GENETIC THEORY IN THE PYTHAGOREAN SCHOOL

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THERE are two reasons for a contemporary geneticist to be interested in Pythagorean speculation on heredity. The first is that scientific genetics originated with this school. The second is that looking at earlier attempts to apply mathematics to complex phenomena may suggest fruitful approaches to a contemporary situation in which new discoveries already promise to upset the elegant simplicity of an atomistic genetic theory. The Pythagoreans, committed to field rather than atomic types of scientific explanation, produced what seems a very different type of theory from our current ones. Looking at such an alternative approach to his subject-matter may have for the geneticist some of the values that studying non-Euclidean geometries has for the mathematician. Thus a summary of the earliest history of genetics may not only remind us of the honor which the Pythagorean scientists deserve, but may suggest new methods of approach to the building of theories. References are given for the reader interested in verifying and evaluating the sources involved, but this discussion will be a summary of the theory as reconstructed without digressions into technical niceties of that reconstruction.

Beginning of Genetics

The Pythagorean research program of applying mathematics to phenomena[1] (begun ca. 529 B.C.[2]) led them to try to find some quantitative key to the interesting phenomena of heredity. This attempt was the beginning of genetics as a science. They seem to have begun with the most interesting, but most complex, phenomena; the hereditary component in complex traits of human "character."[3] This beginning may have been influenced by the theories of the medical school of Crotona, which seems to have regarded the patient as a single psycho-physical whole. Although this emphasis did not preclude study of anatomy and practice of dissection, it did direct attention to the interconnection of parts of an organism, and away from theories treating simple traits separately.[4]

But evidently a complex trait like "honesty" or "courage" is not purely the result of heredity; it must be thought of as a hereditary capacity, elicited by the environment.[5] The only way to measure such capacities is by observing their realization.[6] The key postulate is that if the capacity is not present, no environment can elicit it, though if it is, the environment can always frustrate its realization. Further generalization from their observations led to a second basic concept of this theory: in respect to any trait, the maximum development of a child will be about the average of the capacities of his parents. Third, it seemed clear that human character traits varied through a continuous range, requiring a geometrical, not an arithmetical, type of quantitative representation. The notion of the inherited average as (approximate) maximum was probably supplemented by a physiological theory in which the "seed" was treated as derived from and reflecting the entire parental organism, and the interaction of seeds as a kind of fusion creating a field of potencies.[7] We might note that of the possible notions of relation of parental seeds, the Pythagoreans tried the simplest, that of addition; in later speculation, more complex suggestions were offered; Empedocles introduced a mechanism of "dominance," Plato seems to have seen the need for some provision for "emergence," Aristotle uses terms that suggest the possibility of the two preceding types of relation plus something like "mutation."

The "Genetic Triangle"

Some mathematical device was needed to treat the relations of such "capacities." Apparently, what was done was to equate genetic "potencies" with recently discovered mathematical objects also called "potencies": these were irrational lines, the squares (or higher powers) of which were rational. The mathematical "root" was not conceived as an actual magnitude (for a "magnitude" was defined as an integral number of units), but as an entity that had the power, when it "grew" into a

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square, of becoming rational and commensurable with magnitudes. Consequently, the symbol chosen by the Pythagoreans as the key genetic diagram was a right triangle, one side representing each parent, the hypotenuse their offspring, and the inheritance of a given capacity by the child being determined by its intensity in the parents.[8, 9] In this way, inheritance could be treated as relevant to capacities for complex traits, without being directly causative of the traits in question.

To construct an analogue to this approach today, we would have to use different mathematical tools. If an individual's capacities at a given time were represented by a matrix showing the different probabilities of the possible directions and degrees of their realization, treating the present state as a kind of superimposition of these potential futures, the result would be a modern counterpart of the Pythagorean scheme. And the ontological status of these superimposed "present possible futures" would have just the equivocal character which irrational roots were originally defined as possessing.

Actually, the key genetic symbol of the Pythagoreans was *not simply any right triangle*, but the right triangle with sides of 3, 4, and 5. This symbol embodies several features which we find in their theory. The use of a geometrical figure recognizes the continuous range of traits studied.[10] The selection of 3 and 4 (following the notion of odd numbers as masculine, and even as feminine) reflects the qualitative difference between the parents. (It need not specify any quantitative differences between them.[11]) The use of a right-triangle gives the formula by which mathematical "potencies" may be compared in terms of the "magnitudes" into which they develop; the "Theorem of Pythagoras" being the basic tool for study and construction of irrationals.[12]

Inevitable Deterioration

One defect in this theory is its inability to account for either-or traits, or for inheritance which closely resembles one parental line but differs widely from the other.[13] A second defect, if scales of traits are constructed as Plato's are in the *Phaedrus* and *Republic*, with the best development at the top of the scale, set equal to 1, is the rapid deterioration in human aptitude which the theory predicts.[14] With scales set up in this way, whether the *average* or the *square root of the sum of squares* of parental potencies is used to calculate inherited endowment, the theory predicts an inordinate rapidity of human deterioration. Using the second calculation (which follows the theorem of Pythagoras more exactly) one would expect about one-seventh of each generation to be more degenerate than any of their ancestors, and four generations to eliminate all persons of a top level of ability ranked as 1 on the initial scale.[15]

Of these two difficulties, the second seems to have awaited Plato's invention of eugenics for its discovery; the former, particularly in reference to the sex of offspring, was early recognized and accounts, probably, for suggestions that the determining factor here is not hereditary.[16]

This early venture in genetics has been described both for its interest as an important point in the history of science, and for its value as a very different way of building a theory to explain genetic phenomena. Although the theory seems to us over-ambitious in beginning with complex traits, naïve in assuming that simple arithmetical relations (such as a sum or average) would describe the laws operative, and yet probably hard to apply because of its mathematical complexity, it served as the starting point of subsequent Greek speculation, and provided an incentive for further observation and quantification of phenomena. This line of thought strongly influenced Plato, who, in the awkward position of being unable either to accept the atomistic Hippocratic or Empedoclean theory, or to account for known phenomena by the earlier Pythagorean one, was in much the sort of dilemma toward which geneticists seem to be moving today.[17]

References

(1) For Pythagorean science, see J. Burnet, *Early Greek Philosophy*, 5th edn., and the brief summary in N. P. Stallknecht and R. S. Brumbaugh, *The Spirit of Western Philosophy*, Chap. I.

