BETA DECAY

Prof. Dr. Turan OLĞAR

Ankara University, Faculty of Engineering
Department of Physics Engineering

BETA DECAY

There are three types of beta decay processes

- Negative beta decay (β^-)
- Positive beta decay (β^+)
- Orbital electron capture (ε)

These three types of disintegrations are often called isobaric transformations because they do not involve any change in the mass number ($\Delta A=0$), but there is always a change in the nuclear charge.

In a nucleus, β decay changes both Z and N by one unit: $Z \rightarrow Z \pm 1$, $N \rightarrow N \pm 1$ so that $A = Z + N$ remains constant.

BETA DECAY

The electrons cannot exist inside the nucleus. An important assumption is that the electron or the positron is created at the time the nucleus undergoes decay, while in the process of electron capture by the nucleus the electron disappears by converting its mass into energy.

As shown in Figure 8.1 the stable nuclei lie on a curve that deviates from the $N=Z$ line as Z increases (See Fig. 5.7 or 8.1 in Fundamentals of Nuclear Physics in Atam P. Arya for a plot of N versus Z for all known nuclei).

There are more than a thousand artificial radioactive isotopes that lie on each side of the stability curve. It is evident from Fig. 8.1. that those radioactive nuclei that are located below the stability curve have too many protons and those above the stability curve have too many neutrons.

BETA DECAY

A nucleus that has an excess of neutrons can become stable by increasing its charge, by having more protons. This is achieved through β^- emission in which the positive charge created will convert a neutron into a proton.

If a nucleus has too many protons, the nuclear charge is decreased by β^+ emission, which is equivalent to converting a proton into a neutron. The charge on the nucleus is also decreased if the nucleus captures an orbital electron.

The process of electron capture takes place in the following manner. If the extra-nuclear electrons, in the process of their orbital motion, happen to come close to the nucleus, they may be captured.

BETA DECAY

This is more likely to happen to the K-Shell electrons of the atoms with high Z, because their orbits are very close to the nuclear surface.

If an electron from the K-Shell is captured, the process is called K-*capture*. Less probable processes are *L-capture*, *M-capture*, and so forth.

The vacancy in the K-Shell or L-Shell is filled by the electrons from the outer shells. This results in the emission of K or L x-rays that are characteristic of the product nucleus. Because there is no charged particle emitted in the process of electron capture, the process is observed only through the emission of characteristic x-rays.

BETA DECAY

There is some probability that instead of x-ray emission, the excited K-Shell will de-excite by giving its energy to the L-Shell electron that will be emitted with kinetic energy K_e,

$$K_e = h\nu_k - E_L = (E_K - E_L) - E_L$$
$$= E_K - 2E_L$$

where ν_K is the frequency of the K x-ray and E_K and E_L are the binding energies. These electrons, which sometimes accompany the orbital-electron capture, are called Auger electrons.

The basic decay processes are thus:

$n \rightarrow p + e^-$ Negative beta decay (β^-)

$p \rightarrow n + e^+$ Positive beta decay (β^+)

$p + e^- \rightarrow n$ Orbital electron capture (ε)

ENERGY RELEASE IN β DECAY

The continuous energy distribution of the β-decay electrons was a confusing experimental result in the 1920s. Alpha particles are emitted with sharp, well defined energies, equal to the difference in mass energy between the initial and final states (less the small recoil corrections); all α decays connecting the same initial and final states have exactly the same kinetic energies.

Beta particles have a continuous distribution of energies, from zero up to an upper limit (the endpoint energy) which is equal to the energy difference between the initial and final states. If β decay were, like α decay, a two-body process, we would expect all of the β particles to have a unique energy, but virtually all of the emitted particles have a smaller energy. For instance, we might expect on the basis of nuclear mass differences that the β particles from ^{210}Bi would be emitted with a kinetic energy of 1.16 MeV, yet we find a continuous distribution from 0 up to 1.16 MeV (Figure 9.1).

ENERGY RELEASE IN β DECAY

See Figure 9.1 in Introductory Nuclear Physics by Kenneth S. Krane for the continuous electron distribution from the β decay of ^{210}Bi.

