Ankara University
Department of Geological Engineering

GEO222 STATICS and STRENGTH of MATERIALS

Lecture Notes

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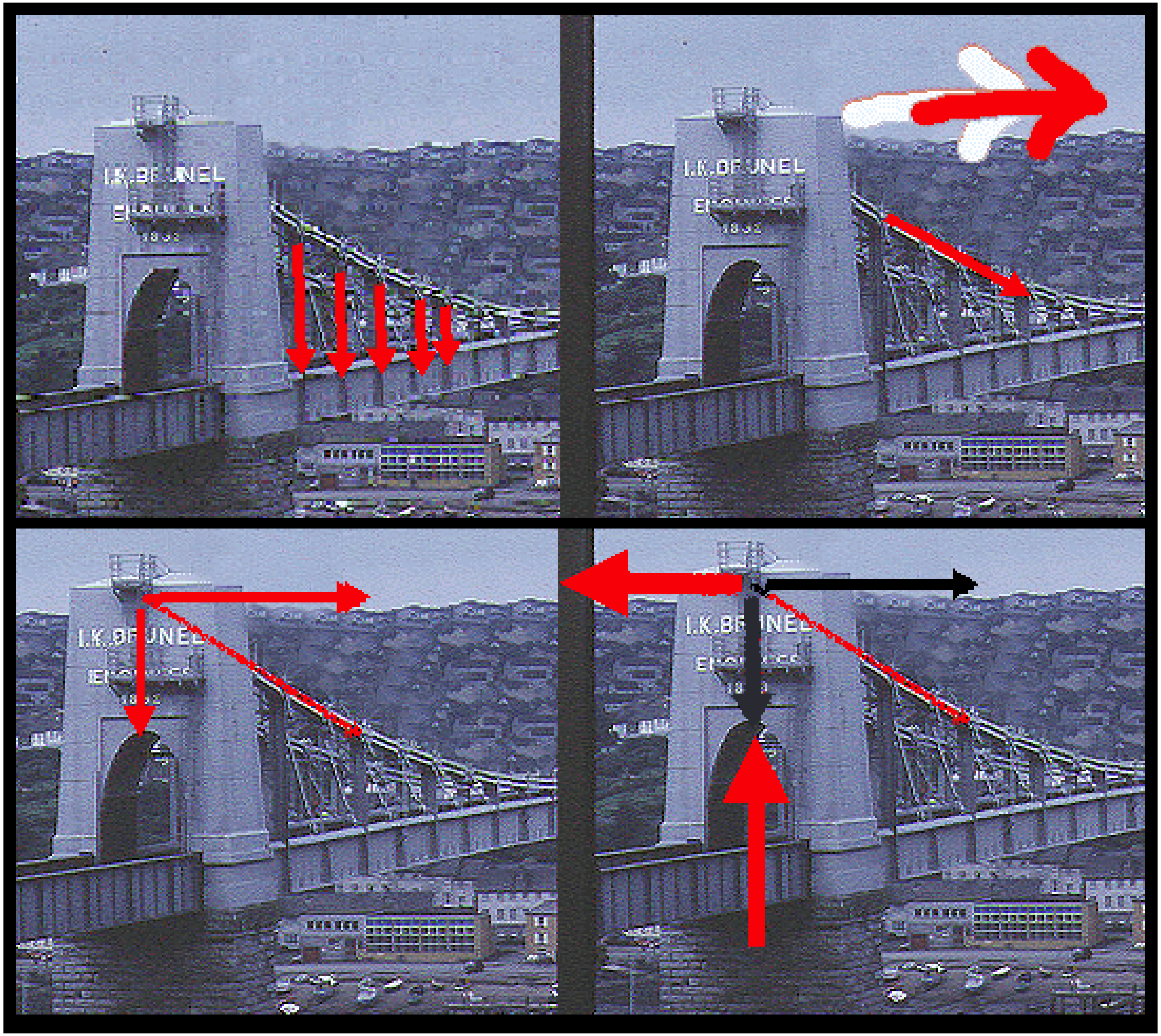
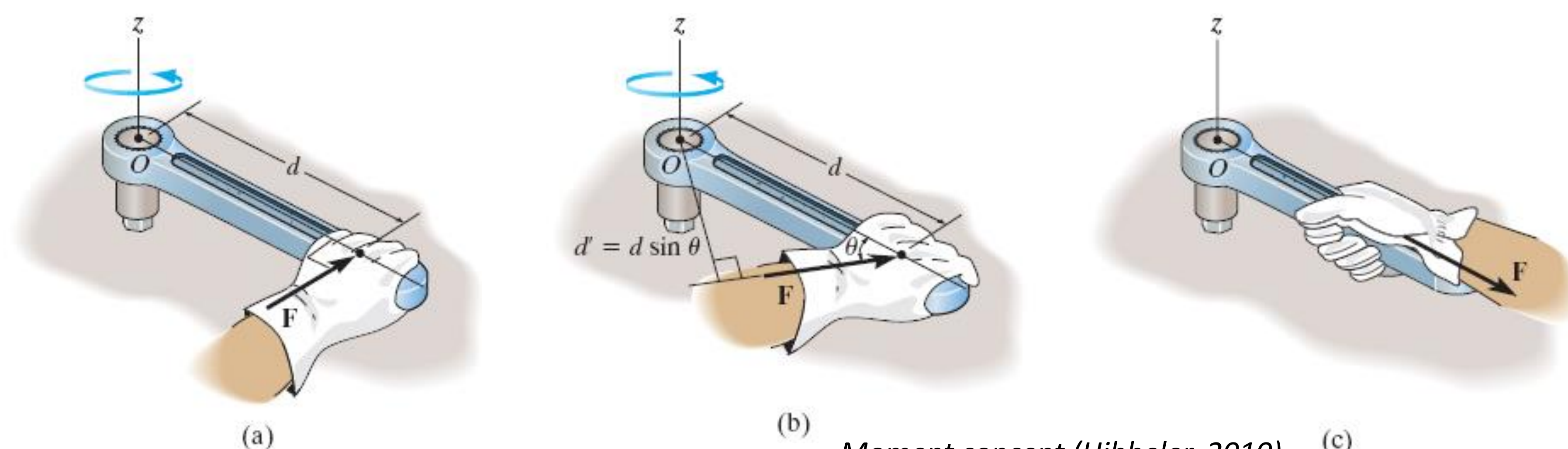
CHAPTER 3. FORCE SYSTEM RESULTANTS



