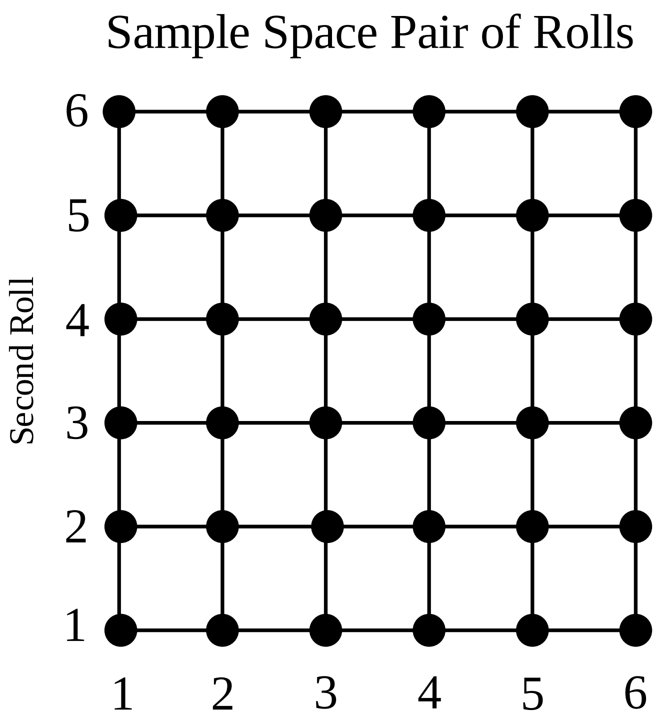
PROBABILISTIC MODELS

Example: Consider the experiment of rolling a pair of <u>fair dice</u>.

<u>Fair dice assumption:</u> probability each outcome: 1/36.

$$S = \{(i,j) : (1,1),(1,2),(1,3),\ldots,(4,3),\ldots,(6,4),(6,5),(6,6)\}$$



$$P(\{\text{the sum is even}\}) = \frac{18}{36} = \frac{1}{2}$$

$$P(\{\text{the sum is odd}\}) = \frac{18}{36} = \frac{1}{2}$$

$$P(\{\text{the first roll is larger than the second one}\}) = \frac{1}{3}$$

Textbook: D. P. Bertsekas, J. N. Tsitsiklis, “Introduction to Probability”, 2nd Ed., Athena Science 2008.

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

PROBABILISTIC MODELS
Continuous Models

Example 1.4. A wheel of fortune, continuously calibrated [0,1]
Experiment: a single spin, numbers in the interval $\Omega = [0,1]$.
fair wheel: equally likely,
What is the probability of the event consisting of a single element?

Answer: It cannot be positive. Remember the additivity axiom, the probability of a single element must be 0.

What is the probability of any subinterval $[a,b]$?

$$P([a,b]) = \frac{\text{Length of Interested Interval}}{\text{Length of Sample Space}} = \frac{b-a}{1} \quad \rightarrow \quad \text{Continuous Uniform Law}$$

Textbook: D. P. Bertsekas, J. N. Tsitsiklis, “Introduction to Probability”, 2nd Ed., Athena Science 2008.

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
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PROBABILISTIC MODELS

Properties of Probability Laws:

1) If $A \subset B$, then $P(A) \leq P(B)$

2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

3) $P(A \cup B) \leq P(A) + P(B)$

4) $P(A \cup B \cup C) = P(A) + P(A^C \cap B) + P(A^C \cap B^C \cap C)$

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Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Equations of Motion

History
Nicolaus Copernicus (1473-1543)
Giordano Bruno (1548-1600)
Galileo Galilei (1564-**1642**)
Johannes Kepler (1571-1630)
Isaac Newton (**1642**-1727)

$$h = V_{0y}t - \frac{1}{2}gt^2 \qquad h_{\max} = \frac{V_{0y}^{\;2}}{2g}$$

$$X_m = V_{0x}.t_u \qquad t_f = \frac{2V_{0y}}{g}$$

$$V_{0x} = V_0\cos(\alpha)\ , \qquad V_{0y} = V_0\sin(\alpha)$$

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Equations of Motion

$$V=\frac{dh}{dt}=d\frac{\left(V_0t-\frac{1}{2}gt^2+h_i\right)}{dt}=V_0-gt$$

$$t_f=\sqrt{\frac{2h_0}{g}}$$

$$t_1=\frac{V_0}{g}$$

$$t_2=\sqrt{\frac{2h_0}{g}}=\sqrt{\frac{2\left(\frac{V_0^2}{2g}+h_i\right)}{g}} \qquad \left(h_{\max}=\frac{V_0^2}{2g}+h_i\right)$$

