PEN205
MODERN PHYSICS

Introduction to Quantum Physics

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If $V \rightsquigarrow C$ → Newtonian mechanics must be replaced by Einstein’s special theory of relativity

↓

solved many experimental and theoretical problems

But for many other problems, however, neither relativity nor classical physics could provide a theoretical answer.

Applying the laws of classical physics to explain the behavior of matter on the atomic scale → unsuccessful

1900-1930 → Quantum mechanics: a new revolution in Physics

Quantum Mechanics was highly successful in explaining the behavior of particles of microscopic size (atoms, molecules and nucleus)

Quantum Mechanics approaches to classical physics when applied to macroscopic systems.

The first explanation of a phenomenon using quantum theory was introduced by Max Planck.

Many subsequent mathematical developments and interpretations were made by


BETTMANN/CORBIS

MAX PLANCK
(1858–1947)


THE ALBERT EINSTEIN ARCHIVES, HEBREW UNIVERSITY OF JERUSALEM

ALBERT EINSTEIN
(1879–1955)


AIP EMILIO SEGRÈ VISUAL ARCHIVES

NIELS BOHR
(1885–1962)


AIP MEGGERS GALLERY OF NOBEL LAUREATES

LOUIS DE BROGLIE
(1892–1987)


BETTMANN/CORBIS

ERWIN SCHRÖDINGER
(1887–1961)


AIP EMILIO SEGRÈ VISUAL ARCHIVES (GIFT OF JOSH LEMMERICH)

MAX BORN
(1882–1970)


AIP EMILIO SEGRÈ VISUAL ARCHIVES (GIFT OF JOSH LEMMERICH)

WERNER HEISENBERG
(1901–1976)

In this lesson, we will see the principles that underline the quantum theory.

Outline

1. Black body radiation and Planck’s hypothesis
2. Photoelectric effect
3. Compton effect
4. Double Slit Experiment
5. Wave properties of particles
6. The Nature of Electromagnetic Waves

1.Black Body Radiation and Planck's Hypothesis

From previous lessons ;

- An object at any temperature emits electromagnetic waves in the form of thermal radiation from its surface
- The characteristics of this radiation depend on the temperature and properties of the object's surface
- Radiation consists of a continuous distribution of wavelengths from all portions of the electromagnetic spectrum
- At room temperature → thermal radiation are mainly in the infrared region → radiation is not detected by the human eye
- As the surface temperature of the object increases → begins to glow visibly red (eg. the coils of a toaster)
- At sufficiently high temperatures → the glowing object appears white (eg. hot tungsten filamen)

a classical viewpoint

↓

thermal radiation originates from accelerated charged particles in the atoms near the surface of the object;

↓

Insufficient to explain the observed distribution of wavelengths in the radiation emitted by a black body

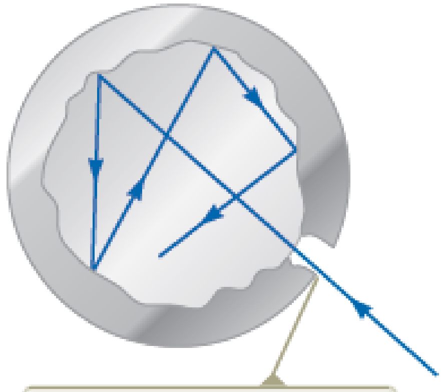
A **black body** is an ideal system that absorbs all radiation incident on it.

The electromagnetic radiation emitted by the black body is called blackbody radiation

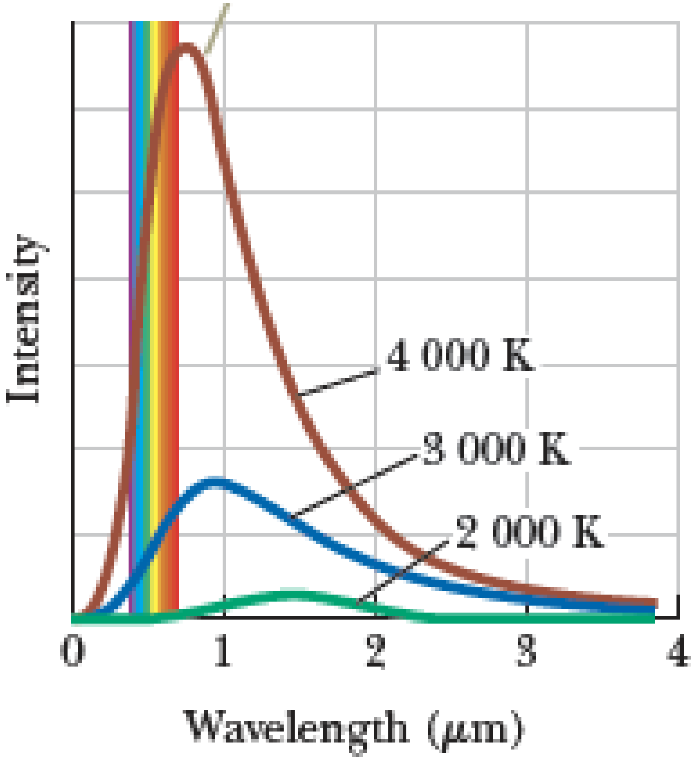
Any radiation incident on the hole from outside the cavity enters the hole and is reflected a number of times on the interior walls of the cavity → the hole acts as a perfect absorber.

The nature of the radiation leaving the cavity through the hole depends only on the temperature of the cavity walls and not on the material of which the walls are made.

very interesting !!



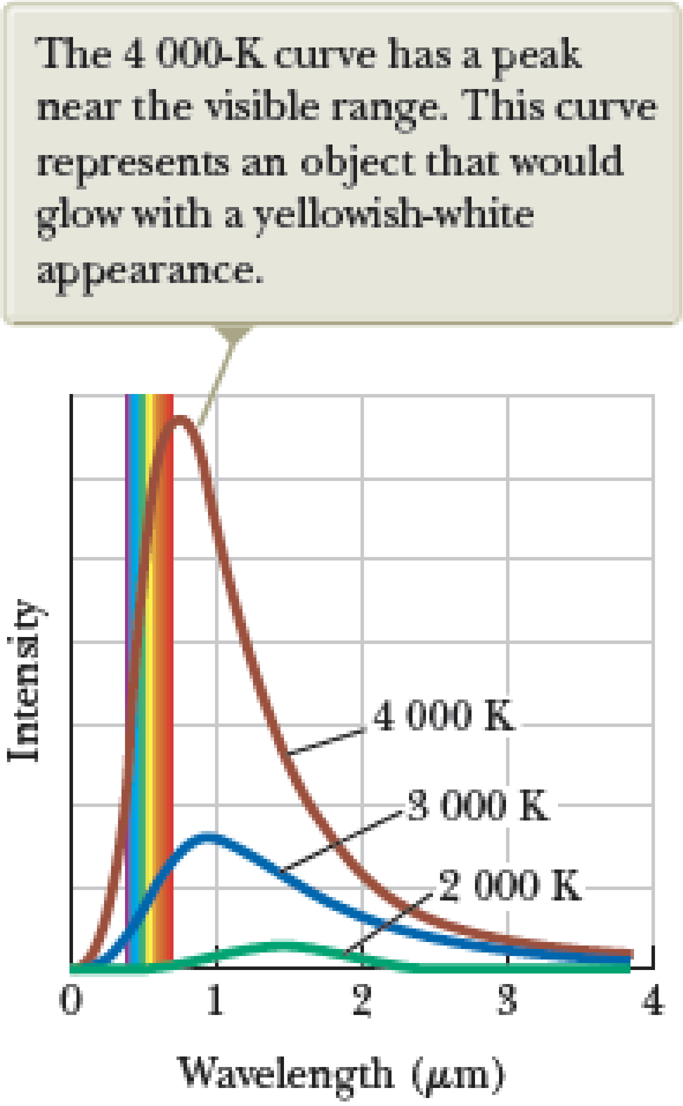
The opening to a cavity inside a hollow object is a good approximation of a black body: the hole acts as a perfect absorber.



→ The wavelength distribution of radiation from cavities was studied experimentally in the late 19th century.

Figure shows how the intensity of blackbody radiation varies with temperature and wavelength.

Two experimental observations;


The 4 000-K curve has a peak near the visible range. This curve represents an object that would glow with a yellowish-white appearance.

1- The total power of the emitted radiation increases with temperature.

Stefan's Law:

$$P = \sigma A e T^4$$

$\sigma = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$

Stefan–Boltzmann constant

surface temperature

power (W) radiated at all wavelengths

surface area

Emission constant

ratio of incoming energy versus emitted energy

e=1 for black body

2- The peak of the wavelength distribution shifts to shorter wavelengths as the temperature increases.

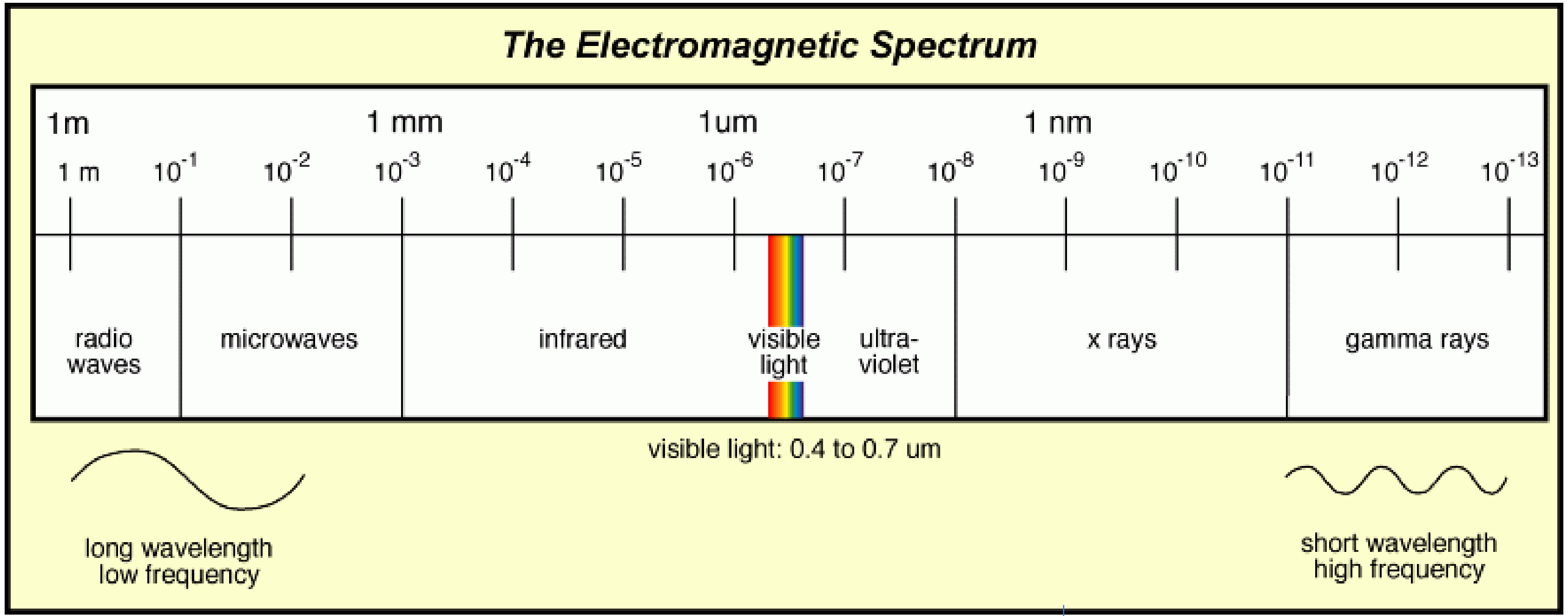
$$\lambda_{\max} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$$

