Ankara University
Department of Geological Engineering

GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

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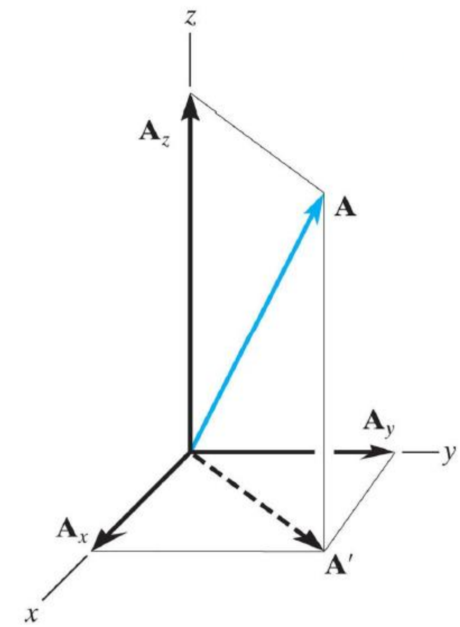
3D Cartesian Vectors

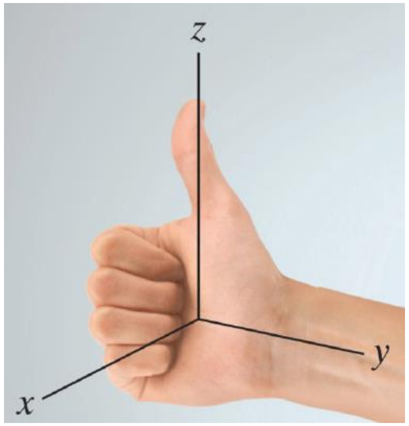
Any vector, say “A” might have components along x, y, z axes. Two successive parallelogram law is applied as “A=A’ + A_z” and “A’ = A_x + A_y”. In order to combine and eliminate A’ the overall sum of components will be:

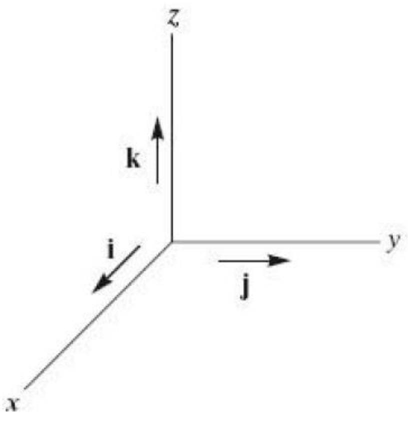
$$A = A_x + A_y + A_z$$



Cartesian Unit Vectors

In 3D, the set of Cartesian unit vectors; “i-j-k” is used to designate the directions of the x,y and z axes*. The sense (or arrowhead) of these* vectors will be represented analytically by a plus or minus sign depending on whether they are directed along positive or negative x, y or z axes.





3D vectors (Hibbeler, 2010)

Cartesian Vector Representation

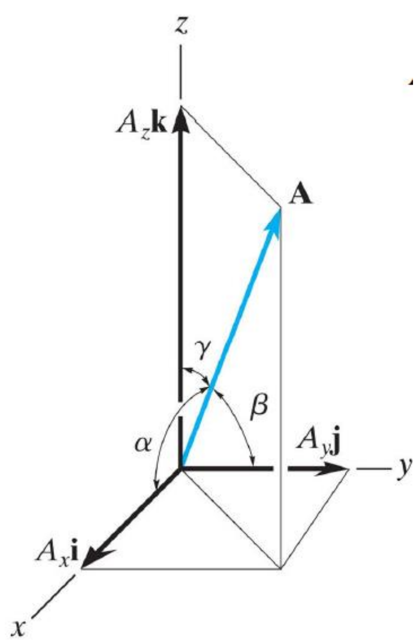
The vector A can be defined as ; **A = (A_x i + A_y j + A_z k)**.

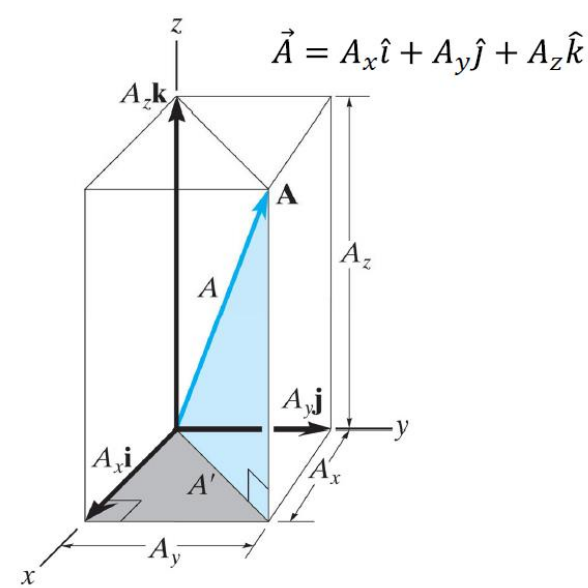
The magnitude of $A' = \sqrt{A_x^2 + A_y^2}$ and $A = \sqrt{A + A_z^2}$.

