Free Fall with Air Resistance

Find an equation for the free fall time (with air resistance) and compare with the equation given below.

An equation for time of flight can be obtained by solving the position-time equation at page 14 in the textbook (Öztürk, 2004) for "*t*".

- Obtain this equation
- MATLAB Symbolic Math Toolbox may be used to obtain this solution

Textbooks: Fikri Öztürk, Levent Özbek, "Matematiksel Modelleme ve Simülasyon", 2004.
Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.

Free Fall with Air Resistance

```
t_Fric =

(m*lambertw(0, -exp(-(y*k^2 + g*m^2)/(g*m^2))) + (y*k^2 + g*m^2)/(g*m))/k
```

```
pretty(t_Fric)
```

```
          /        /      2      2 \ \      2      2
          |        |   y k  + g m  | |   y k  + g m
m lambertw| 0, -exp| - ----------- | | + -----------
          |        |          2    | |       g m
          \        \       g m     / /
----------------------------------------------------
                          k
```


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Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Free Fall with Air Resistance
MATLAB Symbolic Math Toolbox

A simple example applying MATLAB *solve* function

```
Clear
Close all
syms x y
q = solve(y == x^2, x)
pretty(q)
y = 9;
eval(q)
```

A simple example applying MATLAB *simplify* function

```
clear
close all
syms x y
q = (1 - x^2)/(1 - x);
pretty(q)
simplify(q)
```


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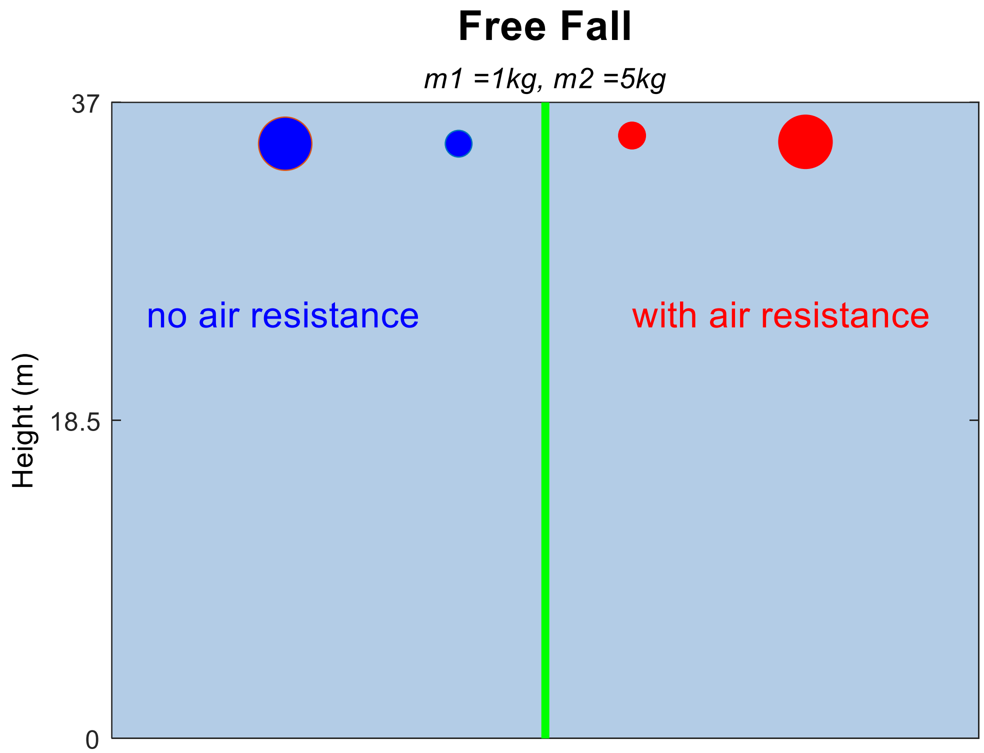
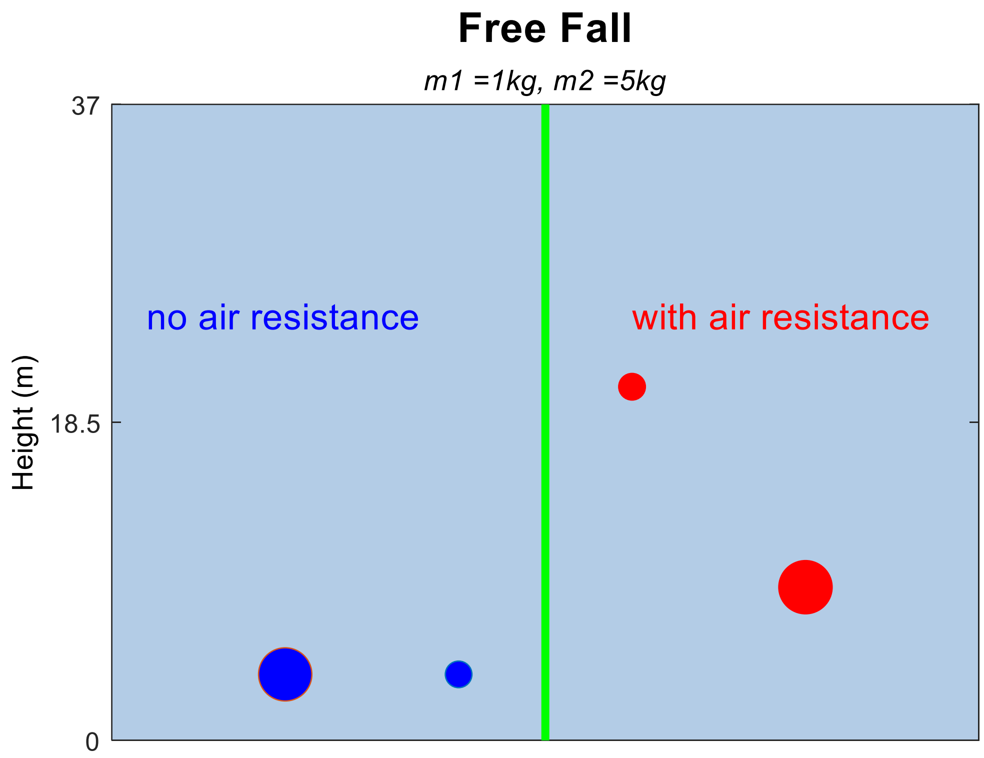
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Free Fall with Air Resistance



Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Free Fall with Air Resistance



For more realistic simulation: You may try to adjust the heights of the objects so that their lower ends are at the same level.