Wien's displacement law

The wavelength at the curve's peak is inversely proportional to the absolute temperature;

↓

as the temperature increases, the peak is "displaced" to shorter wavelengths



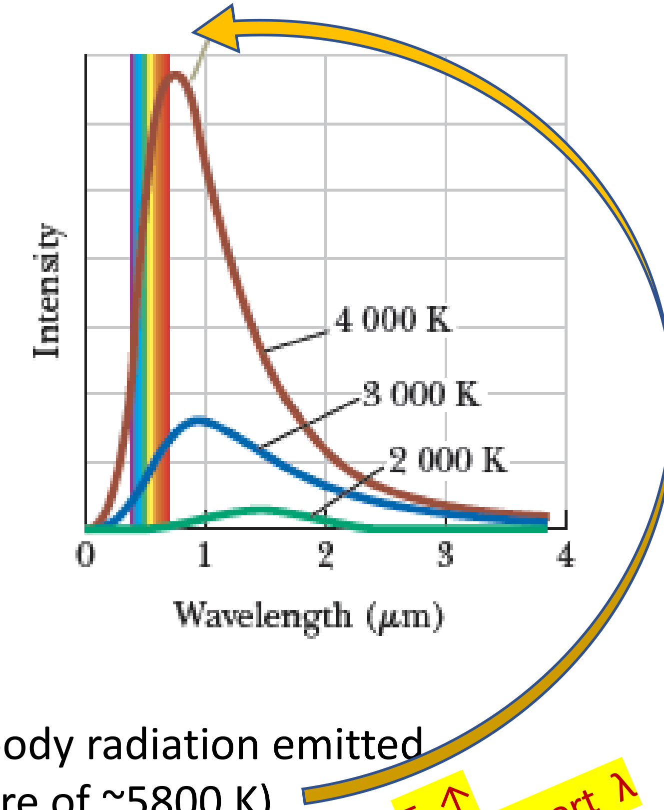
Room temperature ~ 300 K

$$\lambda_{\max} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$$

λ_{max} = 2.898 x 10^{-3} / 300

≈ 10^{-5} m → infrared region → object does not glow

Wien's displacement law is consistent with the behavior at room temperature



peak wavelength of the blackbody radiation emitted by the Sun (surface temperature of ~5800 K)

T ↑ peak shifts to short λ

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{5\,800\ \mathrm{K}} = 0.500\ \mu\mathrm{m}$$

≈ 0.5 x 10^{-6} m

↓

Yellow-green

John Chumack/Photo Researchers, Inc.

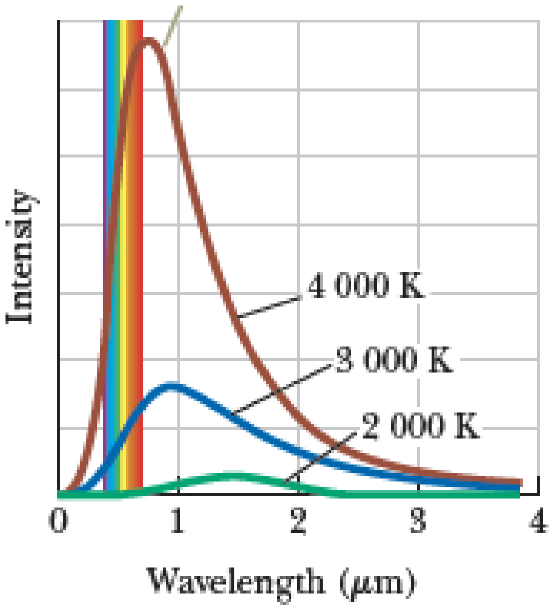
Figure shows two stars in the constellation Orion.

Betelgeuse → glow red
Rigel → glow blue

Which star has a higher surface temperature?

(a) Betelgeuse
(b) Rigel
(c) both the same
(d) impossible to determine

5p to the first correct answer… ☺



A successful theory for blackbody radiation must predict

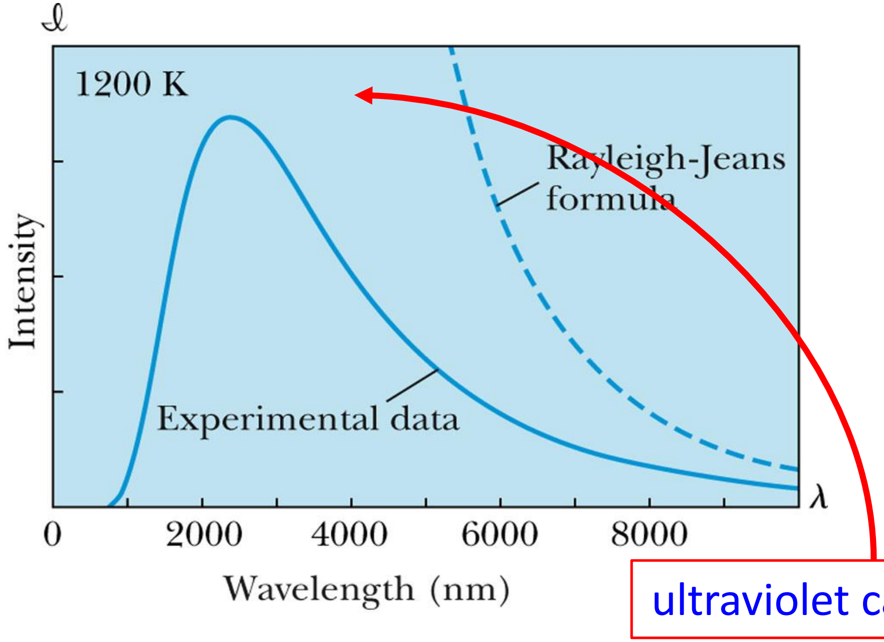
- the shape of the curves
- the temperature dependence expressed in Stefan's law $P = \sigma A e T^4$
- the shift of the peak with temperature described by Wien's displacement law.

$$\lambda_{\max} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$$

Early attempts to use classical ideas to explain the shapes of the curves;

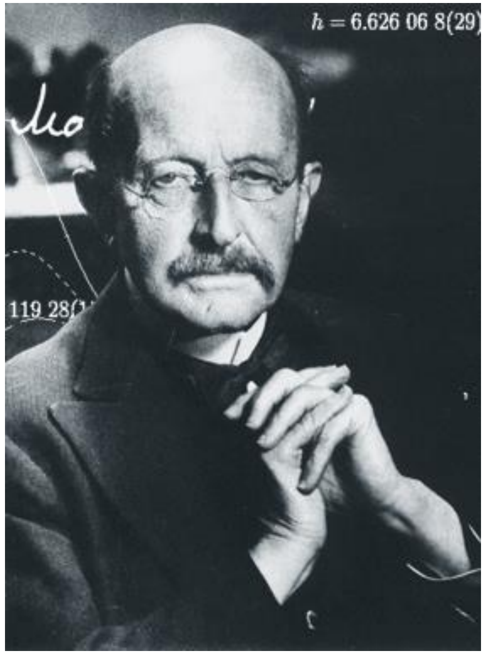
Rayleigh–Jeans law: a classical theory of blackbody radiation

$$I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}$$



ultraviolet catastrophe

agrees with experimental results at large wavelengths
but strongly disagrees at short wavelengths



In 1900, **Max Planck** developed a theory of blackbody radiation that leads to an equation that is in complete agreement with experimental results at all wavelengths.

According to Planck's theory;

1- cavity radiation came from atomic oscillators in the cavity walls

2- The energy of an oscillator can have only certain discrete values

oscillator's frequency

quantum number

$$E_n = nhf$$

Planck's constant

$h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s}$

Energy of each oscillator can have only discrete values → energy is quantized

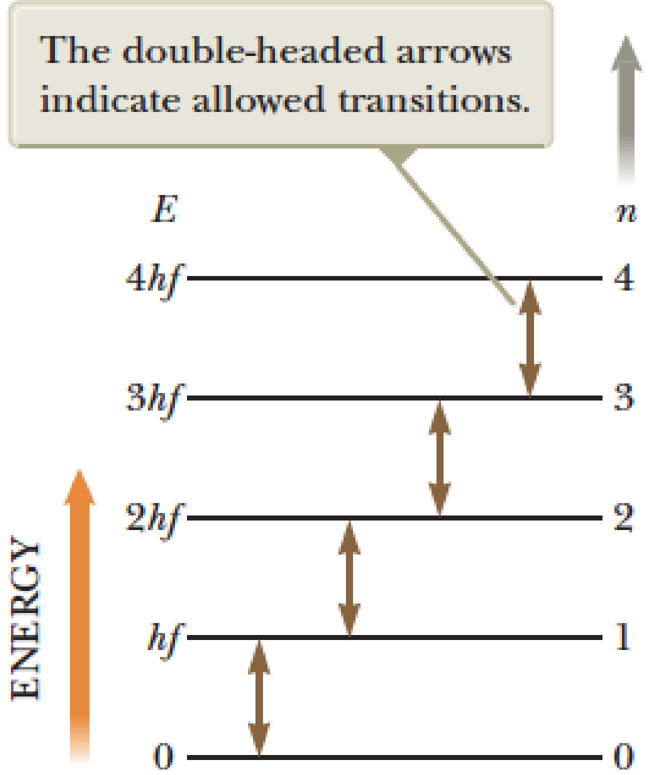
Each discrete energy value corresponds to a different quantum state, represented by the quantum number **n**.

The oscillators emit or absorb energy when making a transition from one quantum state to another.

The entire energy difference between the initial and final states in the transition is emitted or absorbed as a single quantum of radiation.

$$E_n = nhf$$

- ➔ an oscillator emits or absorbs energy only when it changes quantum states.
- ➔ If it remains in one quantum state, no energy is absorbed or emitted.


The double-headed arrows indicate allowed transitions.