An early attempt to account for this "missing" energy hypothesized that the β's are actually emitted with 1.16 MeV of kinetic energy, but lose energy, such as by collisions with atomic electrons, before they reach the detection system. But calorimetric experiments that confined a β source and measured its decay energy by the heating effect showed that the shape of the spectrum shown in Figure 9.1 is a characteristic of the decay electrons themselves and not a result of any subsequent interactions.

To account for this energy release, Pauli in 1931 proposed that there was emitted in the decay process a second particle, later named by Fermi the *neutrino.*

ENERGY RELEASE IN β DECAY

The neutrino carries the “missing” energy and, because it is highly penetrating radiation, it is not stopped within the calorimeter, thus accounting for the failure of those experiments to record its energy.

Conservation of electric charge requires the neutrino to be electrically neutral, and angular momentum conservation and statistical considerations in the decay process require the neutrino to have (like the electron) a spin of 1/2.

Experiment shows that there are in fact two different kinds of neutrinos emitted in β decay. These are called the *neutrino* and the *antineutrino* and indicated by ν and $\bar{\nu}$. It is the antineutrino which is emitted in β^- decay and the neutrino which is emitted in β^+ decay and electron capture.

ENERGY RELEASE IN β DECAY

To demonstrate β-decay energetics we first consider the decay of the free neutron (which occurs with a half-life of about 10 min),

$$n \rightarrow p + e^{-} + \bar{\nu}$$

As we did in the case of α decay, we define the Q value to be the difference between the initial and final *nuclear* mass energies.

$$Q = \left(m_n - m_p - m_e - m_{\bar{\nu}} \right) c^2 \qquad (1)$$

and for decays of neutrons at rest,

$$Q = K_p + K_e + K_{\bar{\nu}} \qquad (2)$$

For the moment we will ignore the proton recoil kinetic energy K_p, which amounts to only 0.3 keV. The antineutrino and electron will then share the decay energy, which accounts for the continuous electron spectrum.

ENERGY RELEASE IN β DECAY

The maximum-energy electrons correspond to minimum-energy antineutrinos, and when the antineutrinos have vanishingly small energies, $Q \cong (K_e)_{max}$. The measured maximum energy of the electrons is 0.782±0.013 MeV. Using the measured neutron, electron, and proton masses, we can compute the Q value:

$$Q = \left(m_n - m_p - m_e - m_{\bar{\nu}} \right) c^2$$

$$= 939.573\, MeV - 938.280\, MeV - 0.511\, MeV - m_{\bar{\nu}} c^2$$

$$= 0.782\, MeV - m_{\bar{\nu}} c^2$$

Thus to within the precision of the measured maximum energy (about 13 keV) we may regard the antineutrino as massless. Because the neutrino is massless, it moves with the speed of light and its total relativistic energy E_ν is the same as its kinetic energy; we will use E_ν to represent neutrino energies.

ENERGY RELEASE IN β DECAY

For the electron, we will use both its kinetic energy K_e , and its total relativistic energy E_e , which are of course related by $E_e = K_e + m_e c^2$.

Let's consider a typical negative β^- decay process in a nucleus:

$$ {}^{A}_{Z}X_N \rightarrow {}^{A}_{Z+1}X'_{N-1} + e^- + \bar{\nu} $$

$$ Q_{\beta^-} = \left[m_N \left({}^{A}_{Z}X \right) - m_N \left({}^{A}_{Z+1}X' \right) - m_e \right] c^2 \qquad (3) $$

where m_N indicates nuclear masses. To convert nuclear masses into the tabulated neutral atomic masses, which we denote as $m({}^{A}X)$, we use

$$ m({}^{A}X)c^2 = m_N({}^{A}X)c^2 + Zm_e c^2 - \sum_{i=1}^{Z} B_i \qquad (4) $$

where B_i, represents the binding energy of the ith electron.