MOMENT OF A FORCE (SCALAR FORMULATION)

When a force is applied on a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency is sometimes called ***"Torque"*** but most often it is the moment of a force or simply the "***Moment***". For example; consider a wrench used to unscrew the bolt in the figure below. If a force is applied to the handle of the wrench it will tend to turn the bolt about point *0*. The magnitude of the moment is directly proportional to the magnitude of "F" and the *perpendicular distance* or "***Moment Arm***"(a). The larger the force or the longer the moment arm, the greater the moment or turning effect. Note that if the force F is applied at an angle $\theta \neq 90$, then it will be more difficult to turn the bolt since the moment arm *($d'=d\sin\theta$)* will be smaller than "d" (b). If "F" is applied along the wrench moment arm will be zero since the line of action of "F" will intersect point *0*. As a result. the moment of "F" about *0* is also zero and no turning may occur (c).



Moment concept (Hibbeler, 2010)

Magnitude

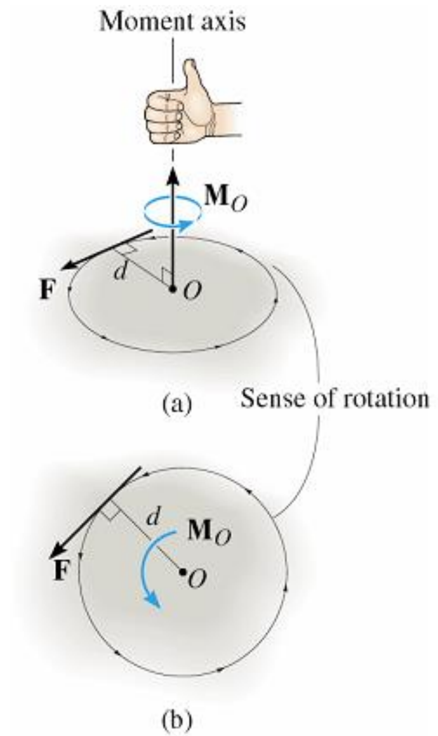
The magnitude of Mo is;

$$M_o = F x d$$

Where "d" is the "Moment arm" or "Perpendicular distance" from the axis at point "O" to the line of action of the force.

Direction

The direction of "Mo" is defined by its "Moment axis" which is perpendicular to the plane that contains the force "F" and its moment arm "d". The right-hand rule is used to establish the sense of M_o. Natural curl of fingers of the right hand as they arc drawn towards the palm represent the tendency for rotation caused by the moment. *As this action is performed the thumb of the right hand will give the directional sense of "M_o".* Moment vector is represented three dimensional by a curl around an arrow. In two dimensions, this vector is represented only by the curl. Since in this case the moment will tend to cause a counterclockwise rotation, the moment vector is actually directed out of the page



Moment concept (Hibbeler, 2010)

Resultant Moment

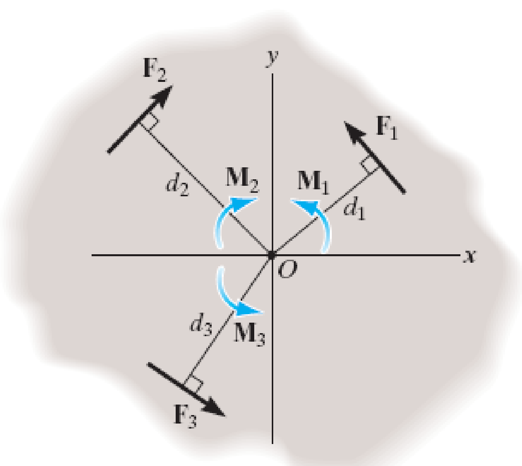
For 2D problems, where all the forces lie within the X-Y plane, the resultant moment about point "0" can be determined by finding the algebraic sum of the moments caused by all the forces in the system.

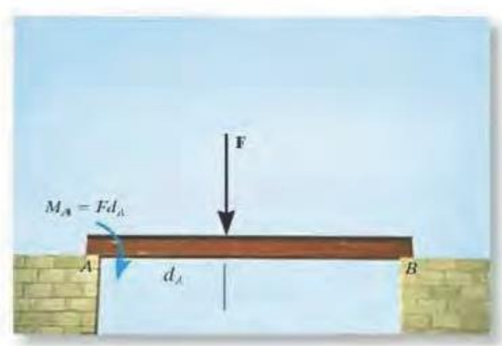
As a convention, we will generally consider positive moments as counterclockwise since they are directed along "+z"(out of page). Clockwise moments will be negative. *Doing this* the directional sense of each moment can be represented by a "+" or "-" sign. Using this sign convention, resultant moment is therefore; (CCW +)

$$(M_R)_0 = F_1 d_1 - F_2 d_2 + F_3 d_3$$

If the numerical result of the sum is a positive scalar. $(M_R)_O$ will be a counterclockwise moment (out of the page) and if the result is negative.

$(M_R)_O$ will be a clockwise moment (into the page).



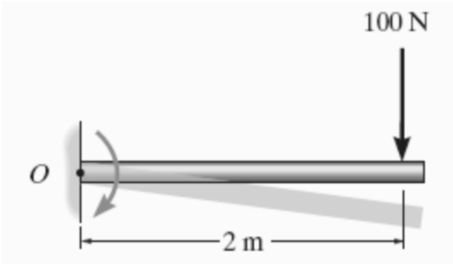


As illustrated by the example problems, the moment of a force does not always cause a rotation. For example, the force **F** tends to rotate the beam clockwise about its support at A with a moment $M_A = Fd_A$. The actual rotation would occur if the support at B were removed.