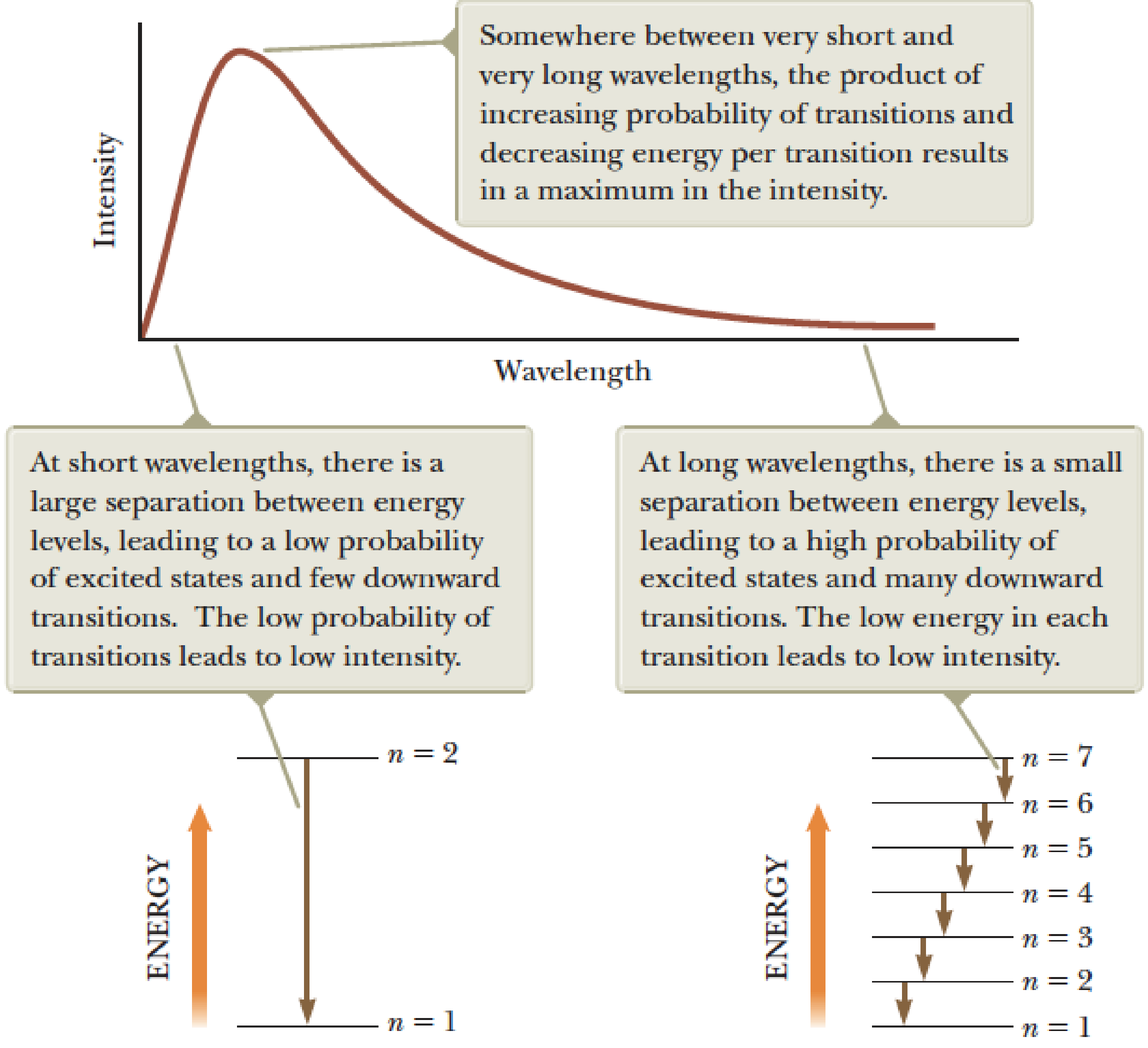
Birth of Quantum Theory

$$\boxed{\Delta E = hf}$$

$$I(\lambda,T) = \cancel{\frac{2\pi c k_B T}{\lambda^4}} \longrightarrow \boxed{I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}}$$

Rayleigh-Jeans
(classic)

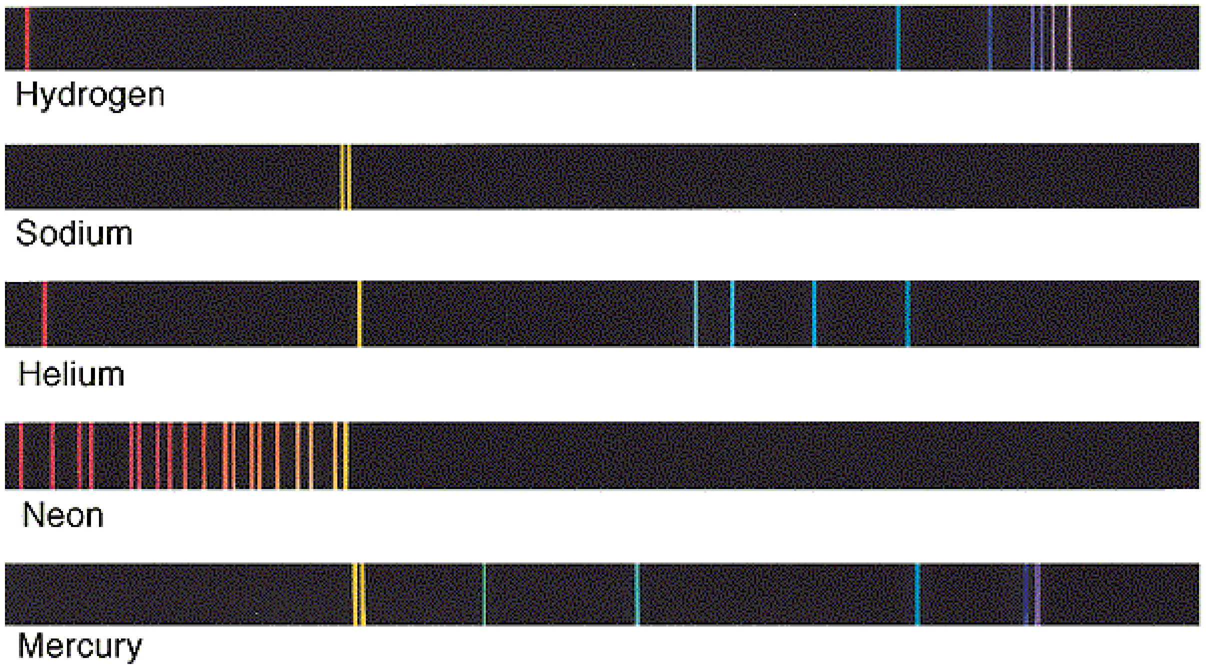
Max Planck
(quantum)


Somewhere between very short and very long wavelengths, the product of increasing probability of transitions and decreasing energy per transition results in a maximum in the intensity.

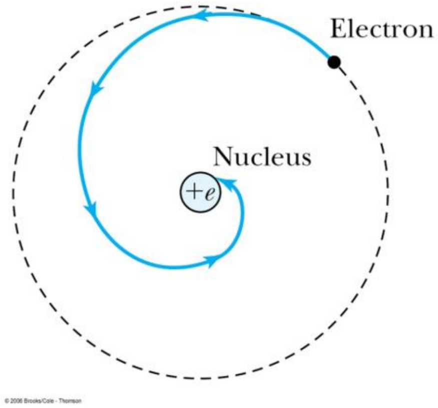
At short wavelengths, there is a large separation between energy levels, leading to a low probability of excited states and few downward transitions. The low probability of transitions leads to low intensity.

At long wavelengths, there is a small separation between energy levels, leading to a high probability of excited states and many downward transitions. The low energy in each transition leads to low intensity.

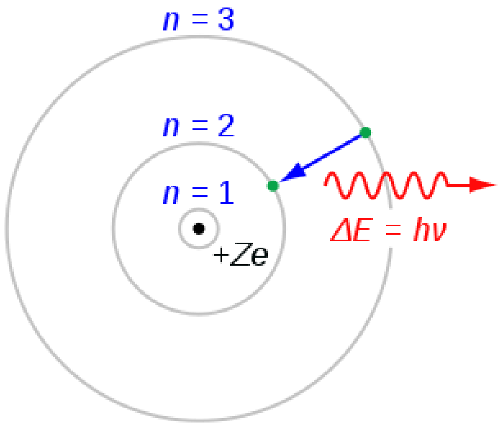
Chemical elements were observed to produce unique wavelengths of light when burned or excited in an electrical discharge.



$$E_n = nhf$$

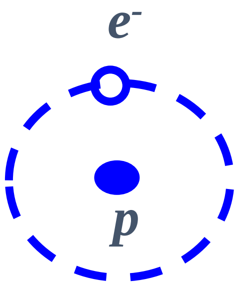


Bohr's Quantization Condition..



Total energy of electron: $$\boxed{E = \frac{1}{2}mv^2 \underbrace{- k\frac{e^2}{r}}_{\text{Coulomb potential energy}}}$$

k= $9\text{x}10^9$ Nm^2/c



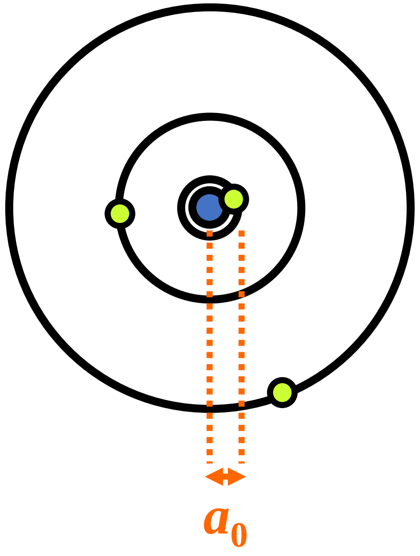
Newton' s 2nd law:

$$\frac{mv^2}{\cancel{r}} = k\frac{e^2}{r^{\cancel{2}}} \Rightarrow mv^2 = k\frac{e^2}{r}$$

$$\Rightarrow \frac{1}{2}mv^2 = k\frac{e^2}{2r}$$

$$E = k\frac{e^2}{2r} - k\frac{e^2}{r} \Rightarrow \boxed{E = -k\frac{e^2}{2r}}$$

Negative sign means electron is bound to proton with that amount of energy:



$$\boxed{r_1 = a_0 = \frac{\hbar^2}{mke^2} = 0.529A^\circ}$$

Bohr's radius

Bohr radius **is the radius of ground state of Hydrogen atom.**

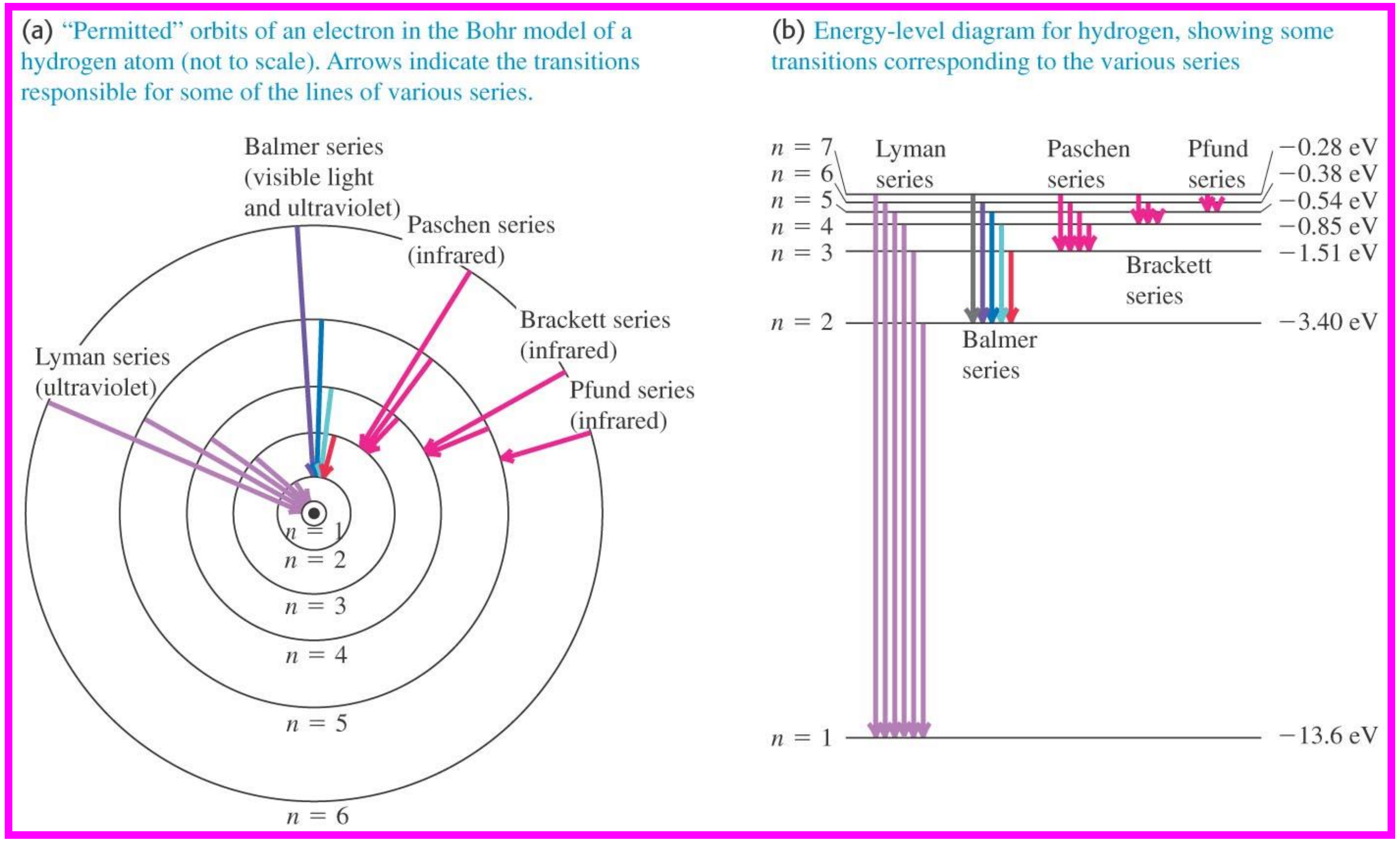
$$\boxed{r_n = a_0 n^2}$$

$$\left.\begin{array}{l} r_n = a_0 n^2 \\ \\ E_n = -k\dfrac{e^2}{2r_n} \end{array}\right\}$$

