Modeling of Thermal Noise and Measurement Error

A Gaussian random variable can be employed for modeling thermal noise and measurement error.

Gaussian PDF

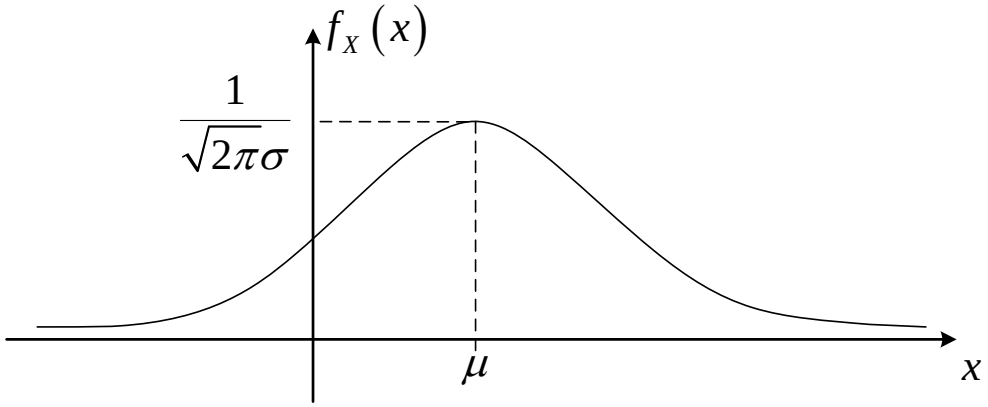
$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \qquad -\infty < x < \infty$$

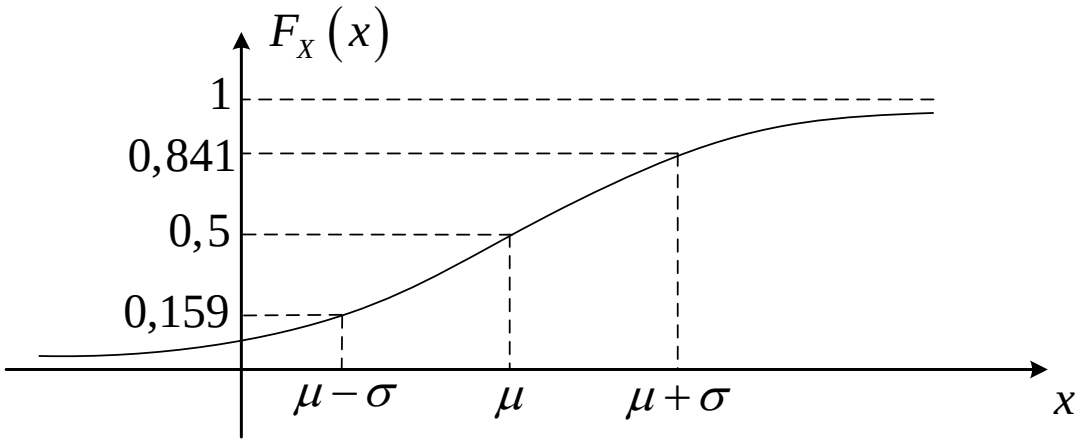
where μ is **mean** and σ is the **standard deviation**.

$X \sim N(\mu, \sigma^2) \rightarrow$ X is Gaussian distributed with mean μ and variance σ^2

Textbooks: Fikri Öztürk, Levent Özbek, "Matematiksel Modelleme ve Simülasyon", 2004.
Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.

Gaussian Random Variables





$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt$$

Textbooks: Fikri Öztürk, Levent Özbek, "Matematiksel Modelleme ve Simülasyon", 2004.
Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.

Gaussian Random Variables

Normality is preserved by linear transformation.

$$Y = aX + b$$

Y is also a normal random variable with mean and variance:

