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PEN205

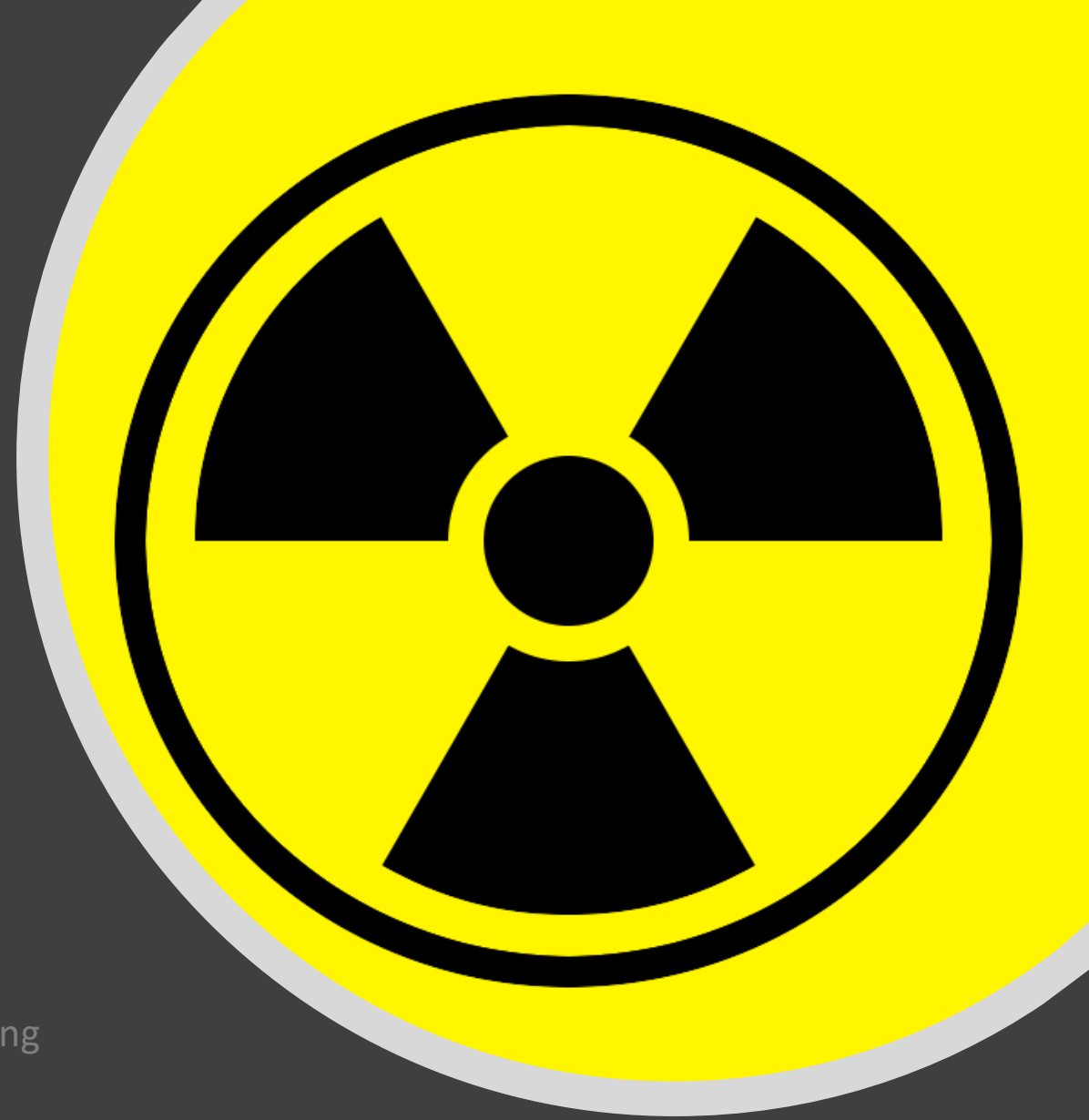
MODERN PHYSICS

Nuclear Structure and Radioactivity - I

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- **OUTLINE**

- Some Properties of Nuclei
- Nuclear Binding Energy
- Nuclear Models
- Radioactivity
- The Decay Processes
- Natural Radioactivity
- Nuclear Reactions
- Nuclear Magnetic Resonance and Magnetic Resonance Imaging