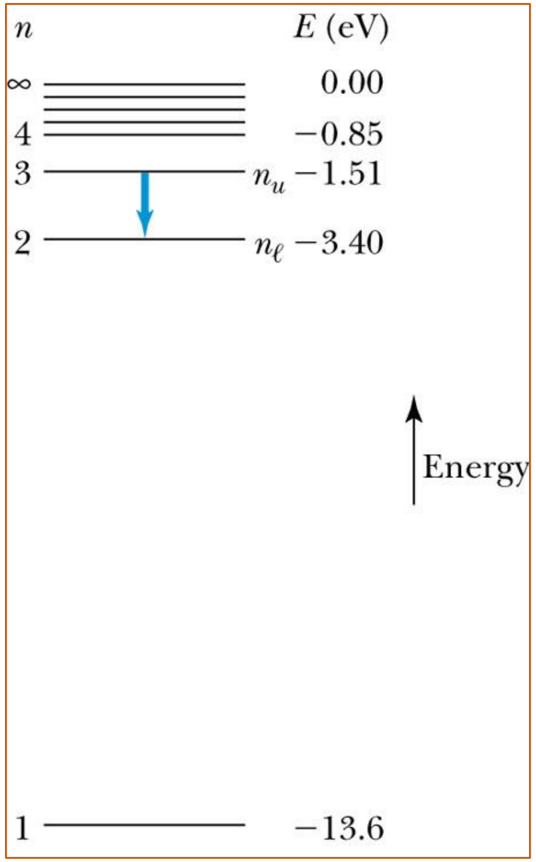
$$E_n = -\underbrace{\frac{ke^2}{2a_0}}_{\textbf{13.6}}\left(\frac{1}{n^2}\right)$$

$$\boxed{E_n = -\frac{13.6}{n^2}\ \text{(eV)}}$$

n=1, 2, 3, ...

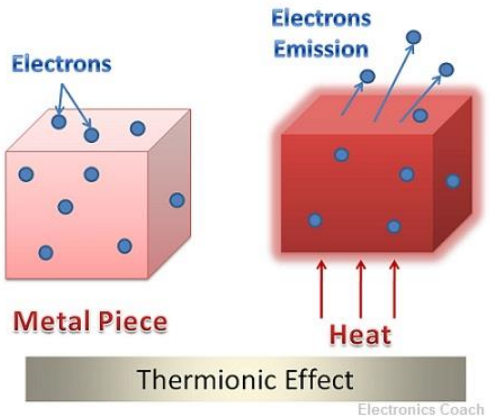

(a) "Permitted" orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series

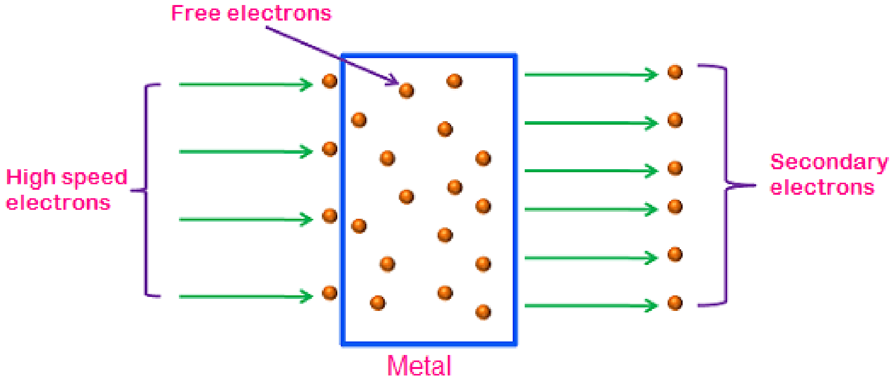


2. Photoelectric Effect

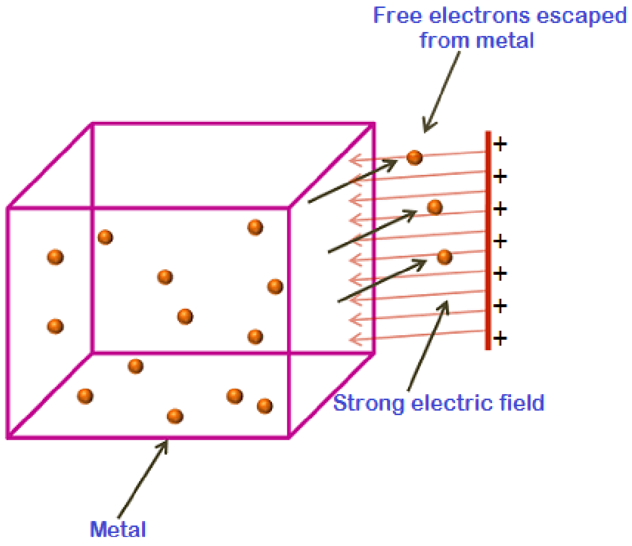
Methods of electron emission:



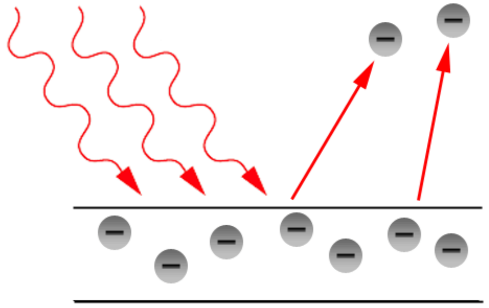
Thermoionic emission: Heat gives energy to electrons to escape.



Secondary electrons: Electron can get enough energy from energy transfer of high energy particles striking a metal surface.

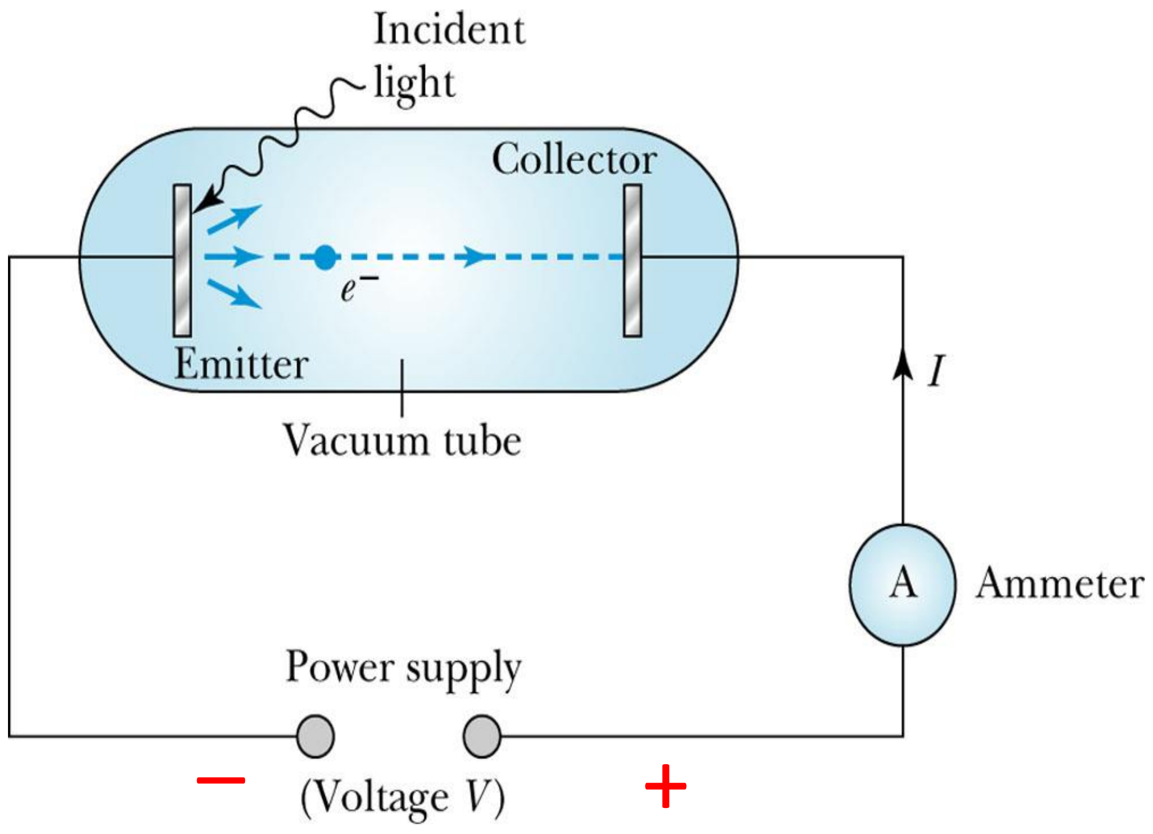


Field emission: Electric field can also give enough energy to electrons to escape from a metal.



Fotoelectric Effect: Electromagnetic radiation coming to the metal transfer energy to electrons and electrons are emitted from the metal's surface. Emitted electrons are called **photoelectrons**.

Experimental setup



When the tube is kept in the dark, the ammeter reads zero, indicating no current in the circuit.

When emitter is illuminated by light having an appropriate wavelength;

⬇

charges flow across the gap between Emitter and Collector

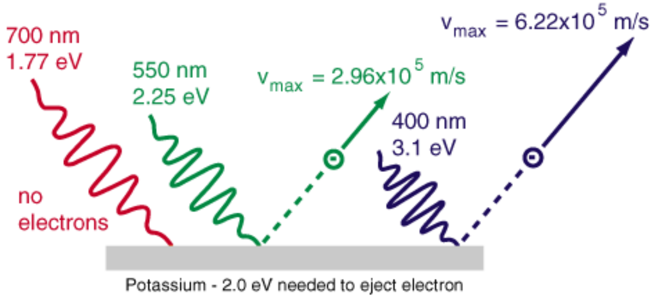
⬇

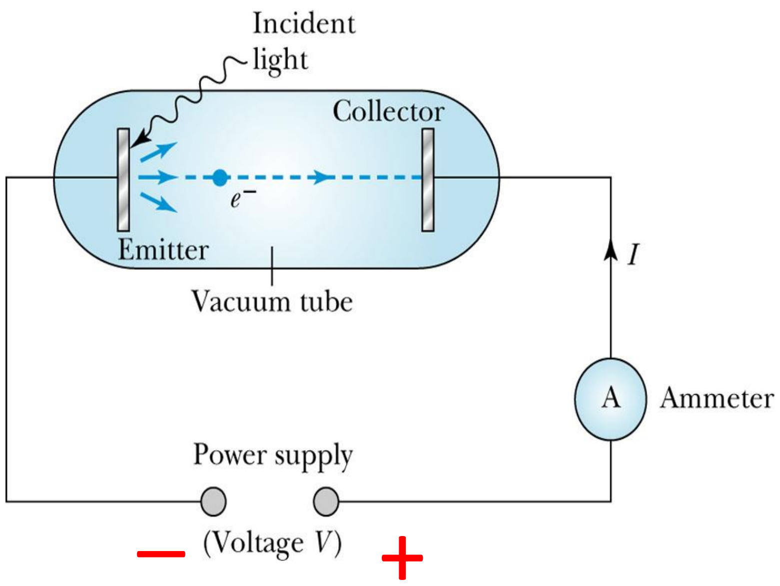
a current is detected by the ammeter.