$$E[Y] = E[aX + b] = aE[X] + b$$

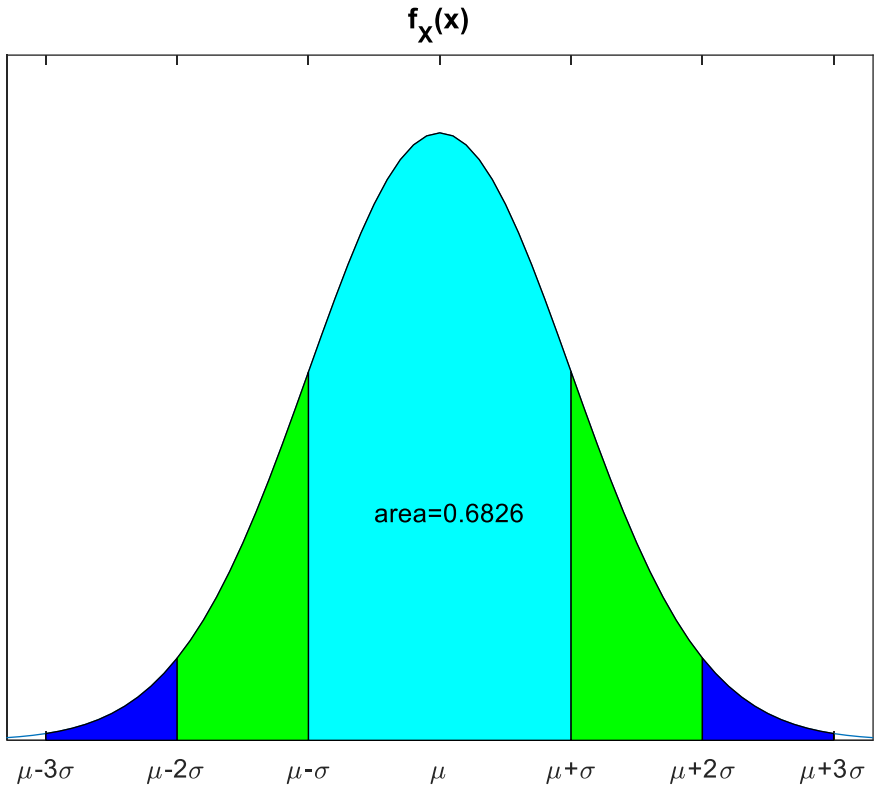
$$\text{var}(Y) = \text{var}(aX + b) = a^2 \,\text{var}(X)$$

Homework: Try to show these properties by simulations.

Textbook: D. P. Bertsekas, J. N. Tsitsiklis, "Introduction to Probability", 2nd Ed., Athena Science 2008.

Textbooks: Fikri Öztürk, Levent Özbek, "Matematiksel Modelleme ve Simülasyon", 2004.
Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.

Gaussian Random Variables



$$P\left(\left|X-\mu\right|\le\sigma\right) = P\left(-1<Y\le 1\right) = 0.68262$$

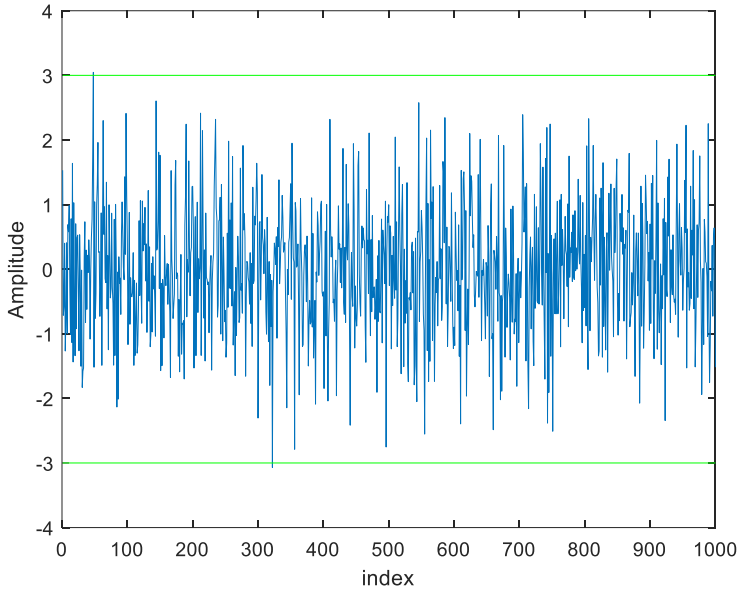
$$P\left(\left|X-\mu\right|\le 2\sigma\right) = P\left(-2<Y\le 2\right) = 0.9545$$

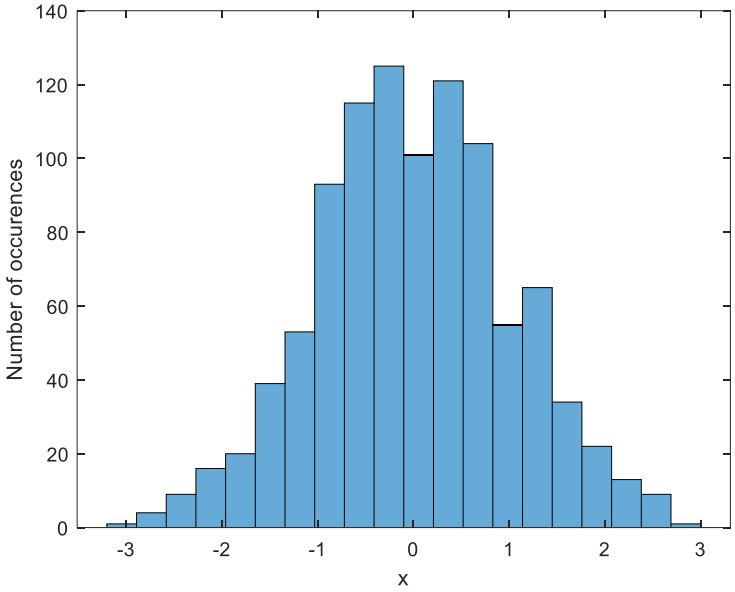
$$P\left(\left|X-\mu\right|\le 3\sigma\right) = P\left(-3<Y\le 3\right) = 0.9973$$