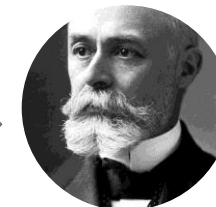


Milestones of Nuclear Science



1895 – Wilhelm Roentgen – discovery of X-rays

1896 – Henri Becquerel – discovery of radioactivity in uranium salts



1898 – Marie Curie and her husband – discovery of Polonium and Radium

1911 – Rutherford, Geiger, Marsden – Scattering of Alpha particles



1930 – Cockcroft, Walton – Observation of nuclear reactions

1932 – Chadwick – The discovery of the neutron



1933 – Joliot Irene Curie – The discovery of artificial radioactivity

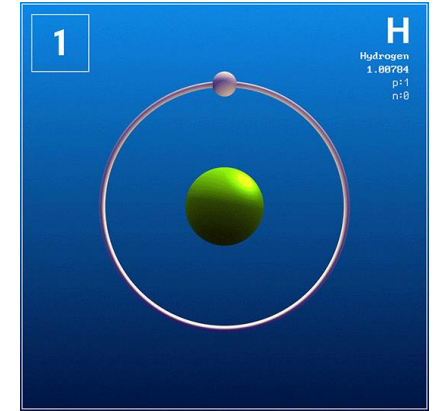
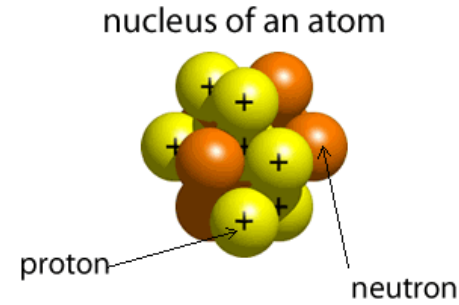
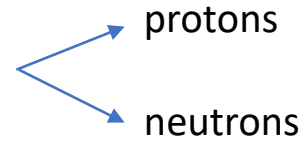
1938 – Hahn and Strassmann – The discovery of nuclear fission



1942 – Fermi and his collaborators – The development of the first controlled fission reactor

Some Properties of Nuclei

All nuclei are composed of two types of particles:



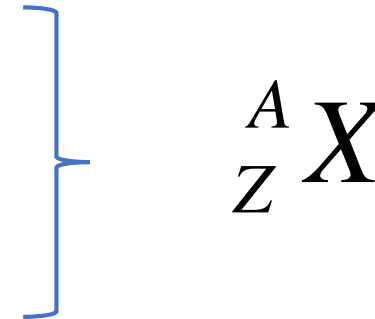
The only exception is the ordinary hydrogen nucleus, which is a single proton.

We describe the atomic nucleus by the number of protons and neutrons it contains

$Z \rightarrow$ the atomic number: the number of protons in the nucleus

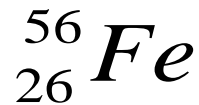
$A \rightarrow$ the mass number: the number of nucleons in the nucleus (neutrons + protons)

$N \rightarrow$ the neutron number: the number of neutrons in the nucleus



\leftarrow The symbol we use to represent nuclei
"nuclide"

Example:



A, the mass number: 56



Protons + neutrons

Z, the atomic number: 26



26 protons



30 neutrons

The nuclei of all atoms of a particular element contain the same number of protons but often contain different numbers of neutrons.



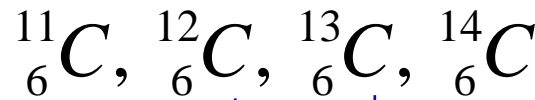
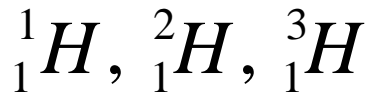
isotopes