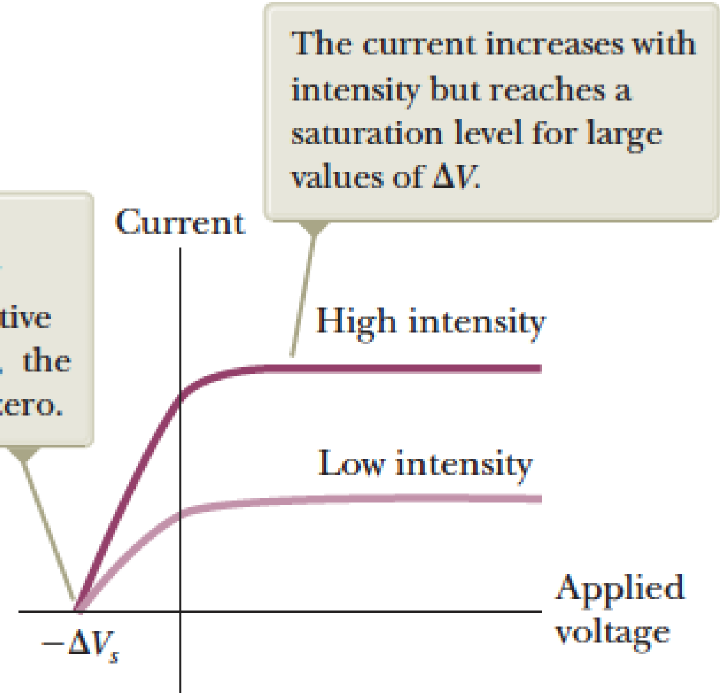
⬇

This current arises from photoelectrons emitted from Emitter and collected at Collector.

$$E_{photon} = h\nu$$





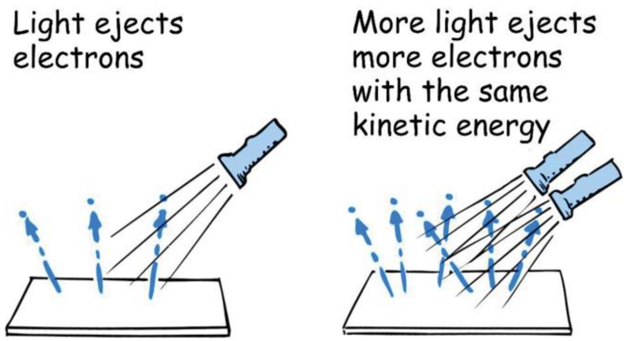

At voltages equal to or more negative than $-\Delta V_s$, the current is zero.

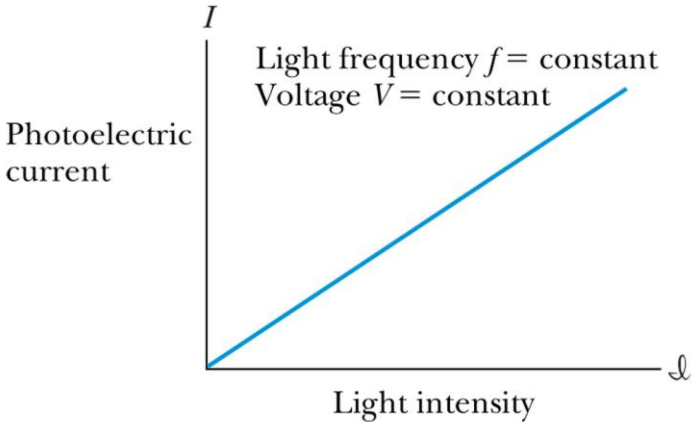
The current increases with intensity but reaches a saturation level for large values of ΔV.

Stopping potential →

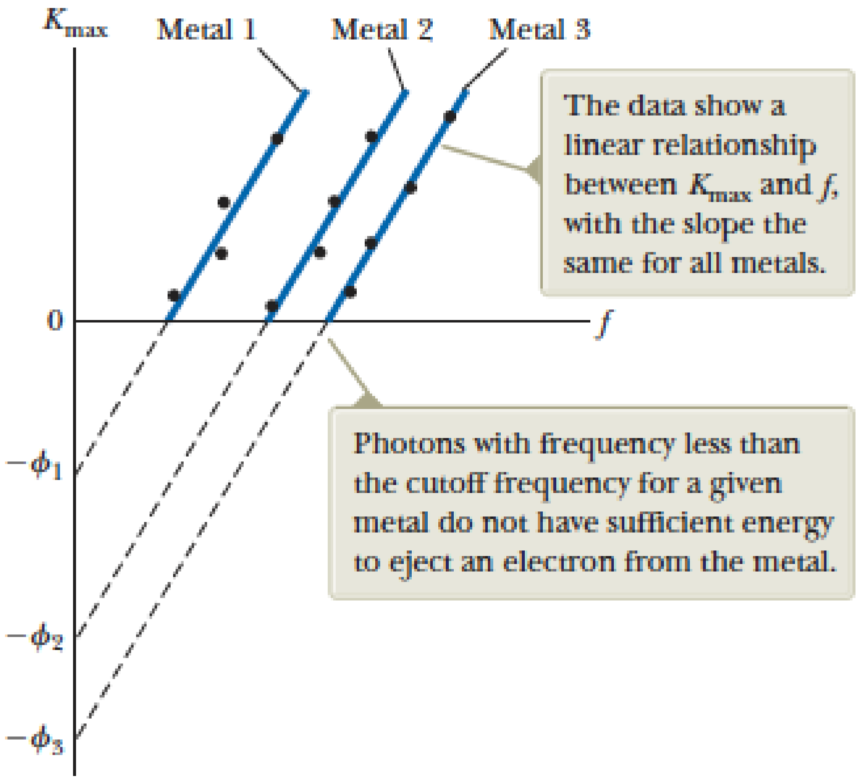
At large values of ΔV , the current reaches a maximum value; all the electrons emitted from E are collected at C, and the current cannot increase further.

the maximum current increases as the intensity of the incident light increases, because more electrons are ejected by the higher-intensity light





Results of Photoelectric Effect



The data show a linear relationship between K_{max} and f, with the slope the same for all metals.

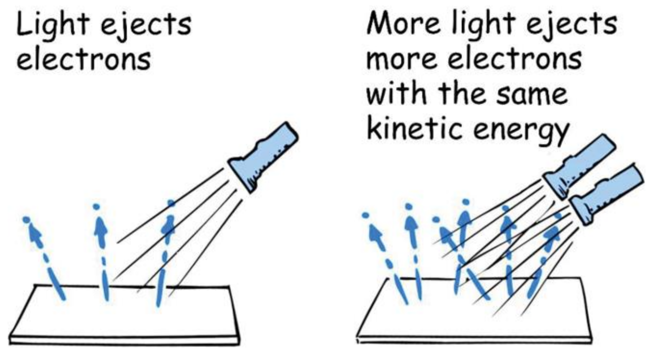
Photons with frequency less than the cutoff frequency for a given metal do not have sufficient energy to eject an electron from the metal.

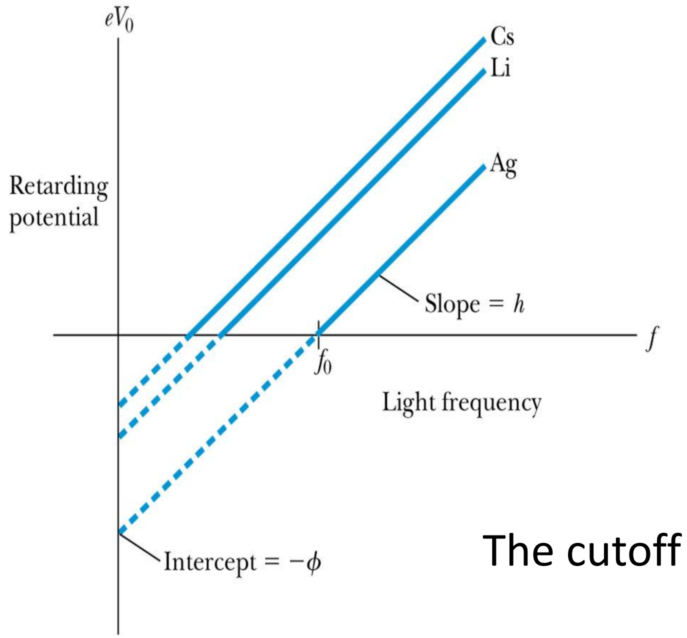
Kinetic energy of photoelectrons are not depend on the intensity of electromagnetic wave.

For a given metal, kinetic energy of photoelectrons depend only on the **frequency** of electromagnetic wave.

If the light intensity is doubled, the number of photons arriving per unit time is doubled, which doubles the rate at which photoelectrons are emitted. The maximum kinetic energy of any one photoelectron, however, is unchanged.

According to the wave theory of classical mechanics, high intensity wave should carry mode energy at unit time to the metal surface. Therefore, electrons emitted from metal should have higher kinetic energies.





No photoelectrons are emitted under a **cutoff frequency**.

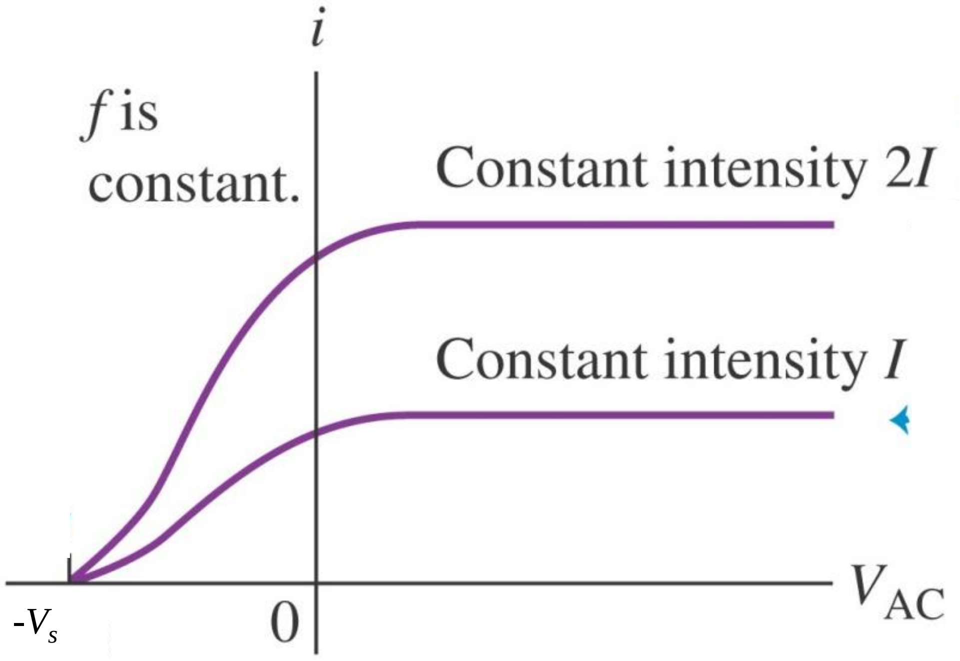
This result contradict with classical wave theory. Because wave theory predicts that electrons can emit at every frequency as long as incoming light has enough intensity.

