Ankara University
Department of Geological Engineering

GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

Assoc. Prof. Dr. Koray ULAMIŞ

Scalar : Any physical quantity specified with a magnitude, i.e. length, mass, time, volume.

Vector : Different than scalar, vector is the quantity with both "*magnitude*" and "*direction*", i.e. force, position and moment. A vector is shown by arrows. The length of the arrow is magnitude, while any angle between the arrow and specified axis is the direction on the line of action. The head of the arrow indicates the sense of direction.

magnitude

sense

initial point

direction

10 lb

A   30

10 lb

A   30
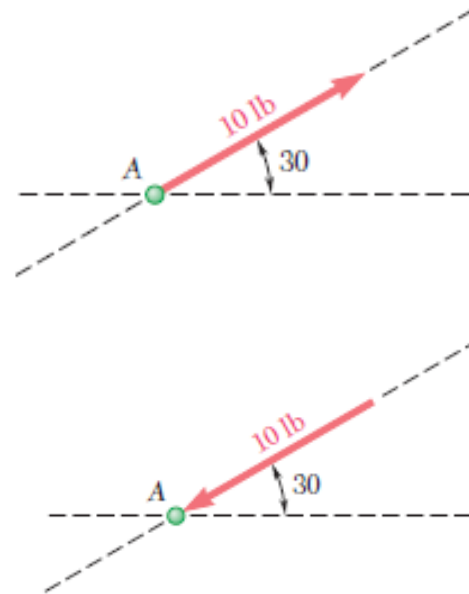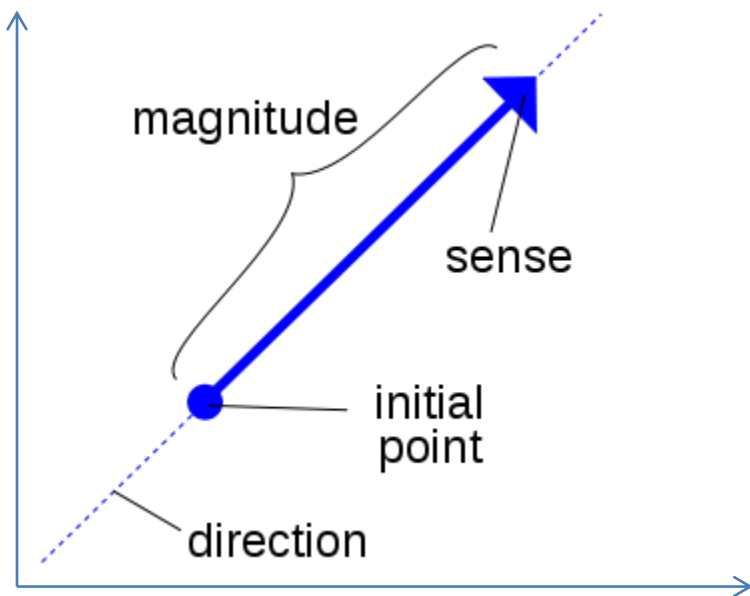
Same force vector with opposite sense

Basic Vector Operations

Multiplication and Division of a Vector by a Scalar : If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. When multiplied by a negative scalar, also the directional sense of the vector changes.
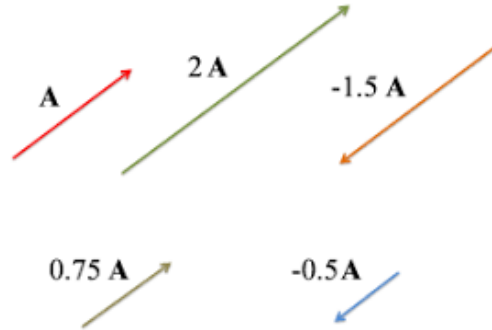
Vector Addition and Subtraction : The resultant vector "*R*" is formed by addition of two separate "*P*" and "*Q*" vectors by "*Parallelogram Law of Addition*". The tails of two vectors are joined to be concurrent. The parallel lines on the opposite of the vectors intersect at "*P*". The line between P and AB intersection point is resultant vector.
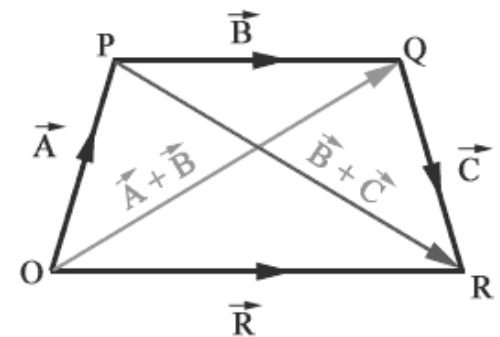
R=P+Q=Q+P (commutative)