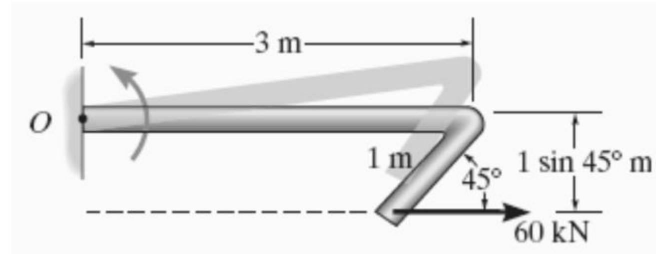
The ability to remove the nail will require the moment of $\mathbf{F}_H$ about point O to be larger than the moment of the force $\mathbf{F}_N$ about O that is needed to pull the nail out.

Moment concept (Hibbeler, 2010)

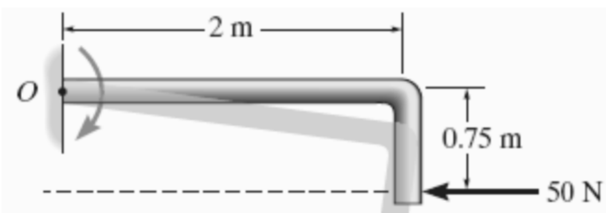
Example 8. Determine the moments about "O"



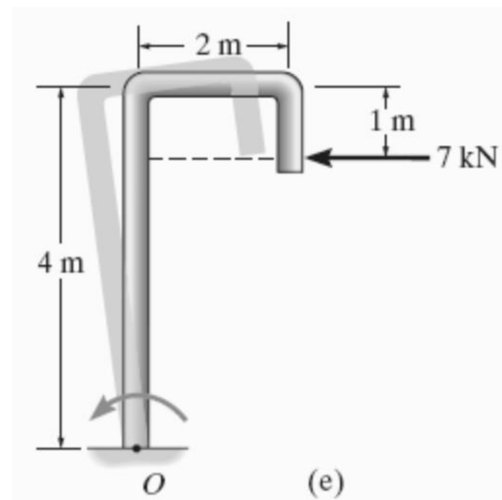
$$(a)M_o = (100N)(2m) = 200N.m(CW)$$



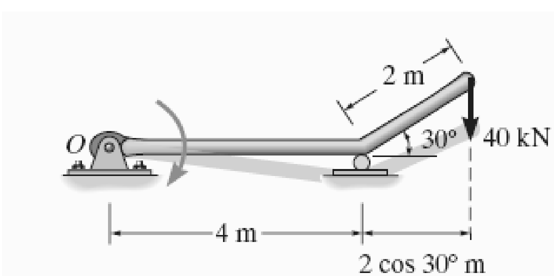
$$(d)M_o = (60N)(1\sin 45^\circ m) = 42.4N.m(CCW)$$



$$(b)M_o = (50N)(0.75m) = 37.5N.m(CW)$$



$$(e)M_o = (7kN)(4m - 1m) = 21.0kN.m(CCW)$$



$$(c)M_o = (40N)(4m + 2\cos 30^\circ m) = 229N.m(CW)$$

CROSS-PRODUCT

The cross product of two vectors "A" and B yields vector "C", which is

$$C = A \times B$$

Magnitude

The *magnitude* of C is defined as the product of the magnitudes of A and B a between their tails. Thus,