The cutoff frequency is related to the work function through the relationship $f_c = \phi/h$

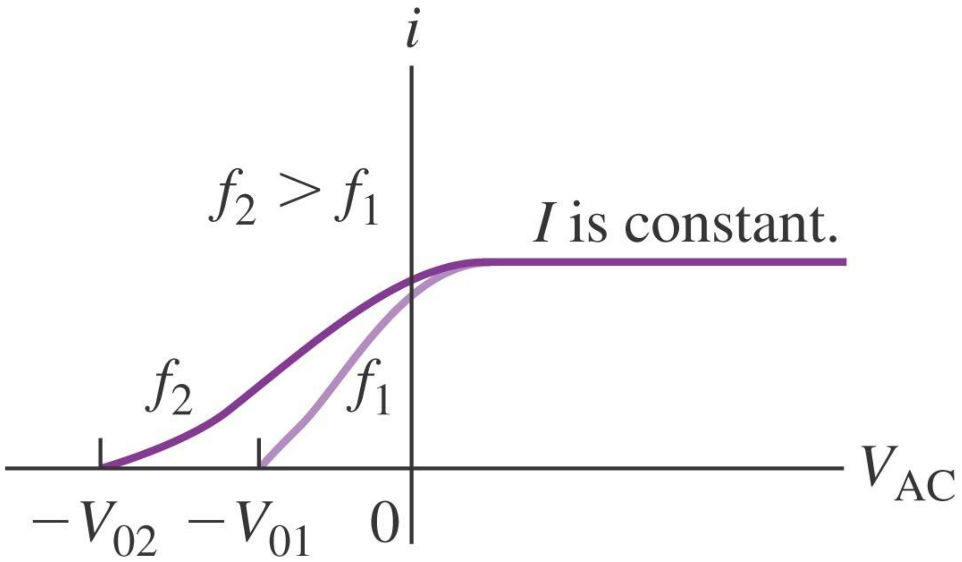
cutoff wavelength → $\lambda_c = \frac{c}{f_c} = \frac{c}{\phi/h} = \frac{hc}{\phi}$

Photoelectrons are created instantaneously upon light strike to the photochatode.

Classical physics predicts that, electrons need some time to absorb and gain enough energy to escape from metal.

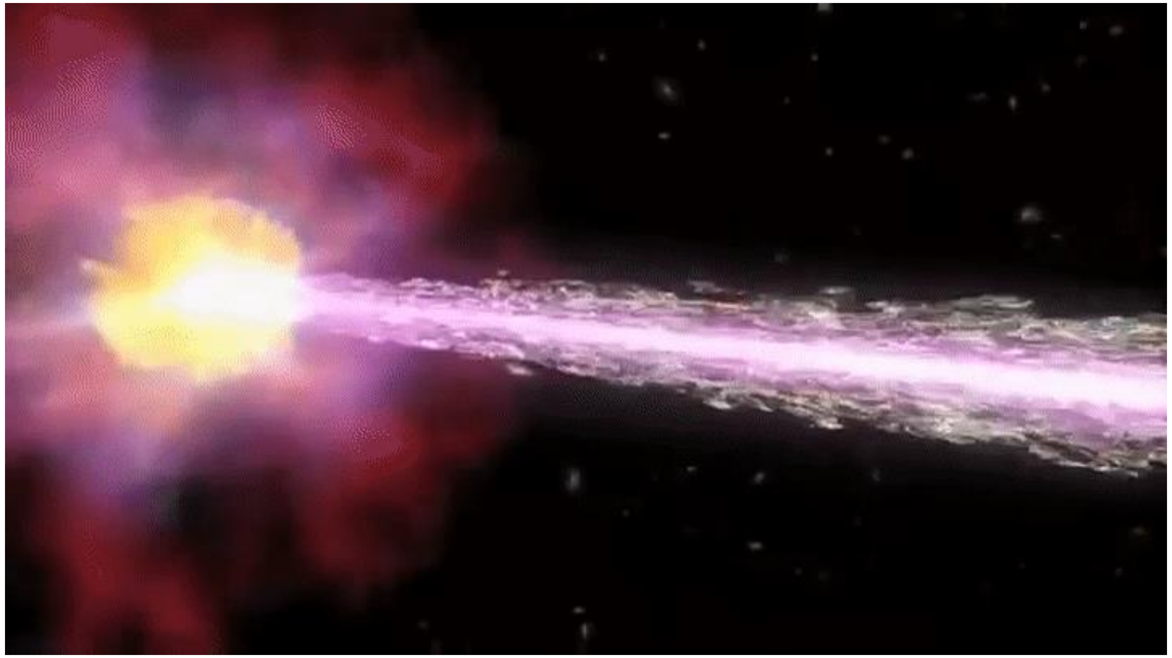


Stopping potential



The stopping potential V_0 (and therefore the maximum kinetic energy of the photoelectrons) increases linearly with frequency: since $f_2 > f_1$, $V_{02} > V_{01}$.

Einstein's Theory of Light : Photons



Einstein created the quantum theory of light, the idea that light exists as tiny packets, or particles, which he called photons.

Energy of each photon:

$$\boxed{E = hf}$$

Alternatively; $E = \hbar w \qquad \hbar \; ^{o} \; h/2p$

According to energy conservation;

Incoming energy (photon) = Outgoing energy (electron)

$$\boxed{hf = \phi + \tfrac{1}{2} m v_{\max}^2}$$

$$K_{\max} = hf - \phi$$

ϕ is the **work function of the metal** (the minimum energy required to remove a delocalized electron from the surface of the metal).

Example:

A sodium surface is illuminated with light having a wavelength of 300 nm. The work function for sodium metal is 2.46 eV.

(A) Find the maximum kinetic energy of the ejected photoelectrons

$$K_{\max} = hf - \phi$$

$$E = hf = \frac{hc}{\lambda} = \frac{(6.63x10^{-34} J.s)(3x10^{8}\, m/s)}{300x10^{-9} m}$$

$$= 6.63x10^{-19} J = 4.14\ eV$$

$$K_{max} = 4.14 - 2.46 = 1.68\ eV$$

(B) Find the cutoff wavelength lc for sodium.

$$\lambda_C = \frac{hc}{\phi} = \frac{(6.63x10^{-34} J.s)(3x10^{8}\, m/s)}{3.94x10^{-19} J}$$

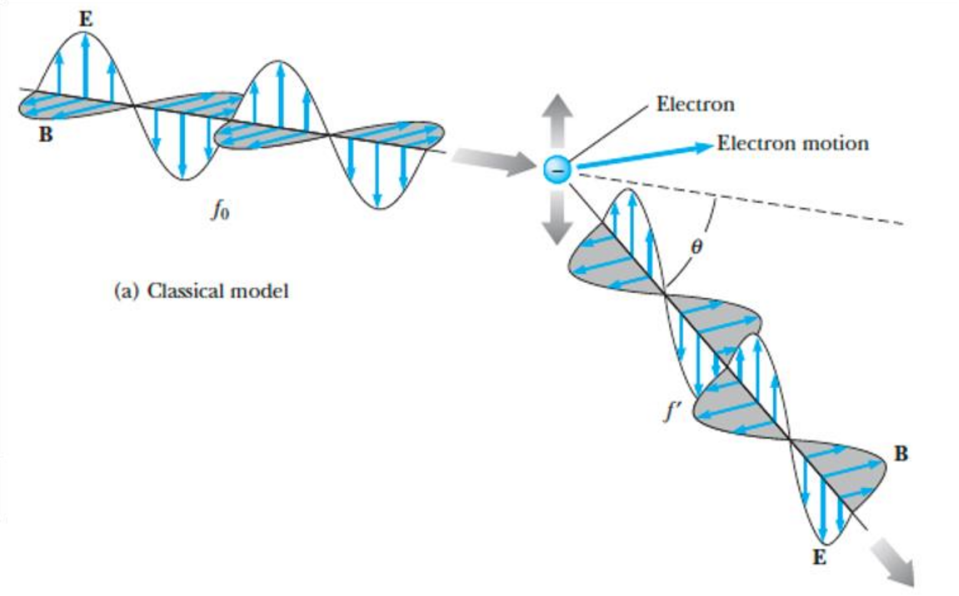
$$= 5.05x10^{-7} m = 505\ nm$$

3. Compton Effect

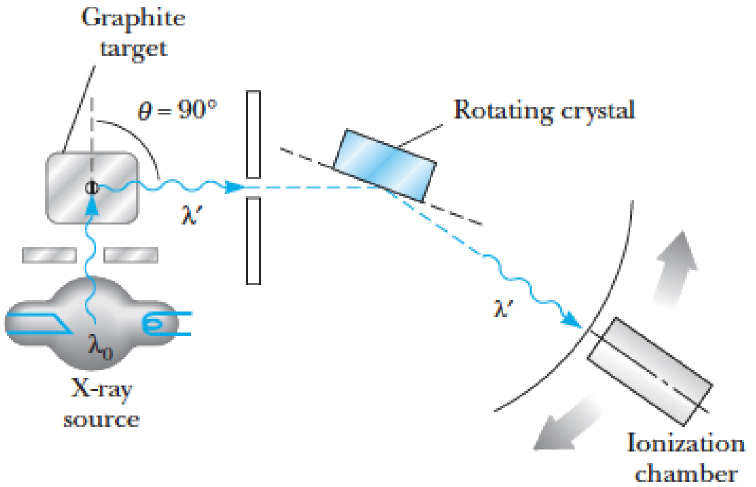
Compton scattering, discovered by Arthur Holly Compton, is the scattering of a photon by a charged particle

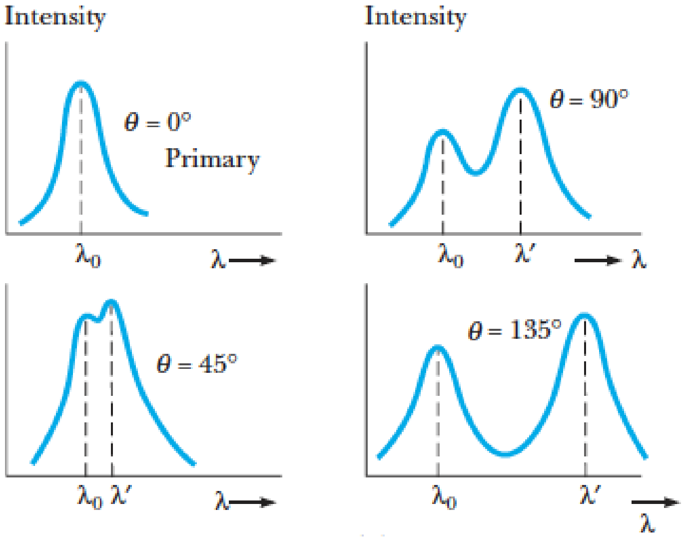
According to classical theory;

- incident radiation of frequency f_0 should accelerate an electron in the direction of propagation of the incident radiation
- it should cause forced oscillations of the electron and re-radiation at frequency f' , where $f' < f_0$
- frequency / wavelength of the scattered radiation should depend on the length of time the electron was exposed to the incident radiation as well as on the intensity of the incident radiation



