CEN 3313

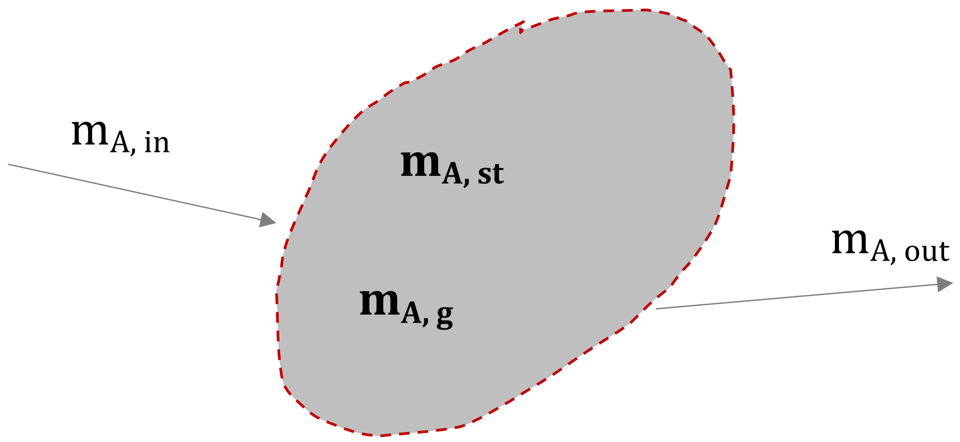
MASS TRANSFER

Assoc. Prof. Ayşe Karakeçili

Assoc. Prof. Berna Topuz

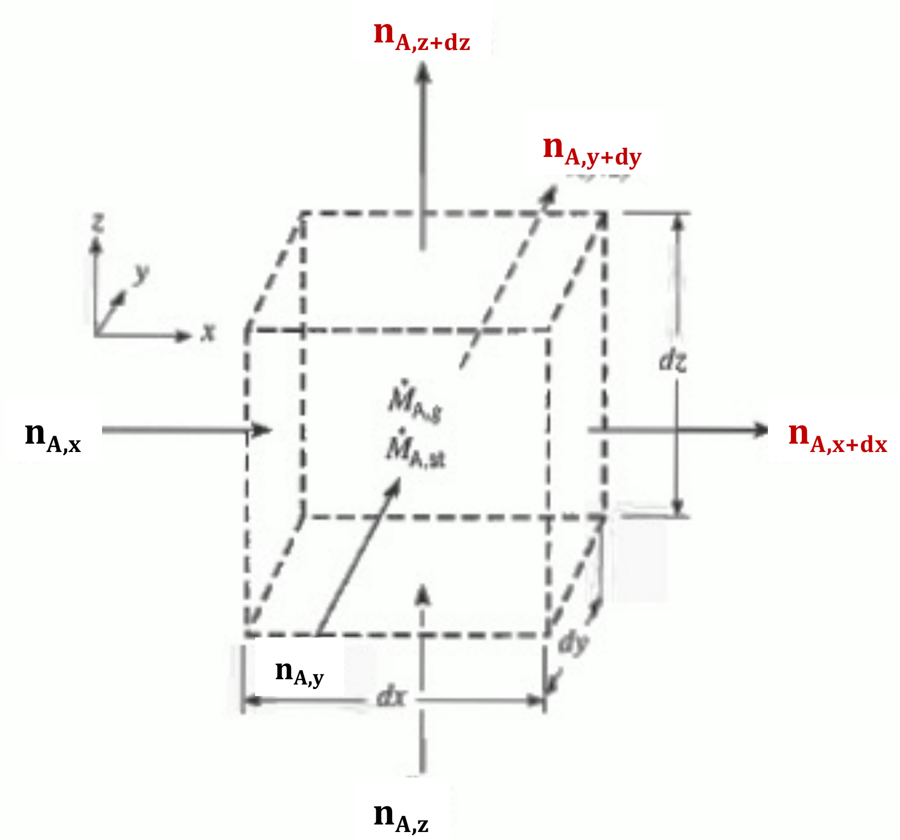
MASS DIFFUSION EQUATION

Conservation of Species in Stationary Medium



{ rate of mass of A in } - { rate of mass of A out } + { rate of generation of A by homogeneous rxn } = { accumulation of species A }