$$t_f=t_1+t_2$$

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Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Equations of Motion

$$x = V_{x0}t = V_0 \cos(\theta_0)t \qquad t = \frac{x}{V_0 \cos(\theta_0)}$$

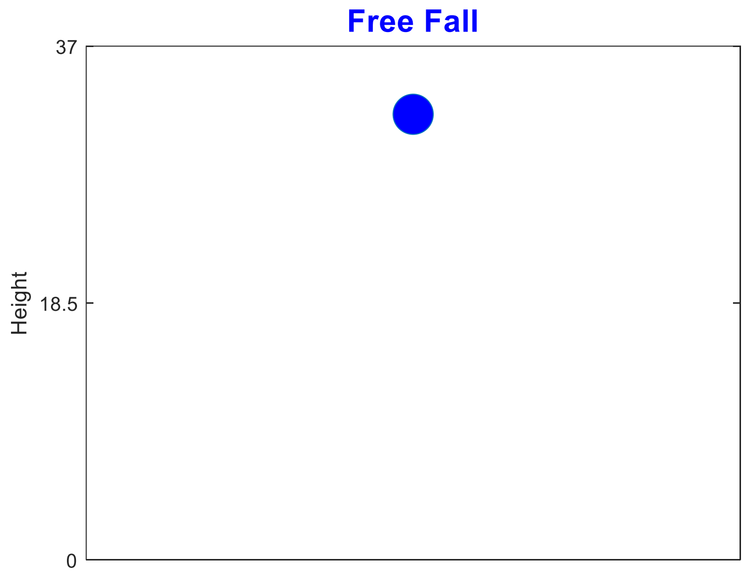
$$y = V_0 \sin(\theta_0)t - \frac{1}{2}gt^2$$

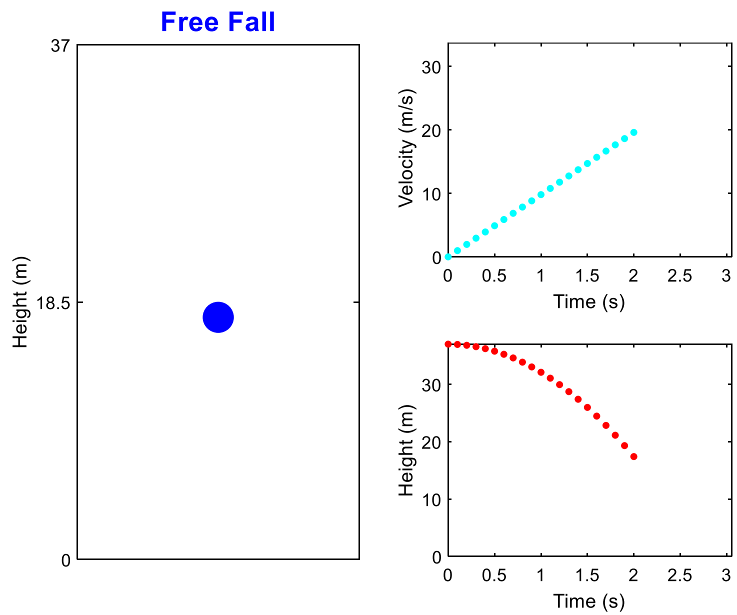
$$y = V_0 \sin(\theta_0)\frac{x}{V_0 \cos(\theta_0)} - \frac{1}{2}g\left(\frac{x}{V_0 \cos(\theta_0)}\right)^2$$