same Z value

Different N and A values

Hydrogen has 2 isotopes

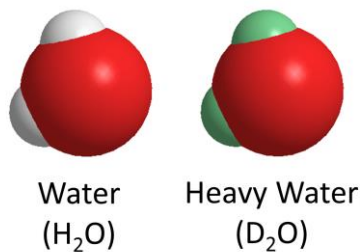
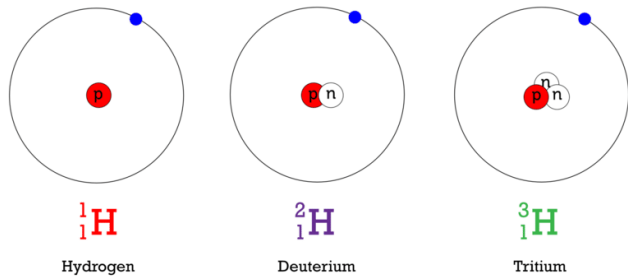
Carbon has 4 isotopes



hydrogen deuterium Tritium
(radioactive)

natural 98,9% Natural 1,1% Do not occur naturally

For most elements other than hydrogen, isotopes are named for their mass number. e.g. carbon -14



- Most elements as they occur naturally on earth are mixtures of several isotopes
- If isotopes/nuclides are radioactive → radioisotopes/radionuclides
- e⁻ number does not change in neutral isotopes → same chemical properties
- Different isotope containing molecules have different sets of vibrational modes, so they have different optical properties in the infrared range.

Charge and Mass

Charge of the proton : e

$$e = 1.602\,177\,3 \times 10^{-19} \text{ C}$$

Charge of the electron : $-e$

Charge of the neutron : *no charge*

Atomic Mass Unit (u): mass of one atom of the isotope $^{12}\text{C} \rightarrow 12\text{u}$

$$\rightarrow 1 \text{ u} = 1.660539 \times 10^{-27} \text{ kg}$$

$$\text{Proton} \rightarrow 1.007276 \text{ u}$$

$$\text{Neutron} \rightarrow 1.008665 \text{ u}$$

$$\text{Electron} \rightarrow 0.000548 \text{ u}$$

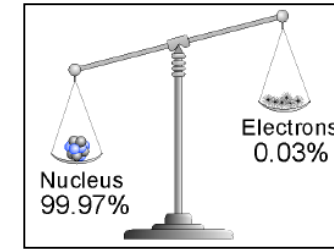
How 6 p and 6 n, each having a mass larger than 1 u, can be combined with 6 e^- to form a carbon-12 atom having a mass of exactly 12 u ?



The bound system of ^{12}C has a lower rest energy than that of 6 separate p and 6 separate n !

Masses of Selected Particles in Various Units

Particle	kg	u	MeV/ c^2
Proton	$1.672\,62 \times 10^{-27}$	1.007 276	938.27
Neutron	$1.674\,93 \times 10^{-27}$	1.008 665	939.57
Electron (β particle)	$9.109\,38 \times 10^{-31}$	$5.485\,79 \times 10^{-4}$	0.510 999
^1_1H atom	$1.673\,53 \times 10^{-27}$	1.007 825	938.783
^4_2He nucleus (α particle)	$6.644\,66 \times 10^{-27}$	4.001 506	3 727.38
^4_2He atom	$6.646\,48 \times 10^{-27}$	4.002 603	3 728.40
$^{12}_6\text{C}$ atom	$1.992\,65 \times 10^{-27}$	12.000 000	11 177.9



Because the rest energy of a particle is given by $E_R = mc^2$, it is often convenient to express the particle's mass in terms of its energy equivalent.

$$E = mc^2 = (1.660\,540 \times 10^{-27} \text{ kg}) \frac{(2.997\,924\,6 \times 10^8 \text{ m/s})^2}{(1.602\,177\,3 \times 10^{-19} \text{ J/eV})}$$

$$= 931.494\,3 \text{ MeV}$$

Mass in MeV/ c^2 units \rightarrow

$$1 \text{ u} \equiv 931.494\,3 \frac{\text{MeV}}{c^2}$$

Mole: A mole corresponds to the mass of a substance that contains 6.023×10^{23} particles of the substance.

$$N_A = 6.02 \times 10^{23} \text{ particles/mol} \rightarrow \text{AVAGADRO CONSTANT}$$

Example: Show that $1 \text{ u} = 1.66054 \times 10^{-27} \text{ kg}$ by using Avagadro (N_A) number.