$$\overline{OR} = \overline{OP} + \overline{PR}$$
or
$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) \longrightarrow \text{(i)}$$

and
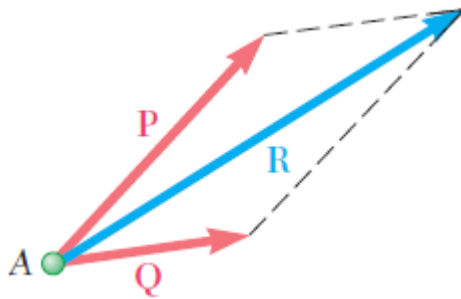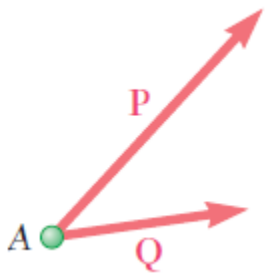
$$\overline{OR} = \overline{OQ} + \overline{QR}$$
or
$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} \longrightarrow \text{(ii)}$$

thus, from (i) and (ii)

$$\boxed{\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}}$$

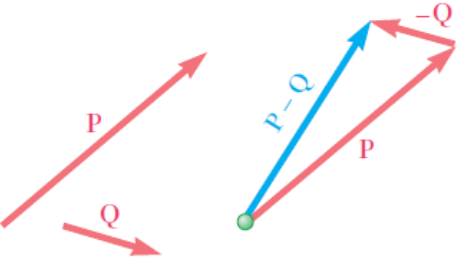*Vector operations (Beer, et al. 2011)*

Vector addition might be considered by "*Triangle rule*" which is a special case of parallelogram. Vector "Q" is added to vector "P" in a "head to tail" fashion.
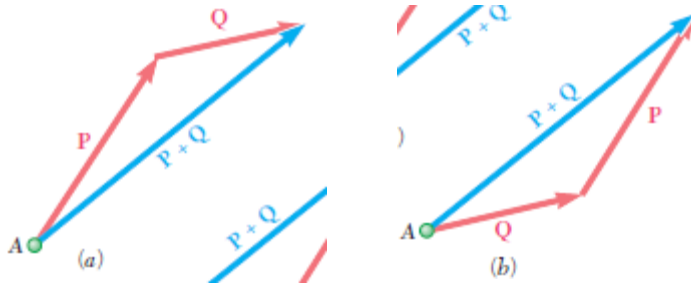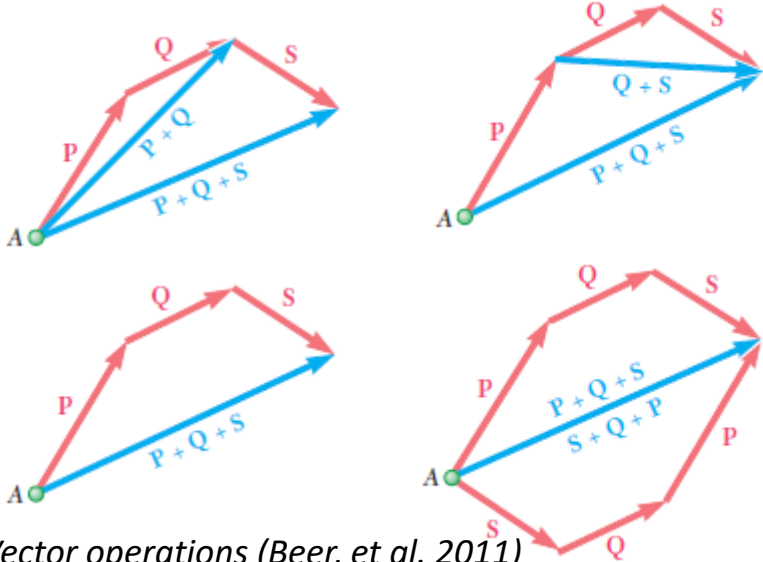