$$y = \tan(\theta_0)x - \left(\frac{g}{2V_0^{\,2} \cos^2(\theta_0)}\right)x^2$$

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Free Fall



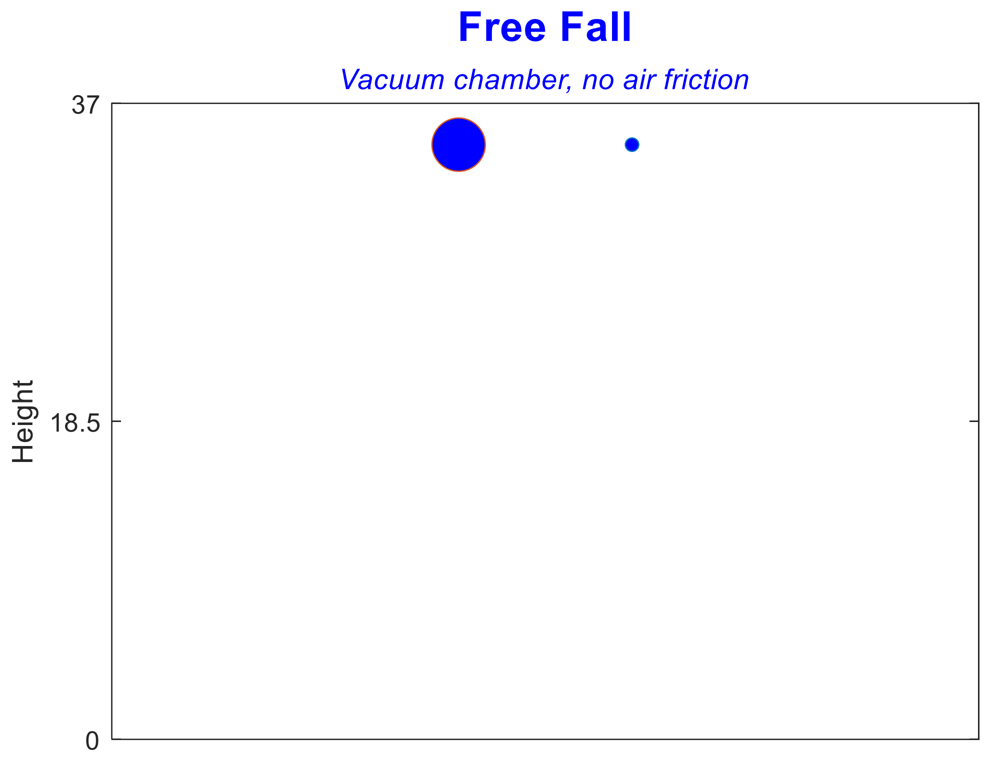


Simulation of free fall.

○ Speed-time and position-time graphs simultaneously with the simulation

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

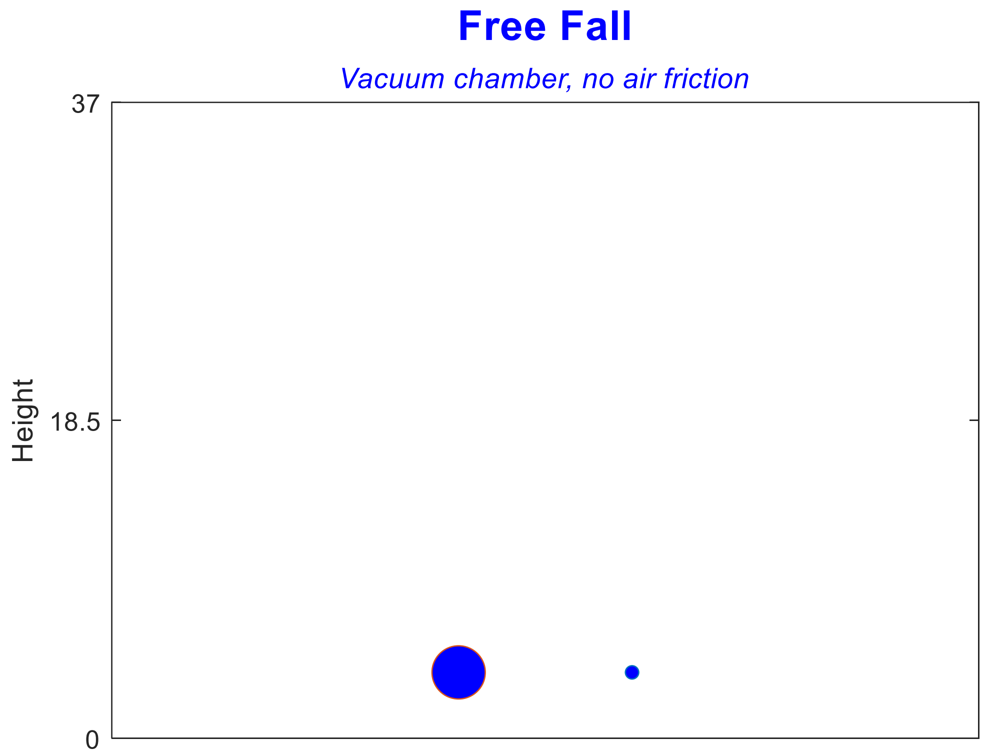
Free Fall



Free fall: two objects of different mass

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Free Fall



For more realistic simulation: You may try to adjust the heights of the objects so that their lower ends are at the same level.

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Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.