12 gram (1 mole) of ^{12}C has N_A number of atoms.

$$N_A = 6,02 \times 10^{23} \text{ atom/mole}$$

$$\text{Mass of one } ^{12}\text{C} \text{ atom} = \frac{0,012 \text{ kg}}{6,02 \times 10^{23}} = 1,99 \times 10^{-26} \text{ kg}$$

$$\text{Mass of one } ^{12}\text{C} \text{ atom} = 12u$$

$$1u = \frac{1,99 \times 10^{-26} \text{ kg}}{12} = 1,66 \times 10^{-27} \text{ kg}$$

The Size and Structure of Nuclei

In Rutherford's scattering experiments;

Setup:

- Positively charged nuclei of helium atoms (alpha particles) were directed at a thin piece of gold foil.
- The gold foil was surrounded with a phosphorescent material that glowed when hit by the alpha particles.

Observations:

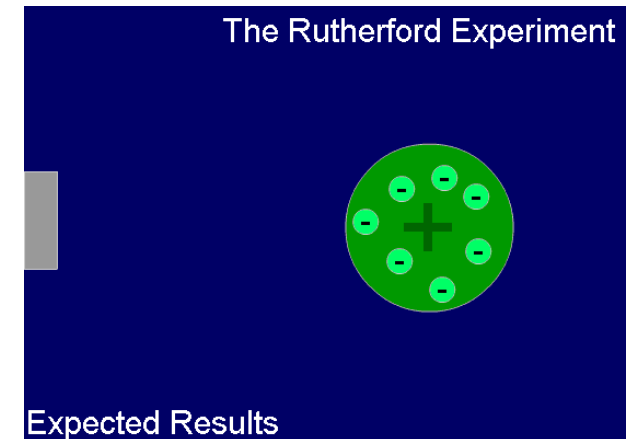
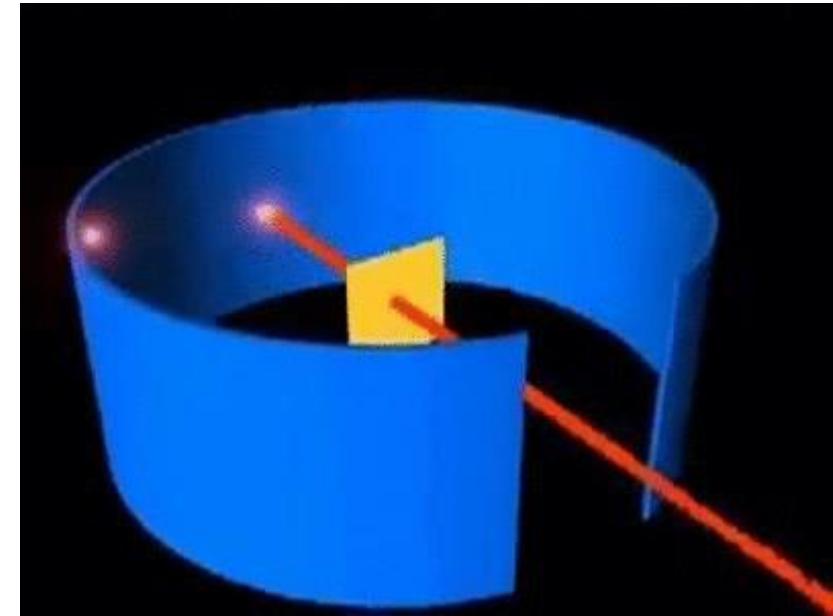
When alpha particles were shot;

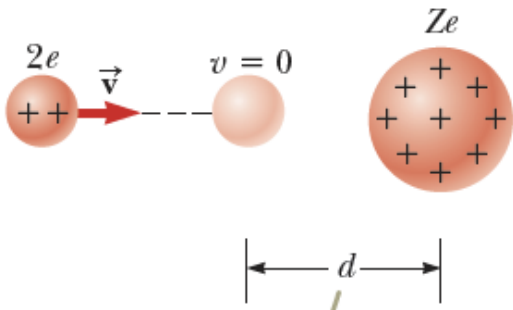
- Nearly all particles pass through the gold foil as if it were not there.
- A few particles were deflected to the side.
- Very few particles were deflected straight back.