Combination of these equations yield;

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction of a cartesian vector is defined by the coordinate direction angles, measured between the tail of A and positive axes.



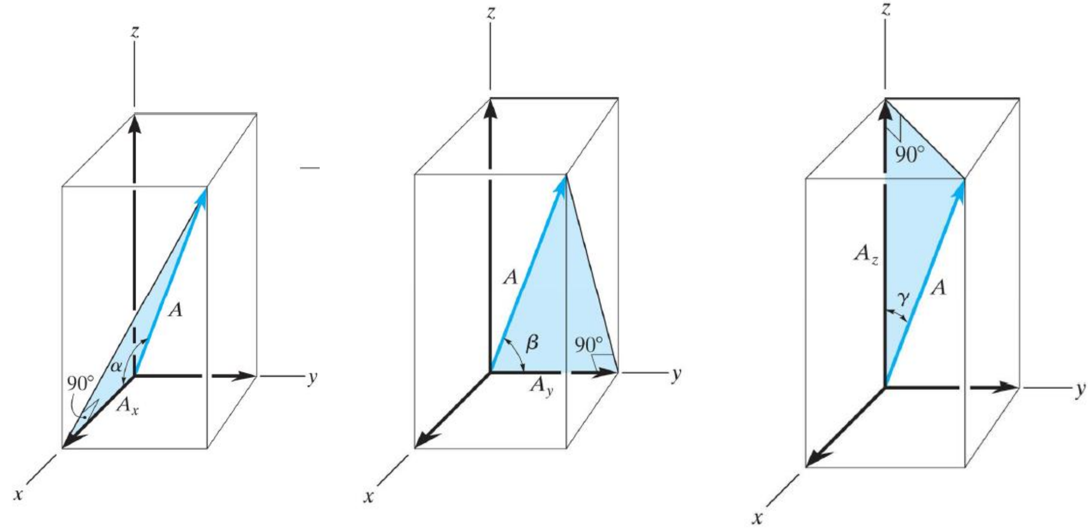


3D vectors (Hibbeler, 2010)

“Direction cosines”

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos\alpha = \frac{A_x}{A}; \cos\beta = \frac{A_y}{A}; \cos\gamma = \frac{A_z}{A}$$



“**Unit Vector** along axes”

$$u_A = \frac{A_x}{A} i + \frac{A_y}{A} j + \frac{A_z}{A} k \qquad u_A = [(\cos\alpha)i + (\cos\beta)j + (\cos\gamma)k]$$

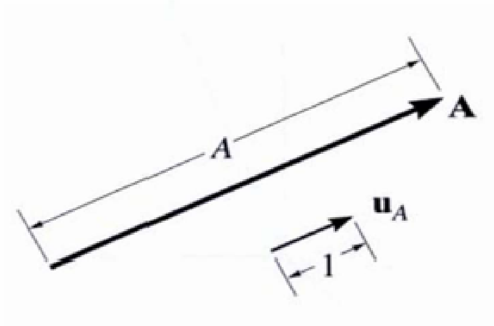
Characteristics of a unit vector:

a) Its magnitude is 1.

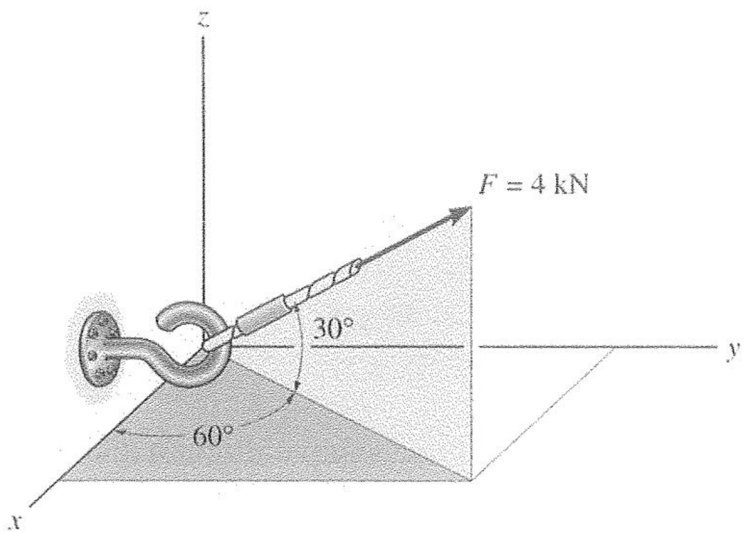
b) It is dimensionless.