ENERGY RELEASE IN β DECAY

In terms of atomic masses,

$$Q_{\beta^-} = \left\{\left[m(^A X) - Z m_e \right] - \left[m(^A X') - (Z+1) m_e \right] - m_e \right\} c^2 + \left\{ \sum_{i=1}^{Z} B_i - \sum_{i=1}^{Z+1} B_i \right\} \qquad (5)$$

Notice that the electron masses cancel in this case and when differences in electron binding energy are neglected, we find

$$Q_{\beta^-} = \left[m\left({}^A X \right) - m(^A X') \right] c^2 \qquad (6)$$

where the masses are neutral atomic masses. The Q value represents the energy shared by the electron and neutrino:

$$Q_{\beta^-} = K_e + E_{\bar{\nu}} \qquad (7)$$

and it follows that each has its maximum when the other approaches zero

$$Q_{\beta^-} = \left(K_e \right)_{\max} = \left(E_{\bar{\nu}} \right)_{\max} \qquad (8)$$

ENERGY RELEASE IN β DECAY

In the case of $^{210}\text{Bi} \rightarrow {}^{210}\text{Po}$ decay, the mass tables give

$$\begin{aligned} Q_{\beta^-} &= \left[m\left({}^{210}Bi \right) - m({}^{210}Po) \right] c^2 \\ &= \left(209.984095u - 209.982848u \right)\left(931.502\ MeV/u \right) \\ &= 1.161\ MeV \end{aligned}$$

In the case of positron decay, a typical decay process is

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}X'_{N+1} + e^{+} + \nu \qquad (9)$$

Expressing Eq. 9 in terms of the atomic masses (and neglecting the binding energy of the electrons) we have

$$ Q_{\beta^+} = \left[m\left({}^{A}X \right) - m\left({}^{A}X' \right) - 2m_e \right] c^2 \qquad (10)$$

ENERGY RELEASE IN β DECAY

For electron-capture processes, such as

$$ {}^{A}_{Z}X_{N} + e^{-1} \rightarrow {}^{A}_{Z-1}X'_{N+1} + \nu \qquad (11) $$

The calculation of the Q value must take into account that the atom *X'* is in an atomic excited state immediately after the capture. Thus the atomic mass of *X'* immediately after the decay is greater than the mass of *X'* in its atomic ground state by B_n , the binding energy of the captured n-shell electron (n = K, L, . . .). The *Q* value is then

$$ Q_{\varepsilon} = \left[m\left({}^{A}X \right) - m\left({}^{A}X' \right) \right] c^2 - B_n \qquad (12) $$

All of the above expressions refer to decays between nuclear ground states. If the final nuclear state *X'* is an excited state, the *Q* value must be accordingly decreased by the excitation energy of the state:

$$ Q_{ex} = Q_{ground} - E_{ex} $$

ENERGY MEASUREMENT OF BETA PARTICLES

The energy measurement involves two types of electrons.

- The electrons emitted in the process of beta decay always have a continuous energy distribution which requires the measurement of the maximum energy, the end-point energy.

- The conversion electrons, which are monoenergetic, are emitted by the following process. In the process of gamma decay, which usually follows beta decay, the nucleus often is de-excited by giving its energy to the orbital electron instead of emitting a gamma ray. These electrons are called conversion electrons. Conversion electrons show up as discrete lines superimposed on the continuous beta spectrum. Thus the measurement of energies requires scanning throughout the range of the whole spectrum.

ENERGY MEASUREMENT OF BETA PARTICLES

An important point for beta decay is the relativistic correction. Because the beta particles emitted from radioactive nuclei have velocities approaching that of the velocity of light, their motion must be described by the special theory of relativity instead of by classical mechanics.

The most precise measurements of the energies and the spectra of beta particles involve the use of magnetic spectrometers. We shall describe the *Semicircular Focusing Spectrometer* method.

This method is the same as that for alpha particles and uses semicircular (180^0) focusing. Because β-particles are much lighter than α-particles, they do not need very high fields.