Result:

- Atoms must be mostly empty space
- Because some particle deflected, there must be a (+) central mass → nucleus

- As the alpha particles moved through the foil, they often passed near a metal nucleus.
- Because of the positive charge on both the incident particles and the nuclei, the particles were deflected from their straight-line paths by the Coulomb repulsive force.





Because of the Coulomb repulsion between the charges of the same sign, the alpha particle approaches to a distance d from the nucleus, called the distance of closest approach.

In such a head-on collision, the mechanical energy of the nucleus–alpha particle system is conserved.

The initial kinetic energy of the incoming particle is transformed completely to electric potential energy of the system when the alpha particle stops momentarily at the point of closest approach before moving back along the same path.

$$\Delta K + \Delta U = 0$$

$$\left(0 - \frac{1}{2}mv^2\right) + \left(k_e \frac{q_1 q_2}{d} - 0\right) = 0$$

$$d = 2k_e \frac{q_1 q_2}{mv^2} = 2k_e \frac{(2e)(Ze)}{mv^2}$$

$$d = \frac{4kZe^2}{mv^2}$$

Rutherford found that the alpha particles approached nuclei to within 3.2×10^{-14} m when the foil was made of gold. Therefore, the radius of the gold nucleus must be less than this value.

Positive charge in an atom is concentrated in a small sphere, which is called the nucleus, whose radius is no greater than approximately 10^{-14} m

Experiments showed that most nuclei are approximately spherical and have an average radius;

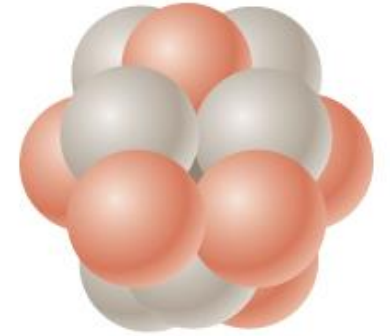
$$r = aA^{1/3}$$

- a : constant ($a = 1.2 \times 10^{-15}$ m)
- A : mass number

Because such small lengths are common in nuclear physics, a convenient unit of length is the *femtometer (fm)*, sometimes called the **fermi** and defined as :

$1 \text{ fm} \equiv 10^{-15} \text{ m}$

A nucleus can be modeled as a cluster of tightly packed spheres, where each sphere is a **nucleon**.



➤ Average radius :

$$r = aA^{1/3}$$

➤ Average volume:

$$V_{\text{nucleus}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi a^3 A$$

➤ Average density :

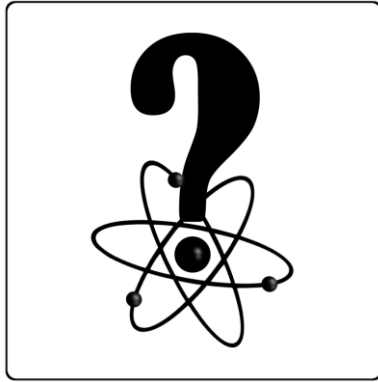
$$\rho = \frac{m_{\text{nucleus}}}{V_{\text{nucleus}}} = \frac{Am}{\frac{4}{3}\pi a^3 A} = \frac{3m}{4\pi a^3}$$

$$\rho = \frac{3(1.67 \times 10^{-27} \text{ kg})}{4\pi(1.2 \times 10^{-15} \text{ m})^3} = 2.3 \times 10^{17} \text{ kg/m}^3$$

This relationship then suggests that all nuclei have nearly the same density.

The nuclear density is approximately 2.3×10^{14} times the density of water ($\rho_{\text{water}} = 1.0 \times 10^3 \text{ kg/m}^3$).

Nuclear Stability



The nucleus consists of a closely packed collection of protons and neutrons!!

How can the nucleus structure remain stable?
Don't the protons in the nucleus repel each other?

✓ Protons repel each other through the Coulomb force.



✓ If the nucleus is still stable there must be a counteracting attractive force!



✓ That force is the **nuclear force** → the nuclear force is a very short range (about 2 fm) attractive force that acts between all nuclear particles.