(2) 529 B.C. is the traditional date of Pythagoras' arrival at Crotona.

(3) Greek literature shows an interest in such inheritance of "character" from Homer on. The interest is natural, and assuming it as a starting-point gives coherence to the Pythagorean doctrines.

(4) The stress on the whole organism is characteristic of such later works as Empedocles' poem and Plato's *Timaeus*, that derive from it; whether it was pre-Pythagorean or an idea taken over from the Pythagoreans, the effect on their genetic theory would be the same.

(5) Or, of course, no hereditary factor may be involved in such traits at all. Plato's "case-studies" in *Republic* VIII-IX show how clearly he recognized the force of environment in forming personality, though there is no telling how far the earlier thinkers (prior to Empedocles) agreed.

(6) Our own measures of "aptitude" are always *performance* tests, and by definition there is no other way for aptitudes to be measured. The "postulates" are treated in (8), below.

(7) The notion of fusion of seeds, which appears in Plato, seems taken by him from Sicilian medicine, from a tradition different from that of Empedocles (in the *Timaeus*, Plato generates "seeds" from the "marrow"

which "coordinates" the parts of an organism).

(8) The basic source here is Plato's passage, *Republic* 546A ff. That passage is about genetics, and in the middle of it appears a 3-4-5 triangle which all scholars identify as a Pythagorean genetic symbol. Some of the extraordinary problems raised by the passage can be found discussed in J. Adam, *Republic,* II, Appendixes to Book VIII. I have assumed that the part of this passage subsequent to the triangle is an elaboration of the Pythagorean diagram, on the basis of Aristotle's comment (*Politics* 1316a) that the triangle is the "principle" explaining all the rest of the construction. I have assumed the first phrase to be Pythagorean in essence, because the triangular figure is grammatically represented as derived from it. If this is right, it may help explain why the effort expended on explaining this text by reference to *Platonic* theories has not produced any generally accepted complete interpretation; it may also explain why Plato presents this theory as a "story," as though not willing to take it entirely literally.

(9) The status of the irrational is discussed in Heath, *History of Greek Mathematics*, I. Plato's passage begins with a derivation of the number governing human generation from the "increase of roots to their squares." Aristotle's reference, *Metaphysics* 1091a, to Pythagoreans who talk about numbers as "generated from seed" preserves a comparable metaphor. The contrast of "roots" and "magnitudes" is made explicitly in Plato, *Theaetetus* 146a. Just before passing on to the triangle in the key *Republic* passage cited, Plato seems to complete his metaphor of "roots growing" with the statement that these increases "reaching three stages with four termini, make everything comparable and in ratio." The Hellenistic commentators believed that it was the fact of its being a right triangle that was the central point of the genetic symbol (see Heath, *Euclid*, I. 2nd edn., "Popular Names for Euclidean Propositions. I. 47"). That would point to the rule that inheritance equals the square root of the sums of the squares of parental endowment, rather than the simpler rule of the average which seems to apply in Plato.

(10) The notion that traits occur in multiples of some elementary quantity, not varying continuously, did not appear until Greek Physics had developed theories of "elements" which were definite quanta.

(11) Aristotle, *Metaphysics* 986a, quotes a Pythagorean table of opposites in which odd numbers are masculine, even numbers are feminine. Plato, *Timaeus* 35 ff., uses series of odd and even numbers to represent qualitative distinctions, and this seems to follow a Pythagorean tradition (see, on this point, A. E. Taylor, *A Commentary on Plato's Timaeus,* notes on 35 ff.).

(12) In spite of uncertainties of interpretation, one is tempted to see evidence that the "sum of squares" rule was used by the Pythagoreans in the triple repetition of an operation of "squaring" and in the reference to "rational diagonals" (nearest integral approximations to the values of roots) in Plato's passage.

(13) See (16), below.

(14) Plato's "scales" are those established by himself in *Phaedrus* 249 and *Republic* 587D. In the Pythagorean table Aristotle quotes, cited above, good and one, evil and many are grouped together, another indication that the best development of a trait would be ranked as 1 and put at the top of any ordinal scale set up by a Pythagorean. Plato himself changed this procedure, at least in his later dialogues, and cites evidence (at 424A) that contradicts it in the *Republic.*

(15) If we take a "generation" of statesmen as about twenty years, this theory would predict a radical change, for the worse, every 80 years. There seems no way of telling how far the Pythagoreans had worked out the social implications of their theory; it was just such implications which made genetics interesting to Plato.

(16) The solution adopted seems to have been to regard sex (and presumably all other either-or traits) as due to environmental factors, hence not hereditary at all. Parmenides and Empedocles both agreed to this suggestion.

(17) In the *Symposium*, Plato makes fun of the Hippocratic theory; in the *Republic* and *Timaeus* he shows his admiration for the elegance of Pythagorean mathematical science, but cites phenomena which their theory will not explain.