Vector Subtraction : The resultant of the difference between P and Q are;

$$R' = P-Q = P+ (-Q)$$

Vector Addition of Forces :
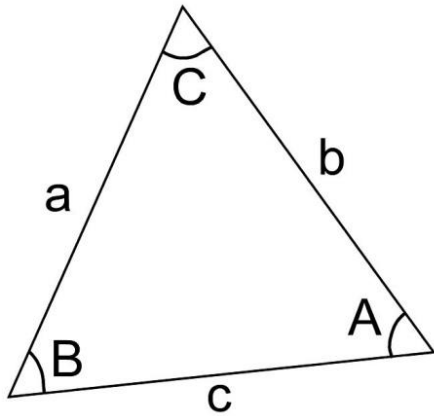

Vector operations (Beer, et al. 2011)

$$P + Q + S = (P + Q) + S = P + (Q + S)$$

Vector addition is "*associative*"

$$P + Q + S = (P + Q) + S = S + (P + Q)$$
$$= S + (Q + P) = S + Q + P$$

Vector addition is "*commutative*"

Sine Law (Rule)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
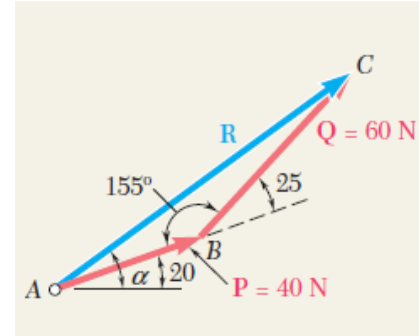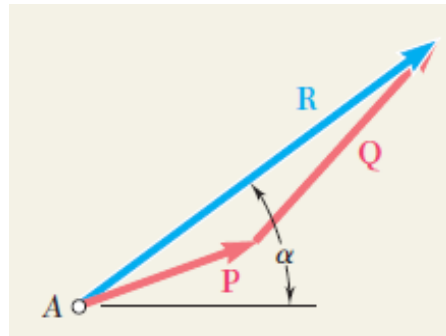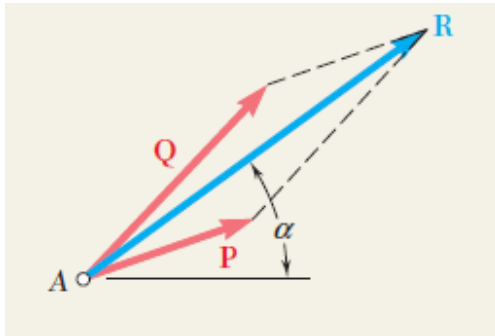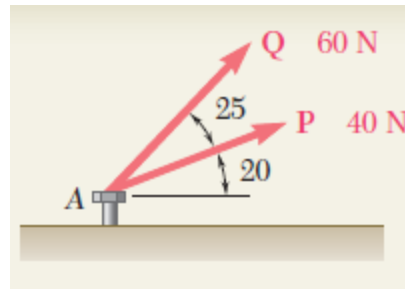
Cosine Law (Rule)

$$a^2 = b^2 + c^2 - 2\,bc\cos A$$

$$b^2 = a^2 + c^2 - 2\,ac\cos B$$

$$c^2 = a^2 + b^2 - 2\,ab\cos C$$

Hint: Cosine rule is used for the resultant magnitude, while sine rule is preferred for direction.