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Generating Gaussian Random Numbers

Homework: Try to obtain these probability values through random number generation.



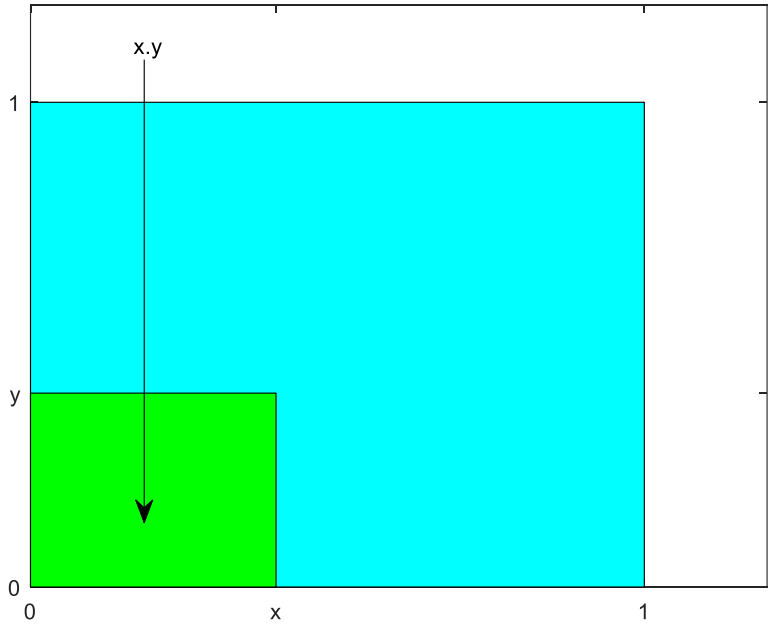


Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Estimating PDF and CDF by Generating Random Samples

Let X and Y be random variables with a joint PDF of the form

$$f_{X,Y}(x,y) = \begin{cases} 1, & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 1, \\ 0, & \text{otherwise} \end{cases}$$



Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Estimating PDF and CDF by Generating Random Samples

Homework

Estimate joint PDF and CDF based on histogram which is created using random numbers.

This homework helps you understand the joint PDF and CDF deeply.

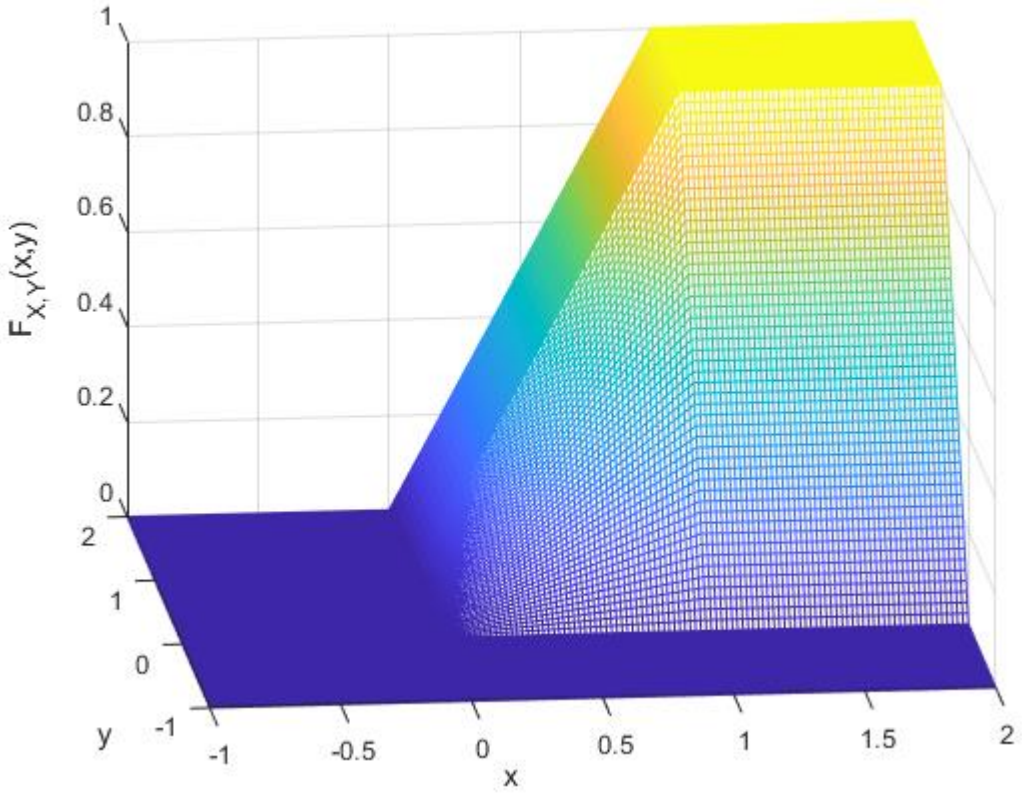
For **each** generated random number, update PDF and CDF estimations based on histogram and visualize the estimation process at **each** step.

