Advanced Quantum Mechanics

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$E > V_0$

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The potential functions are : $V(x) = 0$ when $x < 0$ and $V(x) = V_0$ when $x \geq 0$ Lets write down the general formula of Schrodinger equation again:

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x)$$

As we divided space into two regions lets write Schrodinger equation for every region:

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_I(x)}{dx^2} = E\Psi_I(x), \quad In \quad region \quad I: \quad x<0$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_{II}(x)}{dx^2} + V_0\Psi(x) = E\Psi_{II}(x), \quad In \quad region \quad II: \quad x \geq 0$$

Following these relation we found that our wave-functions are:

$$\Psi_I = Ae^{ik_I x} + Be^{-ik_I x}, \quad k_I = \frac{\sqrt{2mE}}{\hbar}$$

$$\Psi_{II} = Ce^{ik_{II} x} + De^{-ik_{II} x}, \quad k_{II} = \frac{\sqrt{2m(E - V_0)}}{\hbar}, \quad \boxed{D = 0}$$

The current density: $J = \frac{i\hbar}{2m}(\Psi(x)\frac{d\Psi^*}{dx} - \Psi^*(x)\frac{d\Psi}{dx})$. Then the transmission and reflection probability is $R = \frac{|J_{ref}|}{|J_{in}|}$ and $T = \frac{|J_{tra}|}{|J_{in}|}$.

$$R = \frac{|J_{ref}|}{|J_{in}|} = \Big|\frac{-\hbar k_I |B|^2}{m}\frac{m}{\hbar|A|^2 k_I}\Big| = \frac{|B|^2}{|A|^2}$$

$$T = \frac{|J_{tra}|}{|J_{in}|} = \Big|\frac{\hbar k_{II} |C|^2}{m}\frac{m}{\hbar|A|^2 k_I}\Big| = \frac{k_{II}|B|^2}{k_I|A|^2}$$

$$R + T = 1$$

The continuity condition requires that:

$$\Psi_I(0) = \Psi_{II}(0), \quad boundary \quad between \quad regionsI \quad and \quad II$$

Also, the first derivatives of the solution must e continuous at region boundaries:

$$[x = 0] \quad \rightarrow \quad \frac{d\Psi_I(x)}{dx} = \frac{d\Psi_{II}(x)}{dx}$$

then we can write:

$$A + B = C \quad and \quad ik_I(A - B) = ik_{II}C$$

from that relation $C = \frac{2k_I}{k_I + k_2}A$ and $B = \frac{k_I - k_{II}}{k_1 + k_{II}}A$ so our transmission and reflection:

$$R = \frac{(k_I - k_2)^2}{(k_1 + k_2)^2} \quad T = \frac{k_{II}4k_I^2}{k_I(k_I + k_2)^2}$$

Lets transform it:

$$\bar{k} = \frac{k_2}{k_1}\sqrt{1 - \frac{V_0}{E}} \quad R = \frac{(1-\bar{k})^2}{(1+\bar{k})^2} \quad T = \frac{k_{II} 4\bar{k}^2}{(1+\bar{k})^2}$$

$$if \quad E \to V_0, \quad T \to 0$$

$$if \quad E \to \infty, \quad V_0/E \to 0, \quad R \to 0, \quad T \to 1$$