ENERGY MEASUREMENT OF BETA PARTICLES

A beam of electrons is brought to focus by the application of a magnetic field perpendicular to the plane of the motion of the particles. The motion is governed by the equation

$$Bev = \frac{mv^2}{r} \qquad (13)$$

where m is the relativistic mass given by $m_0 / \sqrt{1-\left(v^2 / c^2\right)}$ and r is the radius of curvature.

Rewriting Eq. 13, we get

$$p = eBr \qquad (14)$$

where p is the relativistic momentum. Once the momentum is known, the kinetic energy may be calculated.

ENERGY MEASUREMENT OF BETA PARTICLES

Once the momentum is known, the kinetic energy may be calculated.

$$K_e = mc^2 - m_0c^2 = E - E_0$$

where

$$E = \sqrt{p^2c^2 + E_0^2}$$

Hence

$$K_e = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2 \qquad (15)$$

Either a photographic plate or a Geiger counter may be used for detection.

ENERGY MEASUREMENT OF BETA PARTICLES

The photographic plate has the advantage of recording the whole spectra in one exposure, but it suffers from the disadvantages of lower sensitivity and nonlinearity. For quantitative work the Geiger counter is best. The counter is placed in a fixed position while the magnetic field is varied. The number of β-particles reaching the counter, per unit time, is obtained for different values of *B*. Because r is fixed, each value of *Br* corresponds to different values of *p*. The plot of number versus *Br* gives the momentum distribution curve.

ENERGY LOSS BY ELECTRONS

The process by which electrons lose their energy in traversing a medium is more complicated than the energy loss by heavy charged particles. These complications;

- Because of the small mass and high velocity of the beta particles, it becomes necessary to consider the relativistic effects.
- As contrary to the heavy particles, an electron may lose a large fraction of its kinetic energy in a single collision. This not only causes greater straggling but it also makes it difficult to distinguish between the incident electron and the target electron. The one that has higher energy after collision is called the incident (or primary) electron.
- Collision between the electrons and the atoms, in which the electrons are merely deflected without a loss of energy (i.e. elastic collisions) are frequent occurrence. Such multiple scattering confuses the problem of energy-loss measurement still further.

ENERGY LOSS BY ELECTRONS

- The most important effect that becomes prominent at very high velocities is the energy loss by radiation-so much so that at extremely high velocities of the beta particles, the energy loss by ionization and excitation becomes negligible as compared to the energy loss by radiation.

- The electrons emitted in the process of beta decay do not have a homogenous energy; they have a continuous energy distribution between zero and a maximum.

ENERGY LOSS BY ELECTRONS

All these factors make it difficult to predict theoretically the energy lost by electrons. For relatively low-energy electrons, the energy loss is due mainly to the excitation and ionization of the electrons in the atoms of the stopping material. For high-energy electrons, we shall consider the energy loss by the following processes.

A. ENERGY LOSS BY INELASTIC COLLISIONS

The expression for the stopping power for heavy charged particles derived before as

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{mv^2} NZ \ln\left(\frac{2mv^2}{\bar{I}}\right) \qquad (16)$$

ENERGY LOSS BY ELECTRONS

Eq. 16 must be modified for two different reasons.

- Because the reduced mass of the two-electron system is ½ m, the term log $2mv^2$ must be replaced by log mv^2.

- The identification of the higher energy electron as the primary electron emerging from a collision limits the maximum energy-loss in any collision to ¼ mv^2 instead of ½ mv^2. These corrections lead to the following expression for electron with $E << mc^2$.

$$-\frac{dE}{dx} = \frac{4\pi e^4}{mv^2} NZ \left[\ln\left(\frac{mv^2}{2\bar{I}} \right) + 0.15 \right] \qquad (17)$$