Observe the effect of number of generated random numbers.

Observe the effect of width of the histogram bins.

Textbooks: Fikri Öztürk, Levent Özbek, "Matematiksel Modelleme ve Simülasyon", 2004.
Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.

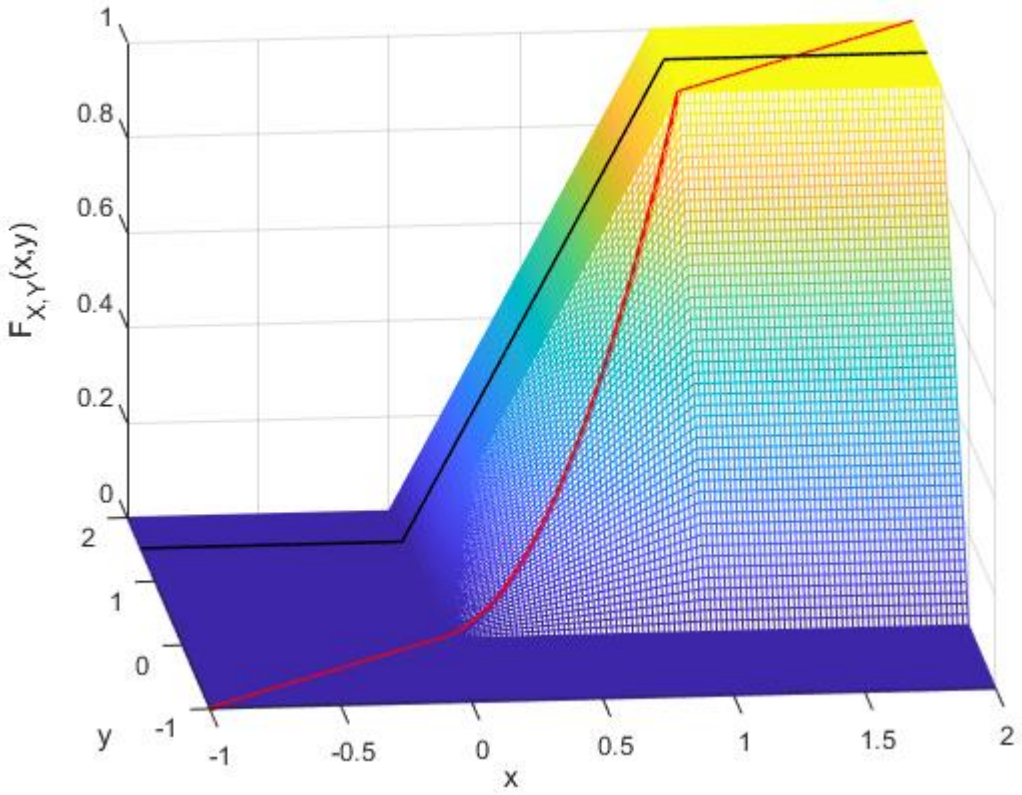
Estimating PDF and CDF by Generating Random Samples



Cumulative Distribution Function (analytical)

Textbooks: Fikri Öztürk, Levent Özbek, “Matematiksel Modelleme ve Simülasyon”, 2004.
Averill M. Law, “Simulation Modeling and Analysis”, McGraw-Hill, 2015.

Estimating PDF and CDF by Generating Random Samples



"Diagonal" and "y=1.5" lines

Textbooks: Fikri Öztürk, Levent Özbek, "Matematiksel Modelleme ve Simülasyon", 2004.
Averill M. Law, "Simulation Modeling and Analysis", McGraw-Hill, 2015.