(a) Classical model



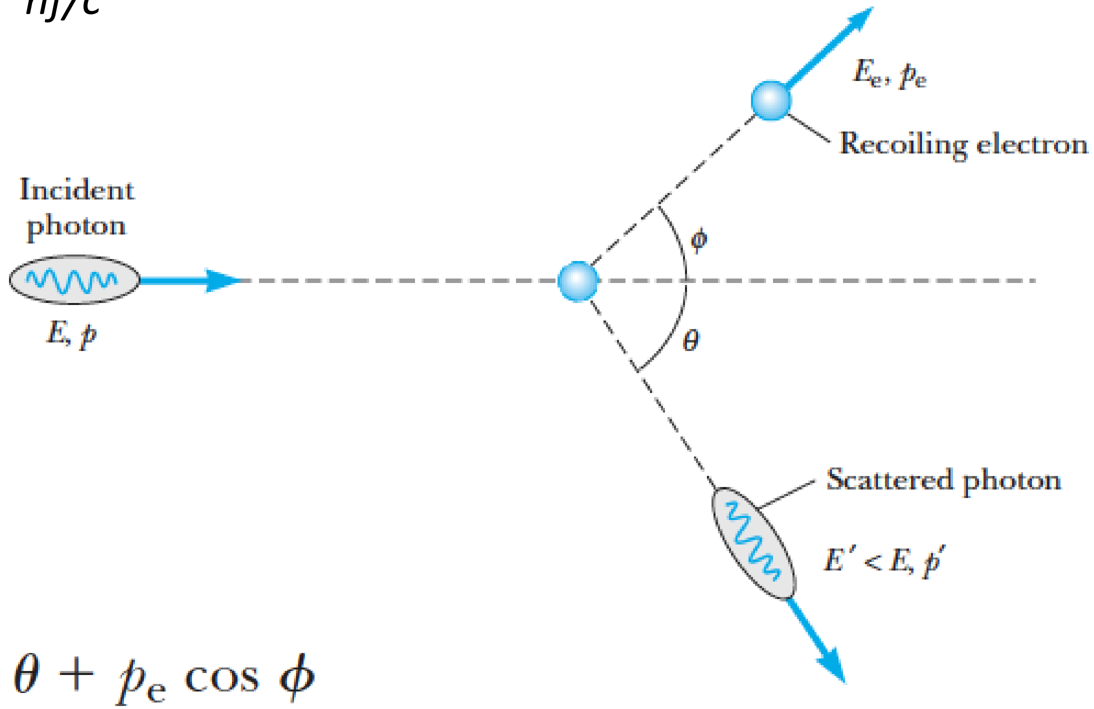


Experiments showed that;

- wavelength shift of x-rays scattered at a given angle is absolutely independent of the intensity of radiation and the length of exposure
- depends only on the scattering angle.

Classical wave theory failed to explain the scattering of x-rays from free electrons !!

Remember: Einstein concluded that a light quantum of energy *E* travels in a single direction (unlike a spherical wave) and carries a momentum directed along its line of motion of *E/c* or *hf/c*



Assuming that x-rays behave like particles, and collides elastically like a billiard ball with a free electron initially at rest;

in elastic collision energy and momentum are conserved;

energy of the incident photon

total relativistic energy of the electron after the collision

$$E + m_e c^2 = E' + E_e$$

rest energy of the electron

energy of the scattered photon

$$p = p' \cos \theta + p_e \cos \phi$$

$$p' \sin \theta = p_e \sin \phi$$

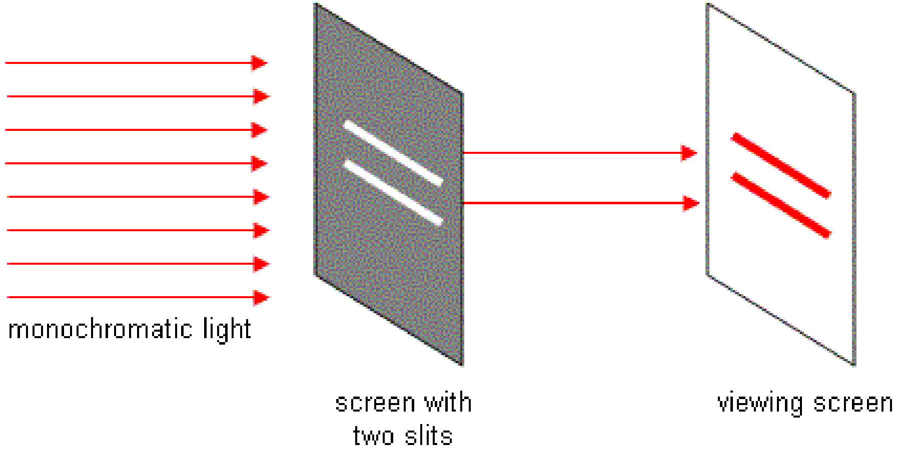
$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos \theta)$$

Compton wavelength of the electron →

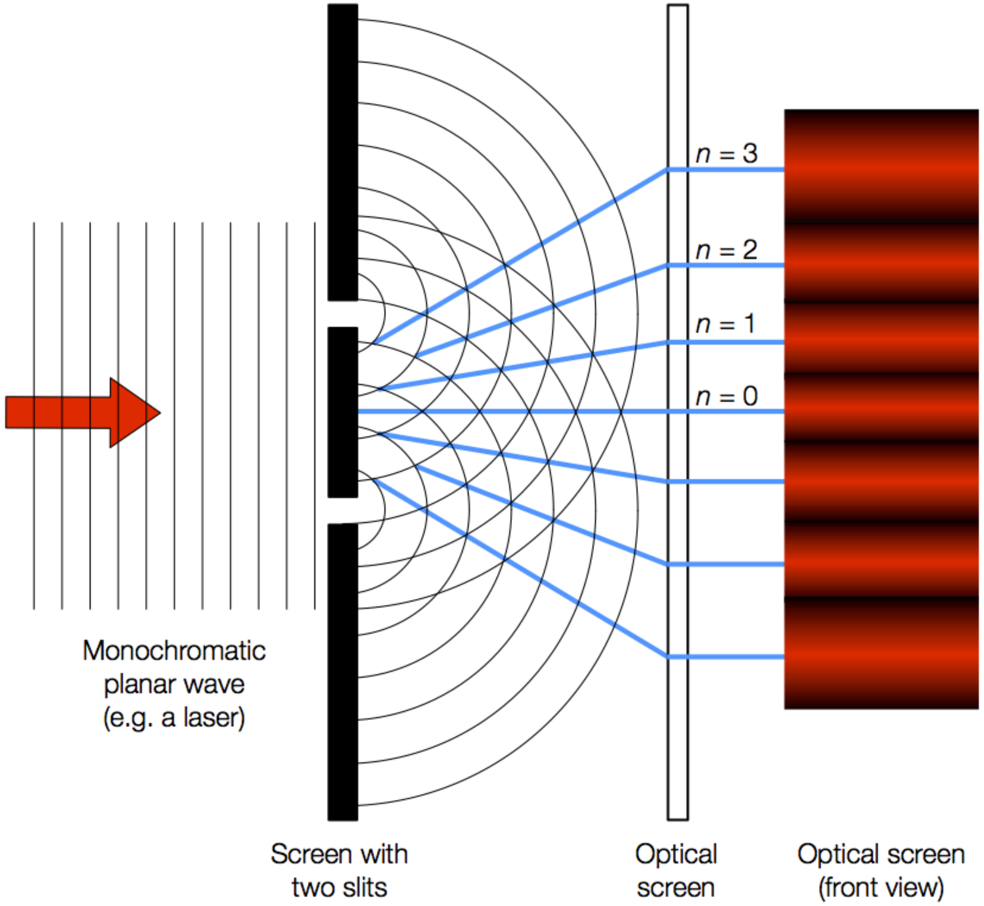
$$\frac{h}{m_e c} = 0.0243 \text{ Å} = 0.00243 \text{ nm}$$

4. The Double-Slit Experiment (with Light)

If light is a particle;

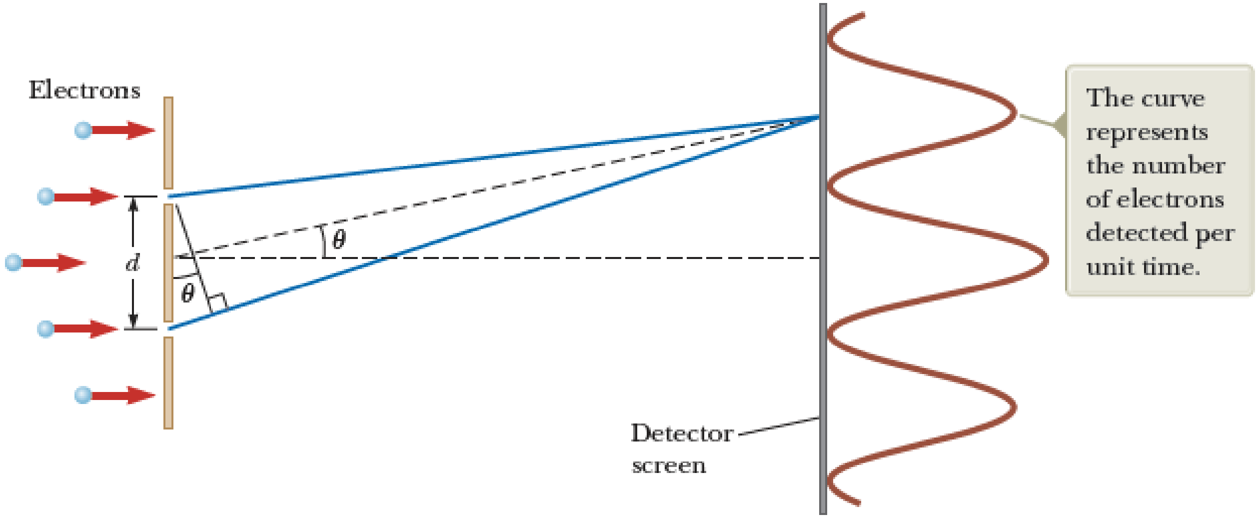


But …



6. The Double-Slit Experiment (with Electrons)

Consider a parallel beam of mono-energetic electrons incident on a double slit



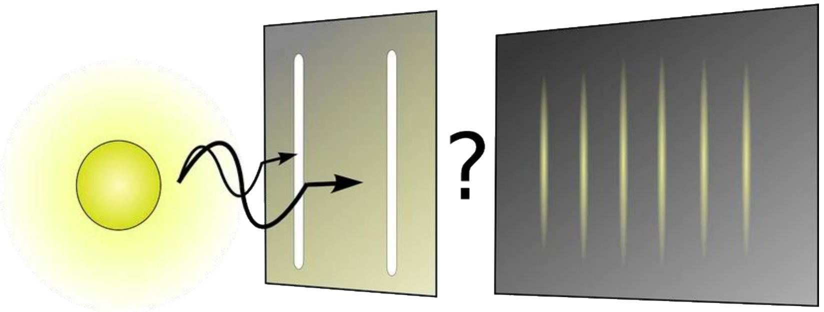
The curve represents the number of electrons detected per unit time.