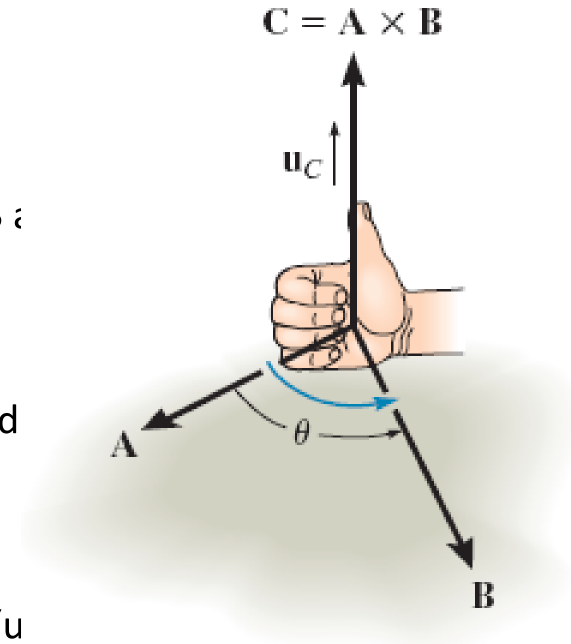
$$C = A \times B \sin\theta$$

Direction

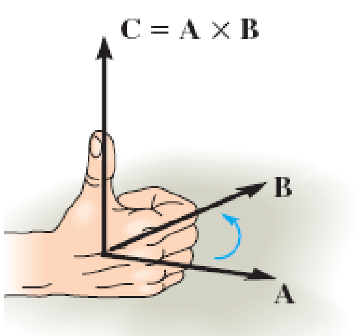
Vector C has a direction, that is perpendicular to the plane containing A and the right-hand rule. Knowing both the magnitude and direction of "C";

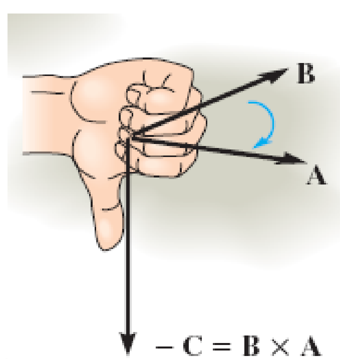
$$C = A \times B = (A \times B \sin\theta) \times u_c$$

where the scalar "*AB* sinθ" defines the Mgnitude of C and the unit vector "u



Laws of Operation





$A \times B \neq B \times A$; instead : $A \times B = -B \times A$ (**<u>Not commutative</u>**)

If the cross product is multiplied by a scalar, result is ***<u>associative</u>***;

$$a(A \times B) = (aA) \times B = A \times (aB) = (A \times B)a$$

The vector cross product is also ***<u>distributive</u>***;

$$A \times (B+D) = (A \times B) + (A \times D)$$

Cross product (Hibbeler, 2010)

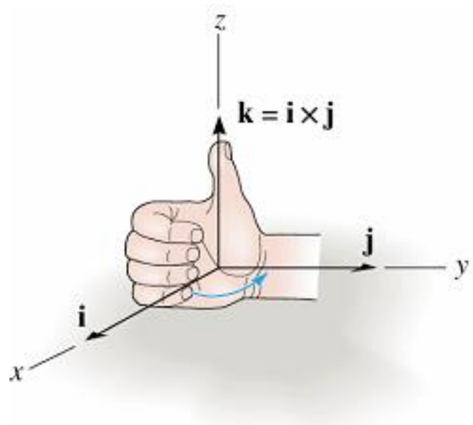
Cartesian Vector Formulation

C = A x B = *(AB* sinθ*)* x u_c

$$i \times j = k \quad i \times k = -j \quad i \times i = 0$$

$$j \times k = i \quad j \times i = -k \quad j \times j = 0$$

$$k \times i = j \quad k \times j = -i \quad k \times k = 0$$



$$\bar{A} \times \bar{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \quad i(A_y B_z - A_z B_y)$$

$$- j(A_x B_z - A_z B_x)$$

$$+ k(A_x B_y - A_y B_x)$$

<u>Hint : A determinant having three rows and three columns can be expanded using three minors, each of which is multiplied by one of the three terms in the first row. There are four elements in each minors.</u>

