Advanced Quantum Mechanics

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The Hilbert space for of angular momentum states for spin 1 is three-dimensional: s, m is $1, 1$, $1, 0$ and $1, 1$ those states are similar to angular momentum l=1 states. All analogy are same. We are going to derive now all these expression below:

$$\boxed{S^2 s, m = s(s+1)\hbar^2 s, m}$$

$$\boxed{S_z s, m = m\hbar s, m}$$

$$\boxed{S_+ s, m = \hbar\sqrt{s(s+1) - m(m+1)} s, m+1} \quad \boxed{S_+ s, m_{max} = 0}$$

$$\boxed{S_- s, m = \hbar\sqrt{s(s+1) - m(m-1)} s, m-1} \quad \boxed{S_- s, m_{min} = 0}$$

$$\boxed{S_x = \frac{S_+ + S_-}{2}}$$

$$\boxed{S_y = \frac{S_+ - S_-}{2i}}$$

And we will also express these operators in matrix form:

$$\boxed{\hat{S}^2 = 2\hbar^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}} \quad \boxed{\hat{S}_z = \hbar \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix}} \quad \boxed{\hat{S}_+ = \hbar\sqrt{2} \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix}} \quad \hat{S}_-$$

$$\boxed{\hat{S}_x = \frac{\hbar}{\sqrt{2}} \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}} \quad \boxed{\hat{S}_y = \frac{\hbar}{i\sqrt{2}} \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix}}$$

Example: Lets write specific Hamiltonian $\hat{H} = A\hat{S}_z + B\hat{S}_x^2$: and find the eigenvalues and eigenvectors of this Hamiltonian

$$\hat{H} = A\hbar \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} + B\frac{\hbar^2}{2} \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = A\hbar \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} + B\frac{\hbar^2}{2} \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$$

$$\hat{H} = \begin{vmatrix} A\hbar + B\hbar^2/2 & 0 & B\hbar^2/2 \\ 0 & B\hbar^2 & 0 \\ B\hbar^2/2 & 0 & B\hbar^2/2 - A\hbar \end{vmatrix}$$

Further look at and study the Section 5.8 from the book Zettili