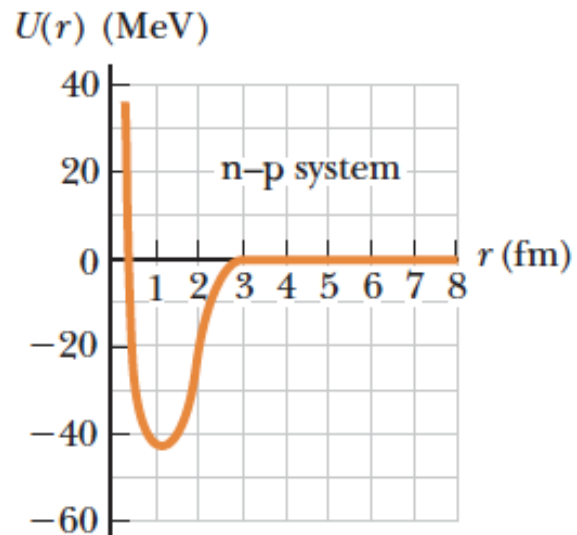
✓ The nuclear force also acts between pairs of neutrons, pairs of protons and between neutrons and protons.



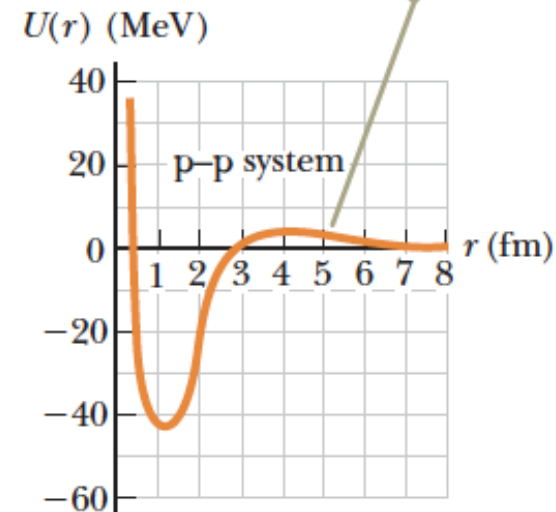
✓ So, the protons attract each other by means of the nuclear force, and, at the same time, they repel each other through the Coulomb force.

These forces are in equilibrium

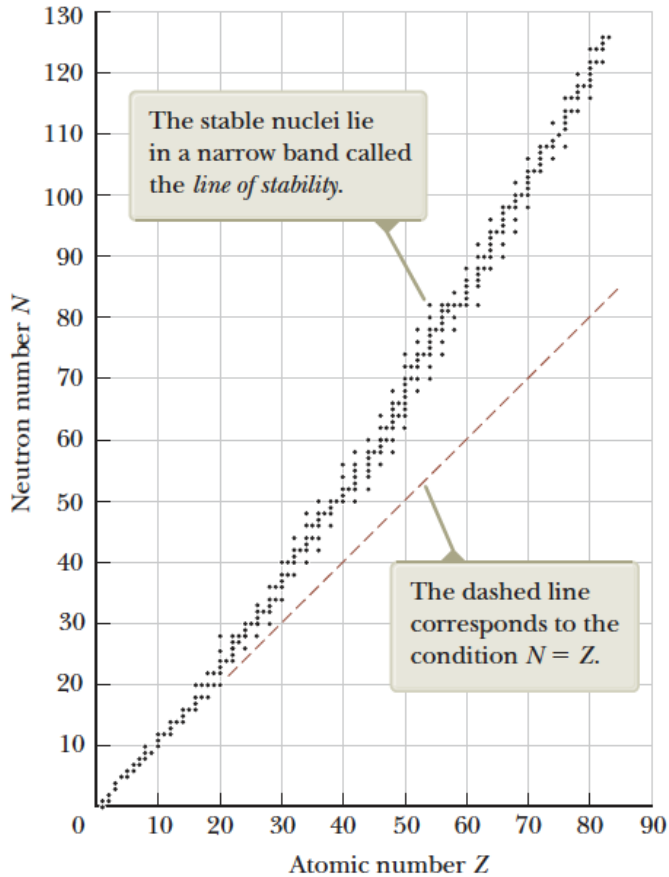
- The nuclear force does not affect electrons
- The nuclear force is independent of charge.
- In other words, the forces associated with the proton–proton, proton–neutron, and neutron–neutron interactions are the same, apart from the additional repulsive Coulomb force for the proton–proton interaction.
- The charge independence of the nuclear force also means that the main difference between the n–p and p–p interactions is that the p–p potential energy consists of a superposition of nuclear and Coulomb interactions.