ENERGY LOSS BY ELECTRONS

For the case of relativistic electrons, the following expression has been derived by H. Bethe,

$$-\frac{dE}{dx}=\frac{2\pi e^4}{mv^2}NZ\left\{\ln\left(\frac{mv^2E}{2\bar{I}^2\left(1-\beta^2\right)}\right)-\left(2\sqrt{1-\beta^2}-1+\beta^2\right)\ln 2+1-\beta^2+\frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2\right\} \qquad (18)$$

where E is the kinetic energy of the incident electron and β=v/c. For the case of slow electrons, in which β<<1, Eq. 18 is almost equivalent to Eq. 17. For the case of extremely relativistic particles Eq. 18 reduces to

$$-\frac{dE}{dx}=\frac{2\pi e^4}{mc^2}NZ\left\{\ln\left(\frac{E^3}{2mc^2\bar{I}^2}\right)+\frac{1}{8}\right\} \qquad for\ \ E>>mc^2 \qquad (19)$$

ENERGY LOSS BY ELECTRONS

B. ENERGY LOSS OF FAST ELECTRONS BY RADIATION (BREMSSTRAHLUNG)

According to classical electromagnetic theory, an accelerated charged particle radiates electromagnetic energy at a rate give by

$$\frac{dE}{dt} = \frac{2e^2a^2}{3c^3} \qquad (20)$$

where a is the acceleration and defined as a= F/m. Whenever a charged particle, such as an electron or proton, moves in the field of the nucleus, it is accelerated and radiates electromagnetic waves; this radiation is called *Bremsstrahlung*. The energy loss by radiation is proportional to Z^2 (where Z is the atomic number of the absorbing material) and increases linearly with energy, while the energy loss by ionization is proportional to Z and increases logarithmically with energy.

ENERGY LOSS BY ELECTRONS

The critical energy, E_c , is defined as the electron energy at which the energy loss by ionization equals the radiative loss. It has been shown by H. Bethe and W. Heitler that

$$E_c \approx 1600\, mc^2 / Z \qquad (21)$$

and the ratio of the radiative loss to the ionization loss is given by

$$\frac{\left(dE/dx\right)_{rad}}{\left(dE/dx\right)_{coll}} \approx \frac{EZ}{1600\, mc^2} \qquad (22)$$

where mc^2=0.51 MeV

ABSORPTION AND RANGE-ENERGY RELATION

Beta particles travel much longer distances in air (or in any gas) than the alpha particles of the same energy. For this reason, metals in the form of thin foils, commonly aluminum, are used for the absorption of beta particles. The exponential law of absorption holds approximately for nuclear beta-rays.

$$I = I_0 \, e^{-\mu x/\rho} \qquad (23)$$

where μ/ρ is an apparent *mass-absorption coefficient* in cm^2/gr, and x is the absorber thickness in mg/cm^2; I_0 is the initial intensity, and I is the intensity after passing through a thickness x of the absorber.

There is a considerable difference in the shapes of the absorption curves for the case of nuclear β-particles (electrons that are produced by nuclear decay and have a continuous energy spectrum) and the homogeneous electrons (that are produced artificially or by conversion).

ABSORPTION AND RANGE-ENERGY RELATION

The nuclear β-particles do not have a linear region in the absorption curve, while the homogeneous electrons' absorption-curves have a long straight portion and a long tail at low intensities going into the background. See Fig 8.12 in Fundamentals of Nuclear Physics by Atam P. Arya for absorption curves of nuclear β-particles and the homogeneous electrons. In Fig 8.12 (a), R_β is the range of the nuclear β-particles. R_β is defined as the point where the absorption curve meets the background.

From Fig 8.12 (b) the range of the homogeneous β-particle is defined as the point where the extension of the straight portion meets the background and is called the *practical range*, R_p , while the point where the curve itself meets the background is called the *maximum range*, R_0.

ABSORPTION AND RANGE-ENERGY RELATION

Once the range has been determined, the next step is to convert it into energy by the use of a proper range-energy relation. Following equations give the empirical relation between range (mg/cm^2) and energy (MeV).

$$R = 412\, E_0^{\,1.265-0.094 \ln E_0} \qquad for \quad E_0 < 2.5\ MeV$$

and

$$R = 530\, E_0 - 106 \qquad for \quad E_0 > 2.5\ MeV \qquad (24)$$

REFERENCES

1. Introductory Nuclear Physics. Kenneth S. Krane

2. Fundamentals of Nuclear Physics. Atam. P. Arya