MASS DIFFUSION EQUATION-Conservation of Species in Stationary Medium



$n_A = \rho_A v_A$: mass flux (mass of A that passes a unit area per unit time)

Control volume fixed in space

Differantial control volume = dxdydz (for cartesian coordinates)

Binary mixture of A & B flowing through

No mass transfer by convection

(rate of mass of A **in**) - (rate of mass of A **out**) + (rate of **generation** of A by homogeneous rxn) = (**accumulation** of species A)

$$\begin{pmatrix}\text{rate of mass} \\ \text{of A } \textbf{in}\end{pmatrix} - \begin{pmatrix}\text{rate of mass} \\ \text{of A } \textbf{out}\end{pmatrix} + \begin{pmatrix}\text{rate of } \textbf{generation} \\ \text{of A by} \\ \text{homogeneous rxn}\end{pmatrix} = \begin{pmatrix}\textbf{accumulation} \\ \text{of species A}\end{pmatrix}$$

$$\left(n_{Ax}\Big|_{x} - n_{Ax}\Big|_{x+\Delta x} \right) \Delta y\, \Delta z + \left(n_{Ay}\Big|_{y} - n_{Ay}\Big|_{y+\Delta y} \right) \Delta x\, \Delta z + \left(n_{Az}\Big|_{z} - n_{Az}\Big|_{z+\Delta z} \right) \Delta x\, \Delta y + \mathbf{r_A} \Delta x\, \Delta y\, \Delta z = \frac{\partial \rho_A}{\partial t} \Delta x\, \Delta y\, \Delta z$$

r_A; rate of genaration of species A per unit volume of mixture

$$- \frac{\partial n_{Ax}}{\partial x} - \frac{\partial n_{Ay}}{\partial y} - \frac{\partial n_{Az}}{\partial z} + r_A = \frac{\partial \rho_A}{\partial t}$$

$$-\frac{\partial n_{Ax}}{\partial x} - \frac{\partial n_{Ay}}{\partial y} - \frac{\partial n_{Az}}{\partial z} + r_A = \frac{\partial \rho_A}{\partial t}$$

$$n_A = -D_{AB}\frac{d\rho_A}{dx}$$

$$\frac{\partial}{\partial x}\left(D_{AB}\frac{\partial \rho_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{AB}\frac{\partial \rho_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_{AB}\frac{\partial \rho_A}{\partial z}\right) + r_A = \frac{\partial \rho_A}{\partial t}$$

$$-\frac{\partial n_{Ax}}{\partial x} - \frac{\partial n_{Ay}}{\partial y} - \frac{\partial n_{Az}}{\partial z} + r_A = \frac{\partial \rho_A}{\partial t}$$

In molar concentration:

$$\frac{\partial}{\partial x}\left(D_{AB}\frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{AB}\frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_{AB}\frac{\partial C_A}{\partial z}\right) + r_A = \frac{\partial \rho_A}{\partial t}$$

for constant D_{AB} and ρ :

$$\left(\frac{\partial^2 \rho_A}{\partial x^2}\right) + \left(\frac{\partial^2 \rho_A}{\partial y^2}\right) + \left(\frac{\partial^2 \rho_A}{\partial z^2}\right) + \frac{r_A}{D_{AB}} = \frac{1}{D_{AB}}\frac{\partial \rho_A}{\partial t}$$

$$\left(\frac{\partial^2 C_A}{\partial x^2}\right) + \left(\frac{\partial^2 C_A}{\partial y^2}\right) + \left(\frac{\partial^2 C_A}{\partial z^2}\right) + \frac{R_A}{D_{AB}} = \frac{1}{D_{AB}}\frac{\partial C_A}{\partial t}$$

References

1. Geankoplis, C.J., Transport Processes and Separation Process Principles, Prentice-Hall, Pearson Education, 2003

2. Incropera F. P., Dewitt D. P. , Bergman T.L., Lavine A.S., Fundamentals of Heat and Mass Transfer, John Wiley & Sons Inc.

3. Middleman S., An Introduction to Mass and Heat Transfer: Principles of Analysis and Design, John Wiley, High Education, 1997.

4. Cussler E.L., Diffusion : Mass Transfer in Fluid Systems, Cambridge University Press, 3rd Edition, 2009.

5. Bird R.B., Stewart W.E., Lightfoot E.N., Transport Phenomena, John Wiley & Sons, 1960.