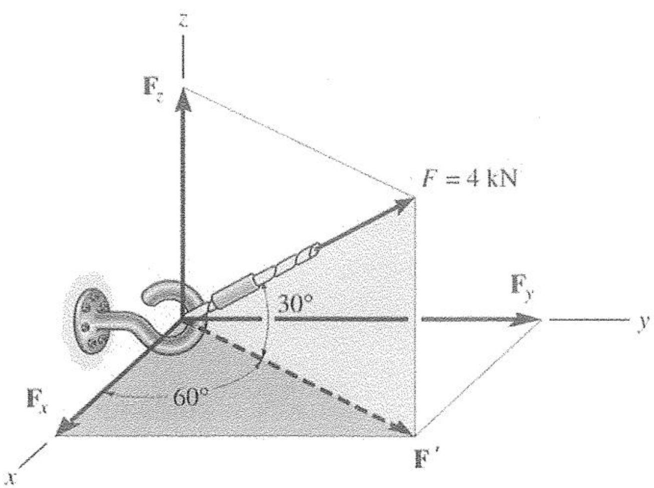
c) It points in the same direction as the original vector $\vec{A}$.

d) Think of $\hat{u}_A$ as direction of vector $\vec{A}$.



Example 6. Please express the force “F” as a cartesian vector.





$$F' = 4cos30 = 3.46\ kN$$

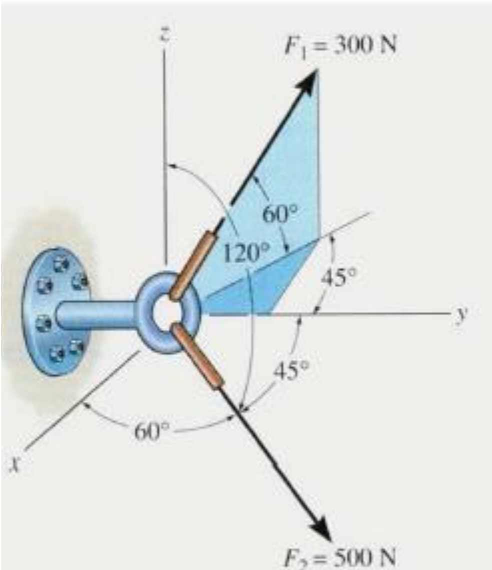
$$F_z = 4sin30 = 2.0\ kN$$

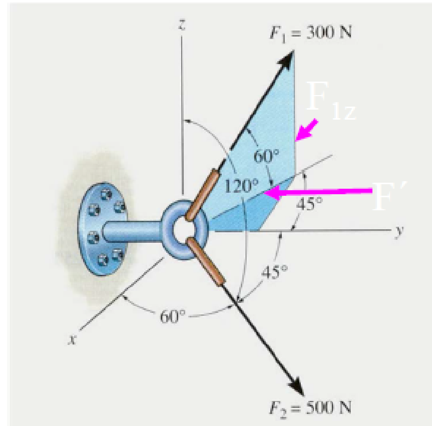
$$F_x = 3.46cos60 = 1.73\ kN$$

$$F_y = 3.46sin60 = 3.0\ kN$$

(Hibbeler, 2010)

<u>Example 7.</u> The screw is subjected to two forces, please find the magnitude and coordinate direction angles of the resultant. Plan: Express the forces with Cartesian vectors, add and find the resultant, determine the direction angles.





First resolve the force $\boldsymbol{F_1}$.

$$F_{1z} = 300 \sin 60° = 259.8 \text{ N}$$

$$F' = 300 \cos 60° = 150.0 \text{ N}$$

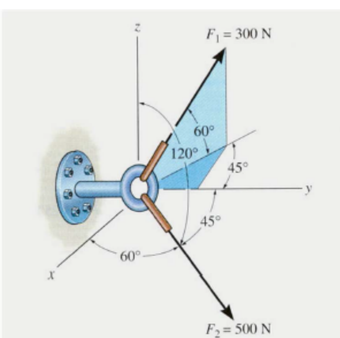
F′ can be further resolved as,

$$F_{1x} = -150 \sin 45° = -106.1 \text{ N}$$

$$F_{1y} = 150 \cos 45° = 106.1 \text{ N}$$

Now we can write :

$$\boldsymbol{F_1} = \{-106.1\, \boldsymbol{i} + 106.1\, \boldsymbol{j} + 259.8\, \boldsymbol{k}\} \text{ N}$$



The force $\boldsymbol{F_2}$ can be represented in the Cartesian vector form as:

$$\boldsymbol{F_2} = 500\{\cos 60°\, \boldsymbol{i} + \cos 45°\, \boldsymbol{j} + \cos 120°\, \boldsymbol{k}\} \text{ N}$$

$$= \{250\, \boldsymbol{i} + 353.6\, \boldsymbol{j} - 250\, \boldsymbol{k}\} \text{ N}$$

$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$$

$$= \{143.9\, \boldsymbol{i} + 459.6\, \boldsymbol{j} + 9.81\, \boldsymbol{k}\} \text{ N}$$

$$F_R = (143.9^2 + 459.6^2 + 9.81^2)^{½} = 481.7 = 482 \text{ N}$$

$$\alpha = \cos^{-1}(F_{Rx} / F_R) = \cos^{-1}(143.9/481.7) = 72.6°$$

$$\beta = \cos^{-1}(F_{Ry} / F_R) = \cos^{-1}(459.6/481.7) = 17.4°$$

$$\gamma = \cos^{-1}(F_{Rz} / F_R) = \cos^{-1}(9.81/481.7) = 88.8°$$

(Hibbeler, 2010)