PROBABILISTIC MODELS

Sequential Models

Some experiments have a sequential character

- **Tree-Based Sequential Description**

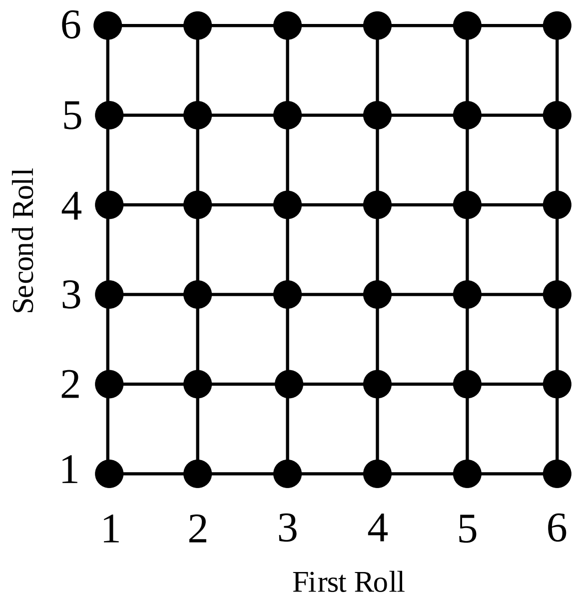
Experiment: Two rolls of a 6-sided die

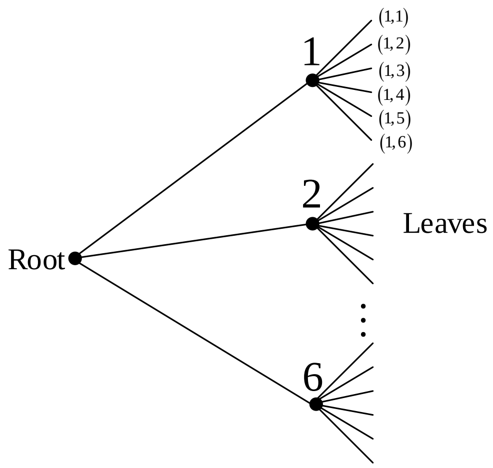
Textbook: D. P. Bertsekas, J. N. Tsitsiklis, "Introduction to Probability", 2nd Ed., Athena Science 2008.

Textbooks: Fikri Öztürk, Levent Özbek, "Matematiksel Modelleme ve Simülasyon", 2004.
Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.

PROBABILISTIC MODELS

Description of the sample space





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PROBABILISTIC MODELS
Probability Axioms

1) **Nonnegativity:**

   $P(A) \geq 0$ for every event *A*.

2) **Additivity:**

   A and *B* are two <u>disjoint events</u>,

$$P(A \cup B) = P(A) + P(B)$$

   More generally,

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \cdots$$

3) **Normalization:**

   Ω : universal set

$$P(\Omega) = 1$$

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PROBABILISTIC MODELS

Some properties

$$1 = P(\Omega) = P(\Omega \cup \varnothing) = 1 + P(\varnothing),$$

which means

$$P(\varnothing) = 0$$

If $A_1, A_2, \text{and}\, A_3$ disjoint events, then

$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) &= P\big(A_1 \cup (A_2 \cup A_3)\big) \\ &= P(A_1) + P(A_2 \cup A_3) \\ &= P(A_1) + P(A_2) + P(A_3) \end{aligned}$$

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PROBABILISTIC MODELS

Discrete Models: Coin tossing, die rolling
Continuous Models: Wheel of fortune, between 0 and 1.

Example 1.2 single coin toss experiment outcomes: heads (*H*) and tails (*T*).

sample space: $\{H,T\}$
events: $\{H,T\},\{H\},\{T\},\varnothing$

"equally likely": equal probabilities

$$P\left(\{H\}\right)=P\left(\{T\}\right)=0.5$$

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PROBABILISTIC MODELS
Example 1.2. (Continued)

Additivity and normalization axioms

$$P\left(\{H,T\}\right) = P\left(\{H\}\right) + P\left(\{T\}\right) = 1$$

Nonnegativity axiom

$$P\left(\text{"Any Event"}\right) \geq 0$$

The probability law:

$$P\left(\{H,T\}\right) = 1, \quad P\left(\{H\}\right) = 0.5, \quad P\left(\{T\}\right) = 0.5, \quad P\left(\varnothing\right) = 0,$$

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PROBABILISTIC MODELS

Discrete Probability Law:

finite number of possible outcomes

$$P\left(\left\{s_1, s_2, \ldots, s_n\right\}\right) = P\left(s_1\right) + P\left(s_2\right) + \cdots + P\left(s_n\right)$$

Uniform Probability Law:

- n possible outcomes, equally likely

$$P\left(A\right) = \frac{\text{number of elements of } A}{n}$$

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PROBABILISTIC MODELS

Example: Discrete Sample Space

Single roll of a 6-sided die.

- Discrete Sample Space:

$$S = \{1, 2, 3, 4, 5, 6\}$$

- Some Events:

$$A = \{\text{a number less than 4 appears}\} = \{1, 2, 3\}$$

$$A = \{\text{an even number appears}\} = \{2, 4, 6\}$$

$$A = \{\text{1 appears}\} = \{1\}$$

$$A = \{\text{a number greater than 6 appears}\} = \varnothing$$

$$A = \{\text{1 or 2 appears}\} = \{1, 2\}$$

$$A = \{\text{1 and 2 appears}\} = \varnothing$$

$$A = \{\text{an integer in the interval [1,6] appears}\} = \{1, 2, 3, 4, 5, 6\}$$