenvalue.

We have solved the wave equation $O\,\Psi = o\Psi$ by finding simultaneously Ψ and o that satisfy the equation.
o is the measure of O for the particle in the state described by Ψ.

Operators

The requirement for two operators to be commuting operators is a very important one in quantum mechanics and it means that we can simultaneously measure the observables represented with these two operators

Fourth Postulate of Quantum Mechanics -

Fourth (Fundamental) postulate of Quantum mechanics:

The time development of the state functions of an isolated quantum system is governed by the time-dependent SWE $\hat{H}\psi = i\hbar\partial\psi / \partial t$, where $\hat{H} = \hat{T} + \hat{V}$ is the Hamiltonian of the system.

Note on isolated system:

The TDSWE describes the evolution of a state provided that no observations are made. An observation alters the state of the observed system, and as it is, the TDSWE can not describe such changes.