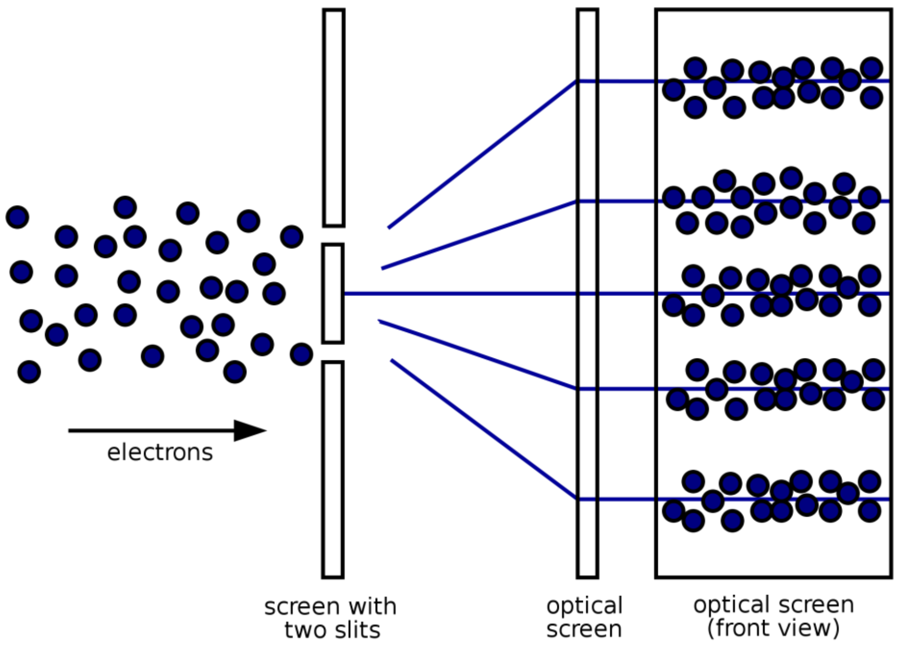
Slit widths are small compared with the electron wavelength

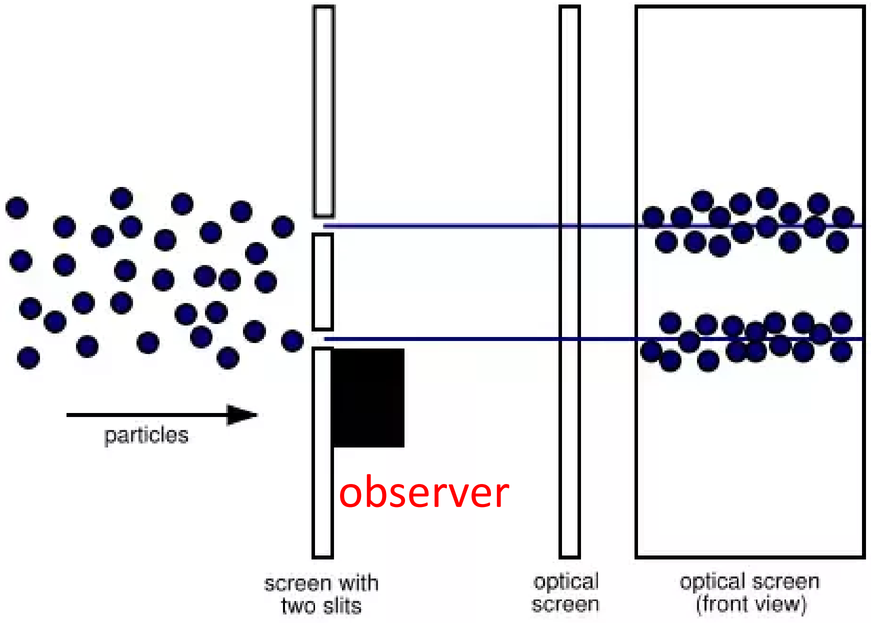
An electron detector screen is positioned far from the slits at a distance much greater than d, the separation distance of the slits

if the electrons behaved as classical particles, this interference pattern would not be expected ➔ electrons behaved like waves !!

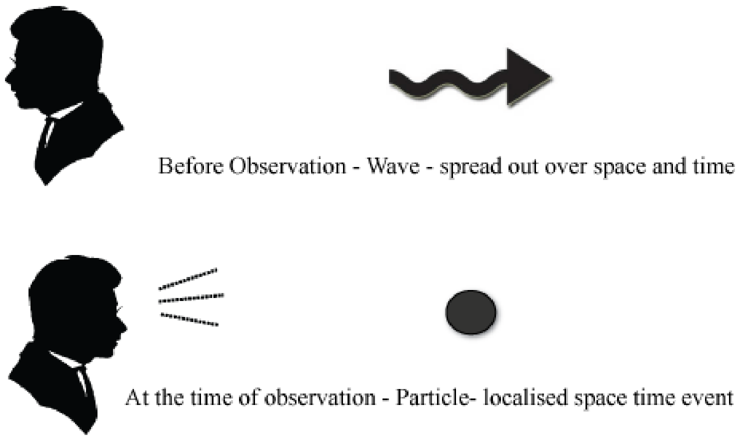
when less number of particles were sent to the slits





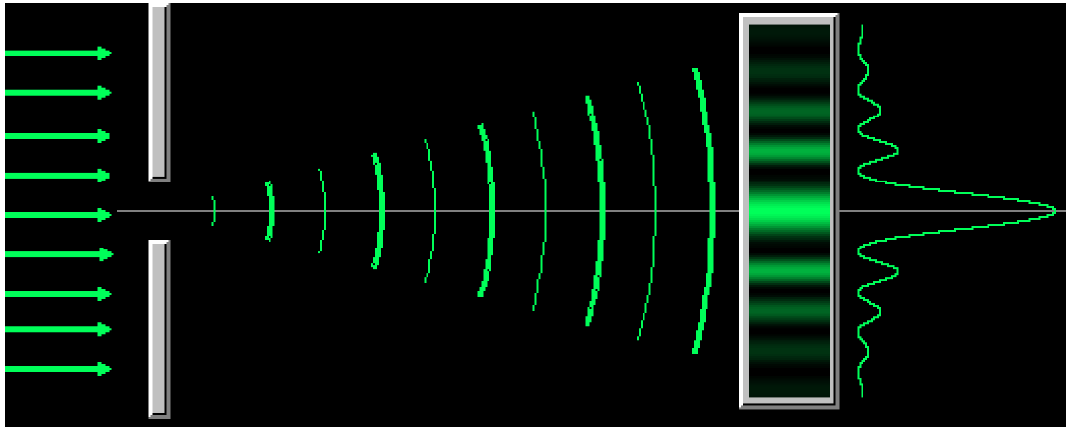


If you try to determine experimentally which slit the electron goes through, the act of measuring destroys the interference pattern. It is impossible to determine which slit the electron goes through. In effect, we can say only that the electron passes through both slits!

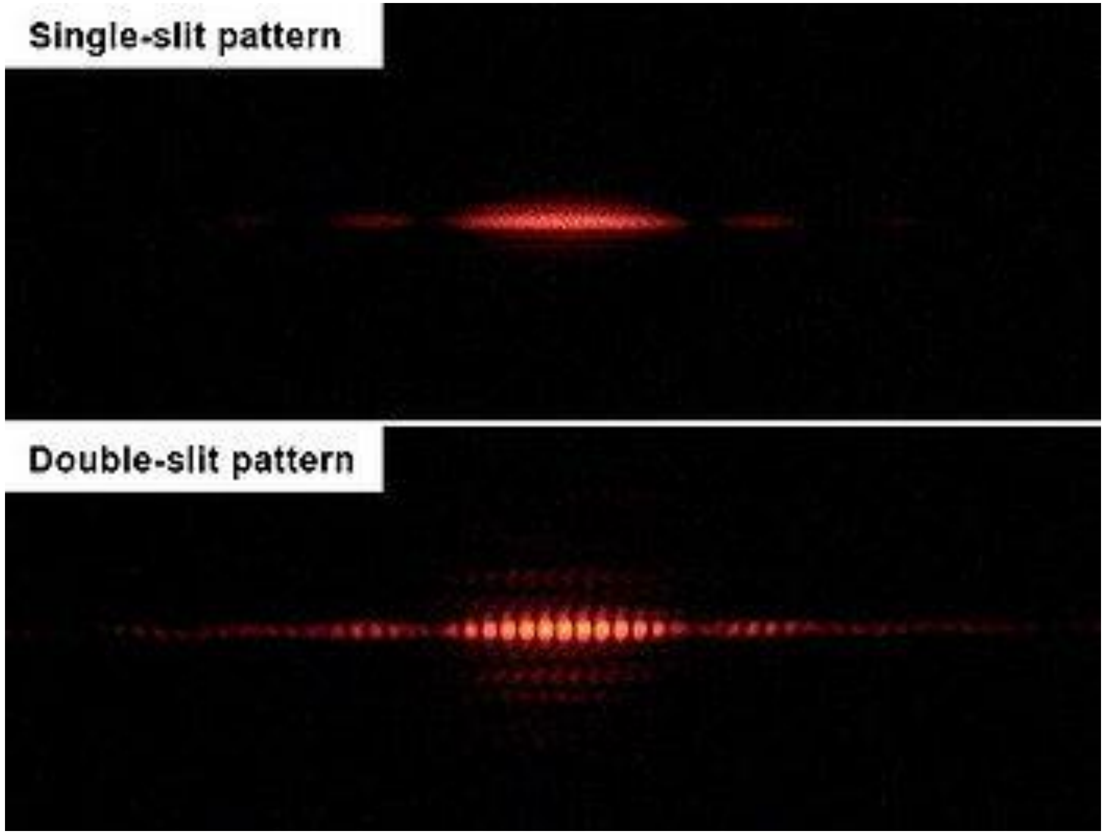


Before Observation - Wave - spread out over space and time

At the time of observation - Particle- localised space time event



When light passes through a single slit whose width w is on the order of the wavelength of the light, then we can observe a single slit diffraction pattern on a screen that is a distance L >> w away from the slit.



many smaller interference fringes with double slits

5. The Wave Properties of Particles

Louis de Broglie
French Physicist (1892–1987)

photon wavelength can be specified by its momentum: $\lambda = h/p$

De Broglie → material particles of momentum p have a characteristic wavelength that is given by the same expression.

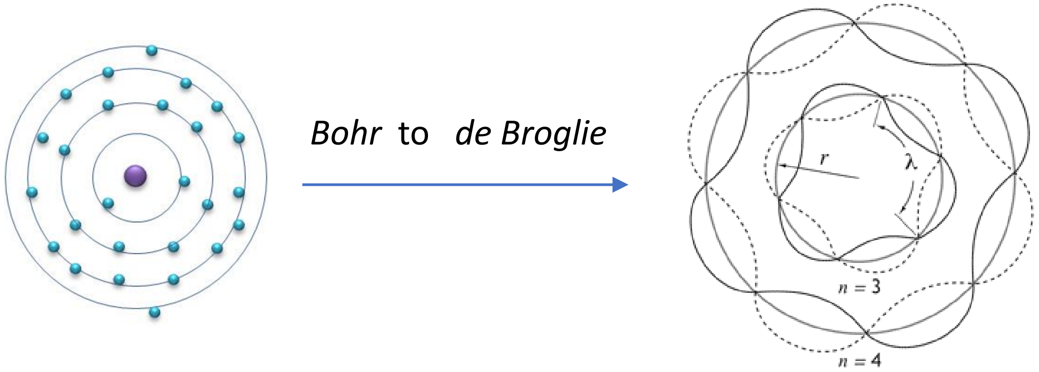
$$\lambda = \frac{h}{p} = \frac{h}{mu}$$

Particles obey the Einstein relation E = hf , where E is the total energy of the particle. The frequency of a particle is then

$$f = \frac{E}{h}$$

The Davisson–Germer Experiment

The Electron Microscope



!!! the wave and particle models of either matter or radiation complement each other !!!

Is light a wave or a particle?

Some experiments can be explained either better or solely with the photon model, whereas others are explained either better or solely with the wave model…

True nature of light is not describable in terms of any single classical picture.

The same light beam ;

- can eject photoelectrons from a metal (meaning that the beam consists of **photons**)
- can also be diffracted by a grating (meaning that the beam is a **wave**).

In other words, the particle model and the wave model of light complement each other.

!!! the wave and particle models of either matter or radiation complement each other !!!