The difference in the two curves is due to the large Coulomb repulsion in the case of the proton–proton interaction.



The existence of the nuclear force results in approximately 270 stable nuclei. There are hundreds of unstable ones.



neutron number N versus atomic number Z

The stable nuclei are represented by the black dots, which lie in a narrow range called the **line of stability**.

$Z < 20 \rightarrow N = Z \rightarrow$ light stable nuclei

$Z > 20 \rightarrow N > Z \rightarrow$ heavy stable nuclei

as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart.

As a result, more neutrons are needed to keep the nucleus stable because neutrons experience only the attractive nuclear force.

Eventually, the repulsive Coulomb forces between protons cannot be compensated by the addition of more neutrons.

This point occurs at $Z = 83$, meaning that elements that contain more than 83 protons do not **have stable nuclei**.

$Z > 83 \rightarrow$ Unstable

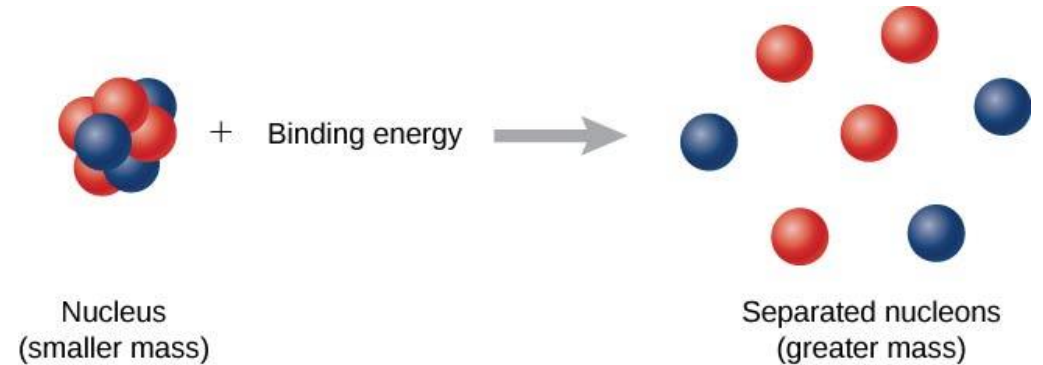
Interestingly, most stable nuclei have even values of A (mass number). In fact, certain values of Z and N correspond to unusually high stability in nuclei. These values of N and Z are called **magic numbers** (Z or N = 2, 8, 20, 28, 50, 82, 126)

Nuclear Binding Energy

- As mentioned before, the total mass of a nucleus is less than the sum of the masses of its individual nucleons.
- Therefore, the rest energy of the bound system (the nucleus) is less than the combined rest energy of the separated nucleons.

This difference in energy is called the **binding energy**.

binding energy → can be interpreted as the energy that must be added to a nucleus to break it apart into its components.