Element "i"

$$\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = i(A_y B_z - A_z B_y)$$

Element "j"

$$\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -j(A_x B_z - A_z B_x)$$

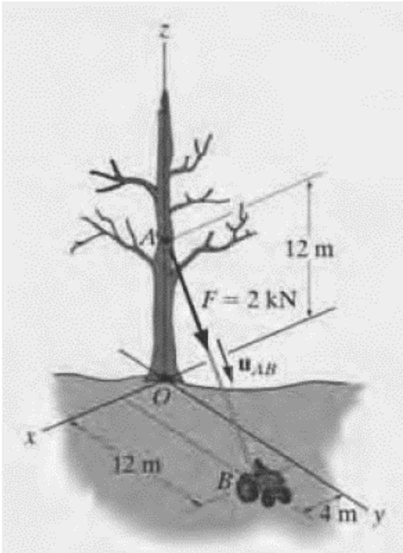
★ Negative sign

Element "k"

$$\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = k(A_x B_y - A_y B_x)$$

Cartesian formulation (Beer, et al. 2011; Hibbeler, 2010)

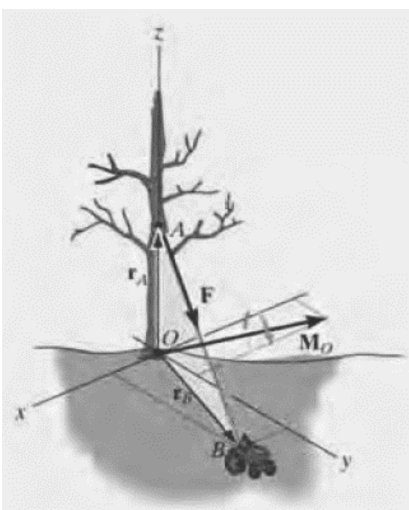
Example 9. Determine the moment produced by "F" about point "O" as Cartesian vector.



$$\mathbf{r}_A = \{12\mathbf{k}\}\text{ m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\text{ m}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\text{ kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\text{ m}}{\sqrt{(4\text{ m})^2 + (12\text{ m})^2 + (-12\text{ m})^2}}\right]$$
$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\text{ kN}$$

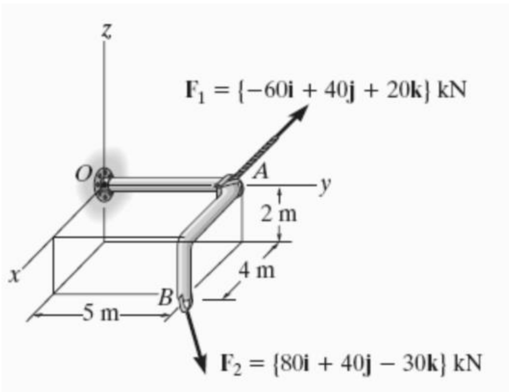
$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$
$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$
$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\text{ kN·m} \qquad \textit{Ans.}$$

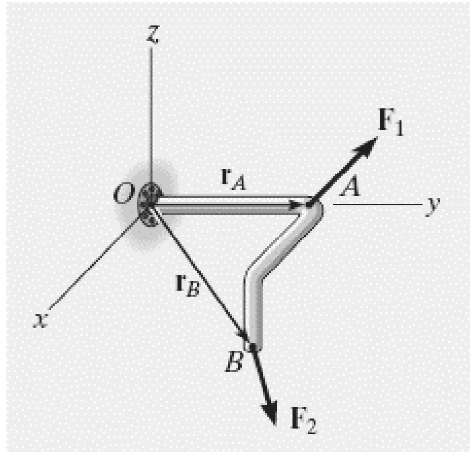


$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 12 & 0 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$
$$= [12(-1.376) - 0(1.376)]\mathbf{i} - [4(-1.376) - 0(0.4588)]\mathbf{j} + [4(1.376) - 12(0.4588)]\mathbf{k}$$
$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\text{ kN·m} \qquad \textit{Ans.}$$