Example 1. The two forces **P** and **Q** act on a bolt A. Determine their resultant.

$$R^2 = P^2 + Q^2 - 2PQ \cos B$$
$$R^2 = (40 \text{ N})^2 + (60 \text{ N})^2 - 2(40 \text{ N})(60 \text{ N}) \cos 155°$$
$$R = 97.73 \text{ N}$$

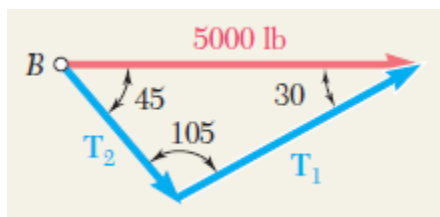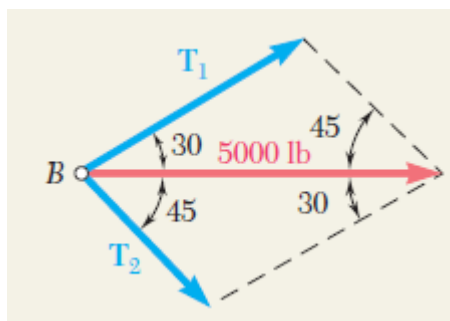$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60 \text{ N}} = \frac{\sin 155°}{97.73 \text{ N}} \qquad \sin A = \frac{(60 \text{ N}) \sin 155°}{97.73 \text{ N}}$$

$$A = 15.04° \qquad \alpha = 20° + A = 35.04°$$

(Beer, et al. 2011)

Example 2. A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine the tension in each of the ropes knowing that $\alpha=45°$
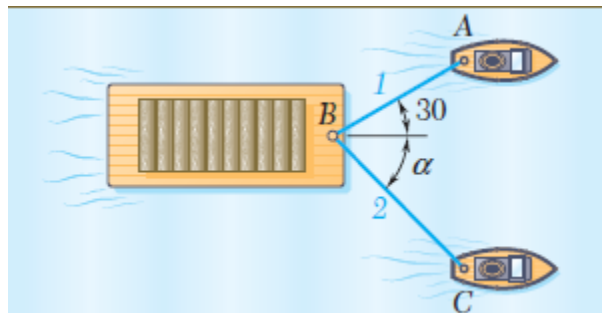
$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000 \text{ lb}}{\sin 105°}$$

$$T_1 = 3660 \text{ lb} \qquad T_2 = 2590 \text{ lb}$$

(Beer, et al. 2011)

Rectangular Components of a Force

Scalar components

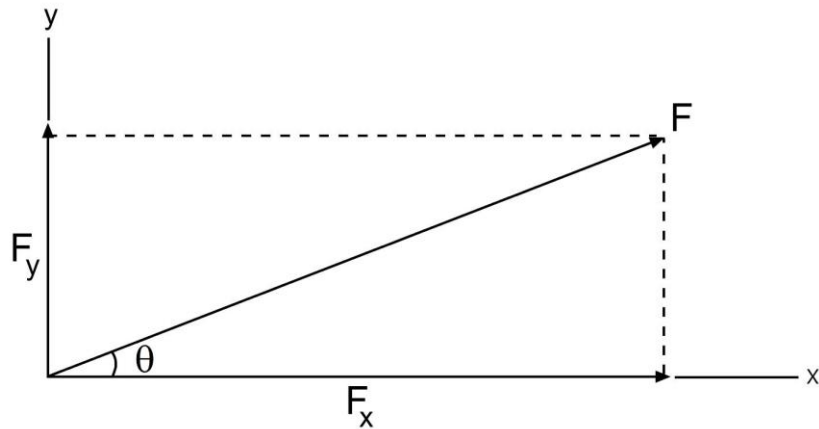$F_x = F\cos\theta$

$F_y = F\sin\theta$

$F = F_x + F_y$

We can determine the components of the resultant force of any number of coplanar forces symbolically by algebraic sum of the *x and y* components of all the forces.