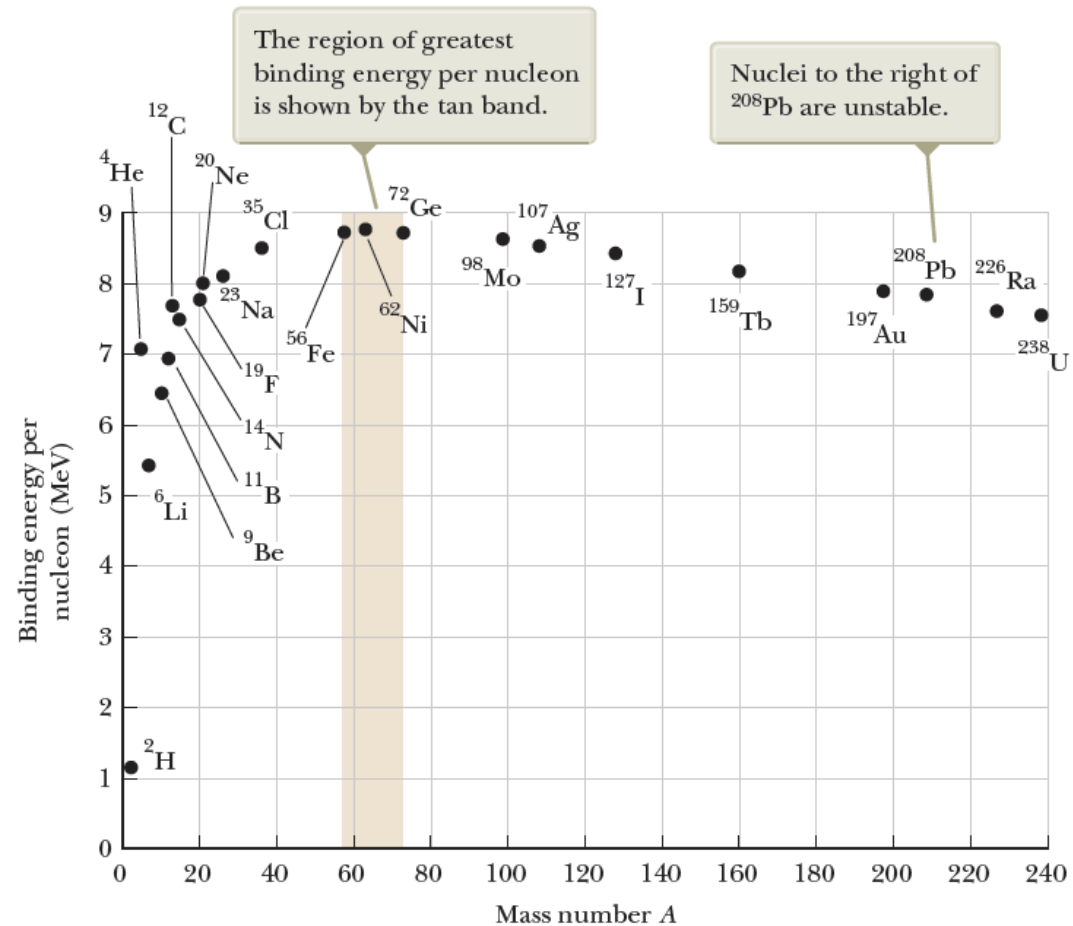
$$E_b = [ZM(\text{H}) + Nm_n - M({}_Z^AX)] \times 931.494 \text{ MeV/u}$$

atomic mass of the neutral hydrogen atom

mass of the neutron

atomic mass of the element X

Let's interpret this plot;



1- Binding energy peaks in the vicinity of $A = 60$.

2- The decrease in binding energy per nucleon for $A > 60$ implies that energy is released when a heavy nucleus splits, or fissions, into two lighter nuclei.

Energy is released in fission because the nucleons in each product nucleus are more tightly bound to one another than are the nucleons in the original nucleus.

3- Binding energy per nucleon is approximately constant at around 8 MeV per nucleon for all nuclei with $A > 50$.

For these nuclei, the nuclear forces are said to be **saturated**, meaning that in the closely packed structure, a particular nucleon can form attractive bonds with only a limited number of other nucleons.

Nuclear Models

- The details of the nuclear force are still an area of active research.
- Several nuclear models have been proposed that are useful in understanding general features of nuclear experimental data and the mechanisms responsible for binding energy.

Two of them are:

The Liquid-Drop Model – Bohr, 1936

- treating nucleons like molecules in a drop of liquid
- nucleons interact strongly with one another and undergo frequent collisions as they jiggle around within the nucleus
- Nuclear properties, such as the binding energy, are described in terms of volume energy, surface energy, compressibility, etc.—parameters that are usually associated with a liquid.
- This model has been successful in describing how a nucleus can deform and undergo fission.

The Shell Model – Bohr, 1936

- The Nuclear Shell Model is similar to the atomic model where electrons arrange themselves into shells around the nucleus.
- Nucleons occupy quantized energy levels.
- These levels are determined with quantum numbers.
- There are only a few collisions among nucleons.
- Because of their $\frac{1}{2}$ spin, protons and neutrons obey Pauli exclusive principle.
- Allowed energy levels of protons are higher than that of neutrons. That is because protons have Coulomb+nuclear potential energies while neutrons only have nuclear potential energy.