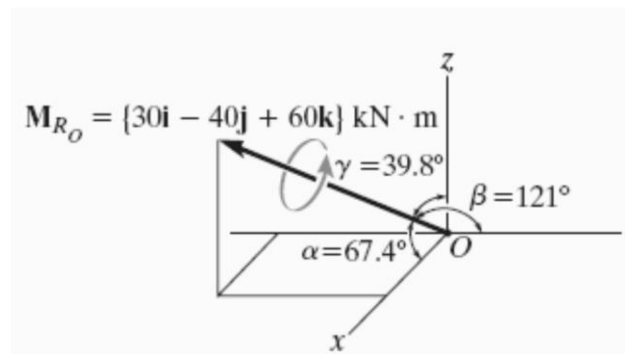
(Beer, et al. 2011)

Ex.10. Determine the resultant moment produced about flange "O" as Cartesian vector.





$$r_A = \{5j\}\,\text{m}$$
$$r_B = \{4i + 5j - 2k\}\,\text{m}$$



$$\vec{M}_O = \sum (r \times F) = r_A \times F + r_B \times F$$

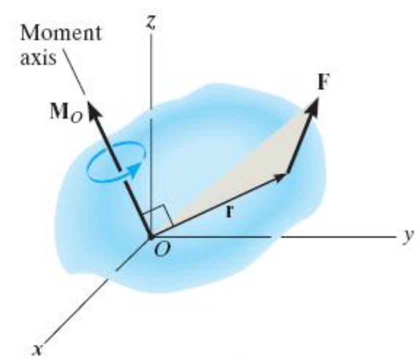
$$= \begin{vmatrix} i & j & k \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

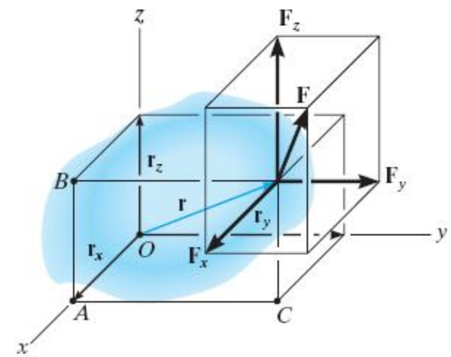
$$= \{30i - 40j + 60k\}\,\text{kN} \cdot \text{m}$$

(Beer, et al. 2011)

Cartesian Vector Formulation

- For force expressed in Cartesian form,
- With the determinant expended,



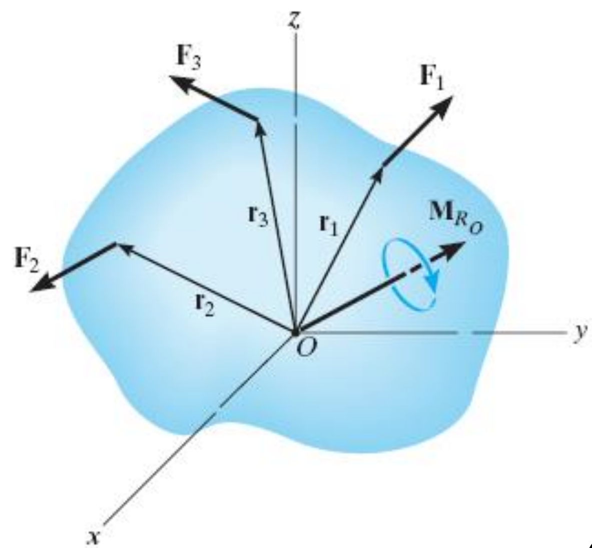


$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - {}_{y}F_x)\mathbf{k}$$

Resultant Moment of a System of Forces

Resultant moment of forces about point O can be determined by vector addition

$$\mathbf{M}_{Ro} = \sum(\mathbf{r} \times \mathbf{F})$$



Cartesian formulation (Hibbeler, 2010)