$$F_{Rx} = \sum F_x \qquad F_{Ry} = \sum F_y$$

Once these components are determined, they may be sketched along the *x and y axes with the proper sense of direction and the resultant* force can be determined from vector addition. Magnitude of F_R is then found from the *Pythagorean Theorem*:

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

Also, the angle *which specifies the "direction of the resultant force"* is;

$$\theta = tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$
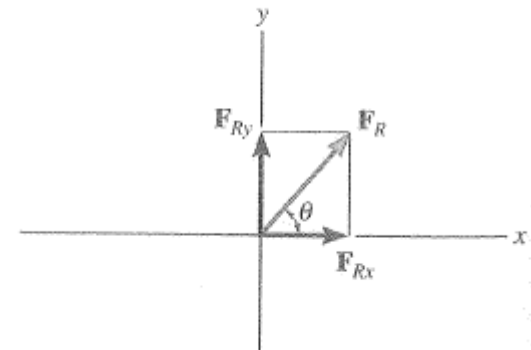
Components of a force (Hibbeler, 2010)

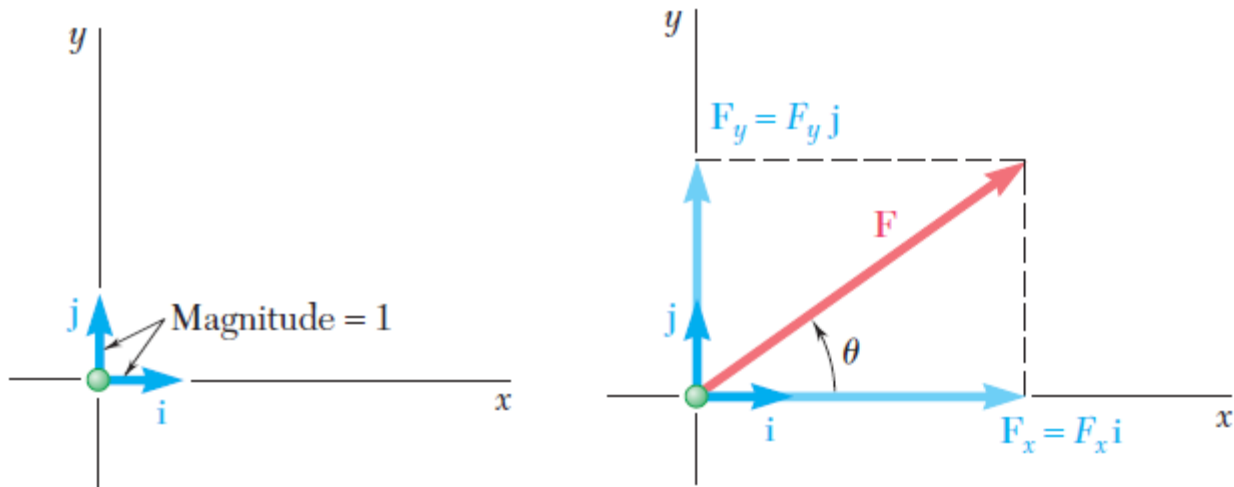**Cartesian Vectors in 2D:** It is also possible to represent the components of a force in terms of Cartesian unit vectors "i" and "j". Each of these unit vector has a dimensionless magnitude of one, and so they may be used to designate the directions of *x, y axes.* Two vectors of unit magnitude, directed respectively along the positive *x* and *y* axes, will be introduced at this point. These vectors are called *unit vectors* and are denoted by **i** and **j** , respectively. Recalling the definition of the product of a scalar and a vector, note that the rectangular components **Fx** and **Fy** of a force **F** may be obtained by multiplying respectively the unit vectors **i** and **j** by appropriate scalars.

