Advanced Quantum Mechanics

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Lets continue from the first part: we found the relations $a_1 = 0$, $b_1 = 0$ and $c_1 = 0$.Then, $a_2 = \hbar\sqrt{2}$, $b_2 = 0$ and $c_2 = 0$. Also, $a_3 = 0$, $b_3 = \hbar\sqrt{2}$ and $c_3 = 0$.

$$\boxed{\hat{L}_+ = \hbar \begin{vmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{vmatrix}}$$

Lets use same analogy for L_-:

$$L_-1,1 = 0 \quad L_+1,0 = \hbar\sqrt{2}\begin{vmatrix}1\\0\\0\end{vmatrix} \quad L_+1,-1 = \hbar\sqrt{2}\begin{vmatrix}0\\1\\0\end{vmatrix}$$

Lets be more clear lets assume that :

$$L_{-} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_2 \end{vmatrix}$$

if:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_2 \end{vmatrix} \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_2 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} = \hbar\sqrt{2} \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_2 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

we found the relations $a_1 = 0$, $b_1 =\sqrt{2}$ and $c_1 = 0$.Then, $a_2 = 0$, $b_2 = 0$ and $c_2 =\sqrt{2}$. Also, $a_3 = 0$, $b_3 = 0$ and $c_3 = 0$.

$$\boxed{\hat{L}_- = \hbar \begin{vmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{vmatrix}}$$

Using same analogy we can find that:

$$\boxed{\hat{L}^2 = 2\hbar^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}$$

Further study and solve the following exercises from the book by Zettili Study Problem 5.1 pg 310 Study Problem 5.4 pg 312