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Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Model Parameter Estimation
Example-I

The following observations belong to a system which can be modeled by a quadratic polynomial and measurement errors are Gaussian. Estimate the coefficients of the polynomial.

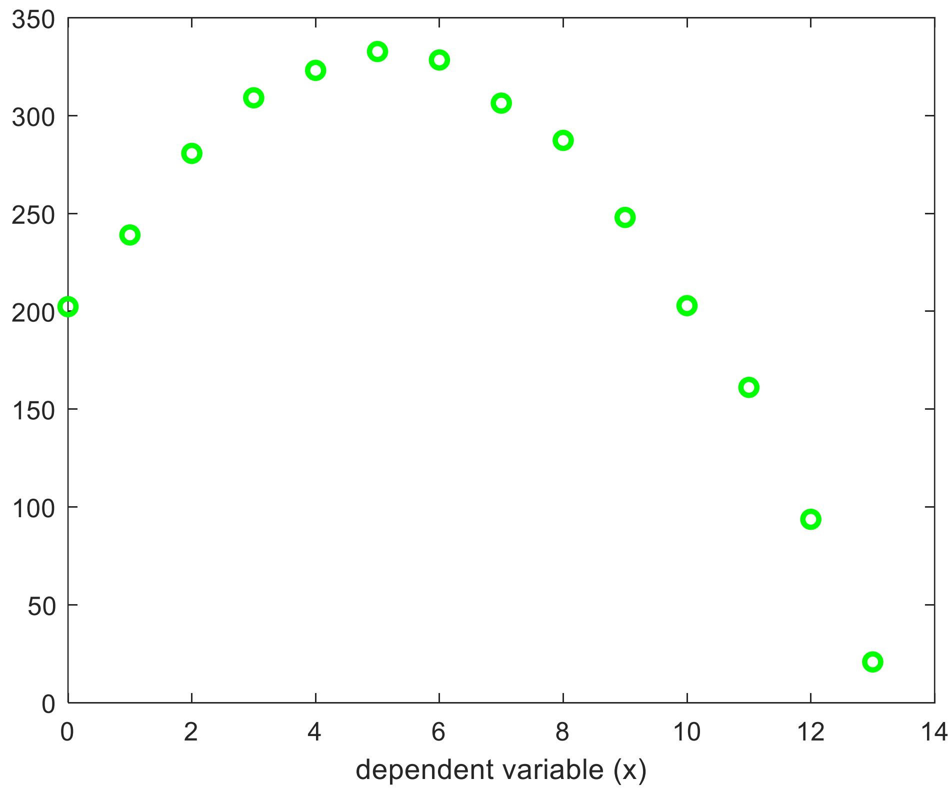
Assume that

- independent variables (x) are observed without error
- measurement errors in the dependent variables (y) are i.i.d and Gaussian.

x_i	Y_i	x_i	Y_i
0	202.36	7	306.40
1	239.03	8	287.36
2	280.71	9	247.97
3	309.12	10	202.89
4	323.15	11	161.11
5	332.78	12	93.68
6	328.45	13	20.78

Textbooks: Fikri Öztürk, Levent Özbek, "Matematiksel Modelleme ve Simülasyon", 2004.
Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.

Model Parameter Estimation
Example-I (Continued)



Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Model Parameter Estimation
Example-I (Continued)

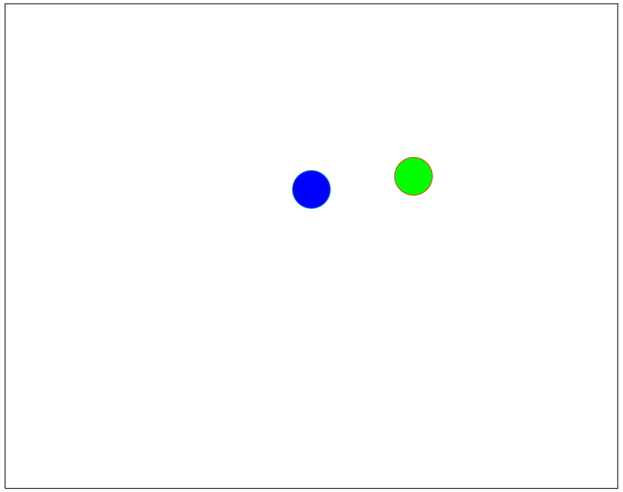
If it is given that the observations in the table belong to a motion in one dimension and the dependent and independent variables are the observations of the height of the object and the time, respectively,

- Estimate the initial velocity of the object and gravitational acceleration
- Give the mathematical model of the motion.
- Animate the motion.
- Plot speed-time and position-time graphs simultaneously with the animation.

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Model Parameter Estimation
Example-I (Continued)

- In the animation of the motion, at the measurement times, plot the object both in the position given by the mathematical model and in the measurement position in two different colors.



Positions of the object at the measurement times: (blue) given by the mathematical model and (green) measurement.

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Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.