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j}$$
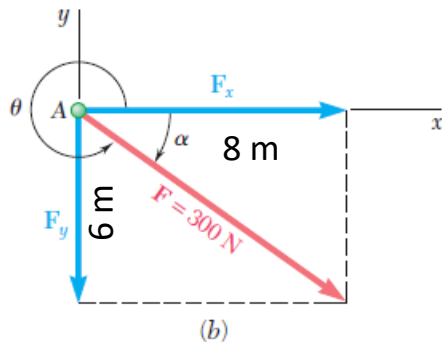
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

Cartesian vectors (Hibbeler, 2010)

Example 3. A force of 800 N is exerted on a bolt "A". Determine the horizontal and vertical components of the force ($\alpha=35°$)
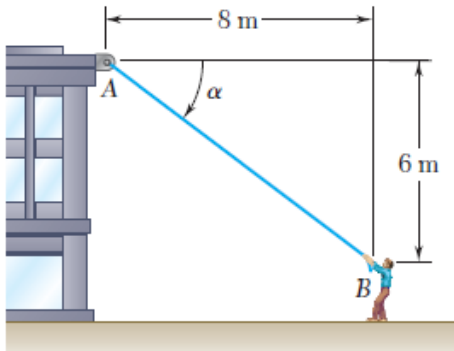
$$F_x = -F \cos \alpha = -(800 \text{ N}) \cos 35° = -655 \text{ N}$$
$$F_y = +F \sin \alpha = +(800 \text{ N}) \sin 35° = +459 \text{ N}$$

Example 4. A man pulls with a force of 300 N on a rope attached to a building. What are the horizontal and vertical components of the force exerted by the rope at point A ?

(b)

$$F_x = +(300 \text{ N}) \cos\alpha \qquad F_y = -(300 \text{ N}) \sin\alpha$$

$$\cos \alpha = \frac{8 \text{ m}}{AB} = \frac{8 \text{ m}}{10 \text{ m}} = \frac{4}{5} \qquad \sin \alpha = \frac{6 \text{ m}}{AB} = \frac{6 \text{ m}}{10 \text{ m}} = \frac{3}{5}$$

Scalar notation

$$F_x = +(300 \text{ N})\tfrac{4}{5} = +240 \text{ N} \qquad F_y = -(300 \text{ N})\tfrac{3}{5} = -180 \text{ N}$$

Vector notation

$$\mathbf{F} = (240 \text{ N})\mathbf{i} - (180 \text{ N})\mathbf{j}$$
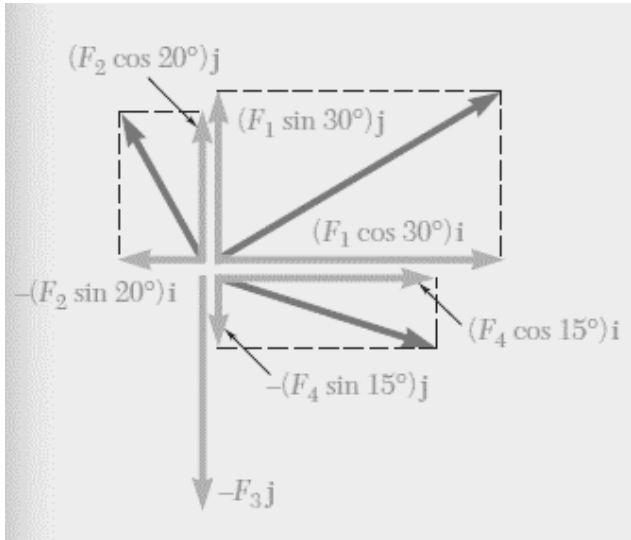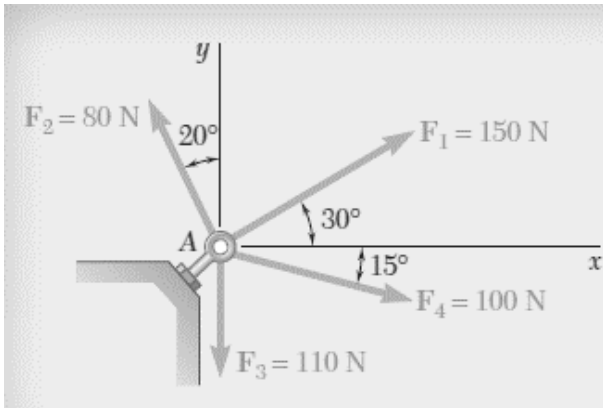
(Beer, et al. 2011)

Coplanar Force Resultants: We can use either of the two methods just described to determine the resultant of several coplanar forces. *To do this,each force is first resolved into its x and y components* and then the respective components are added using *scalar algebra since* they are collinear. The resultant force is then formed by adding the resultant components using the parallelogram law.
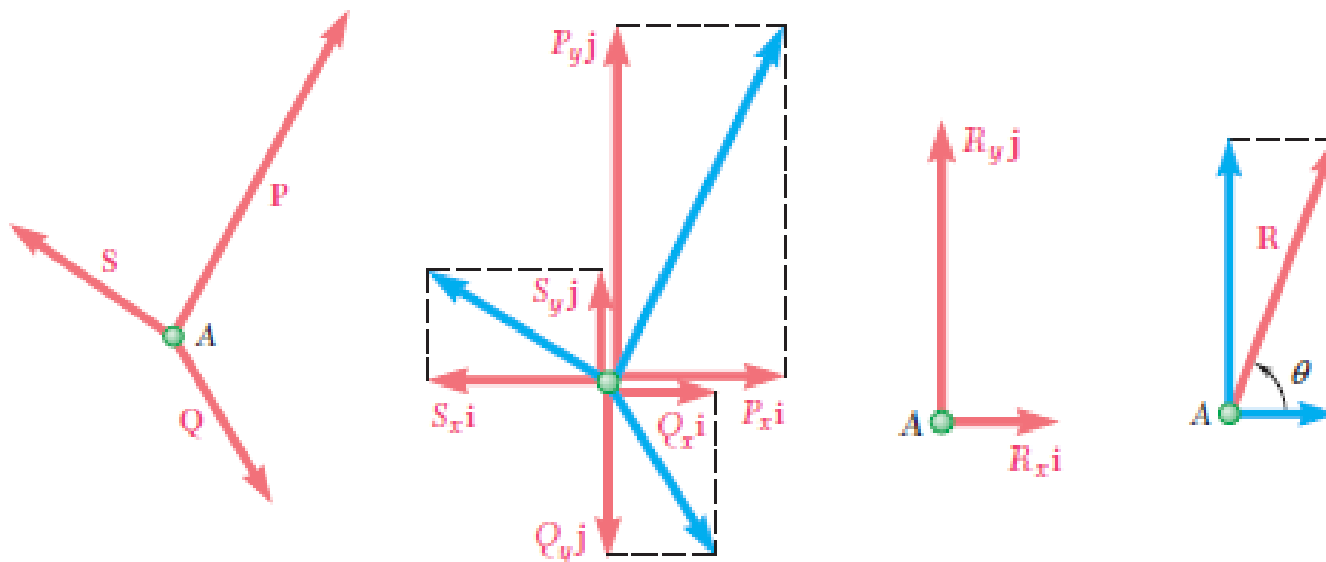
$$R = P + Q + S$$

$$R_x i + R_y j = P_x i + P_y j + Q_x i + Q_y j + S_x i + S_y j$$
$$= (P_x + Q_x + S_x)i + (P_y + Q_y + S_y)j$$

Coplanar force resultants (Beer, et al. 2011)

Example 5. Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.

Force	Magnitude, N	x component	y component
F1	150	+129.9	+75.0
F2	80	-27.4	+75.2
F3	110	0	-110
F4	100	+96.6	-25.9
Sum		+199.1	+14.3

R= (199.6²+14.3²)⁰·⁵=200.11 N (Scalar)

$$R= (199.6^2+14.3^2)^{0.5}=200.11 \text{ N (Scalar)}$$

$$R=\{199.6i+14.3j\}N \text{ (Vector, cartesian)}$$

$$\theta = tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right| = 4.1°$$

(Beer, et al. 2011)